Mathematical details of time-lapse AVO framework

Shahin Jabbari*, Kris Innanen
sjabbari@ucalgary.ca

Introduction

The study described here focuses on applying the perturbation theory in time-lapse amplitude variation with offset (Time-lapse AVO) method to model a framework to describe the difference data from a baseline survey to monitor survey in a reservoir. Reflection coefficients are derived for the baseline and monitor survey using Zoeppritz equations to calculate the reflection coefficient for difference data.

Procedure for deriving A-R from Zoeppritz equations

Consider an incident P wave striking on the boundary of a planar interface between two elastic media with rock properties VP0, VS0, 0 an

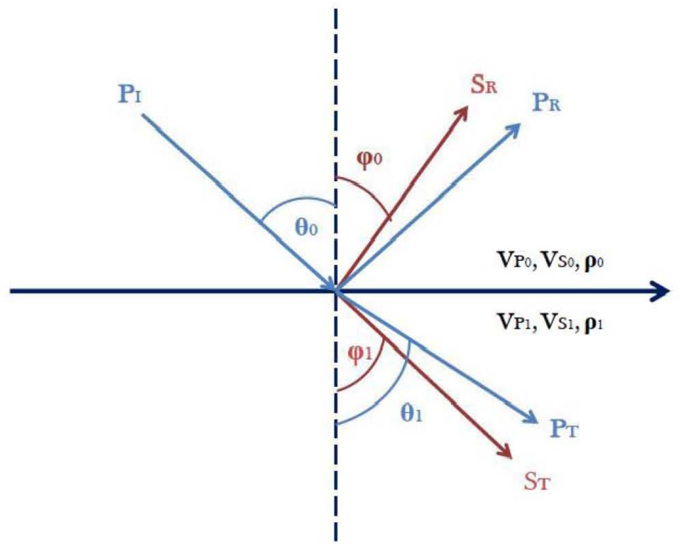

Figure 1: Displacement amplitude of an incident P-wave with related reflected and transmitted P and S waves.

Setting boundary conditions in the problem leads to Zoeppritz equations:

$$P\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = b_P \qquad R_{PP}(\theta) = \frac{\det(P_P)}{\det(P)} \qquad b_P \equiv \begin{bmatrix} X \\ \sqrt{1-X^2} \\ 2B^2X\sqrt{1-X^2} \\ 1-2(BX)^2 \end{bmatrix}$$

where

$$X = Sin(\theta_0),\quad A \equiv \frac{\rho_1}{\rho_0},\quad B \equiv \frac{V_{S_0}}{V_{P_0}},\quad C \equiv \frac{V_{P_1}}{V_{P_0}},\quad D \equiv \frac{V_{S_1}}{V_{P_0}},\quad E \equiv \frac{V_{P_1}}{V_{S_0}},\quad F \equiv \frac{V_{S_1}}{V_{S_0}}.$$

Then

$$P \equiv \begin{bmatrix} -X & -\sqrt{1-(BX)^2} & CX & \sqrt{1-(DX)^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-(CX)^2} & -DX \\ 2B^2X\sqrt{1-X^2} & B\left(1-2(BX)^2\right) & 2AD^2X\sqrt{1-(CX)^2} & AD\left(1-2(DX)^2\right) \\ -\left(1-2(BX)^2\right) & 2B^2X\sqrt{1-(BX)^2} & AC\left(1-2(DX)^2\right) & -2AD^2X\sqrt{1-(DX)^2} \end{bmatrix}$$

We next seek a way to expand these solutions about the contrasts across the interface. Now lets introduce perturbation parameters the wave experience traveling from medium one to two (Figure 1):

$$a_{VP} = 1-\frac{V_{P_0}^2}{V_{P_1}^2},\quad a_{VS} = 1-\frac{V_{S_0}^2}{V_{S_1}^2},\quad a_\rho = 1-\frac{\rho_0}{\rho_1},$$

The elastic parameters are expressed in terms of perturbations parameters:

$$A = \left(1-a_\rho\right)^{-1},\quad C = \left(1-a_{VP}\right)^{-\frac{1}{2}},\quad D = B\times\left(1-a_{VS}\right)^{-\frac{1}{2}}$$

$$(1-a_\rho)^{-1} = 1+a_\rho+a_\rho^2+\ldots \quad (1-a_{VP})^{-\frac{1}{2}} = 1+\frac{1}{2}a_{VP}+\frac{3}{8}a_{VP}^2+\ldots \quad (1-a_{VS})^{-\frac{1}{2}} = 1+\frac{1}{2}a_{VS}+\frac{3}{8}a_{VS}^2+\ldots$$

Substituting these parameters in Zoeppritz equations and expanding them about X and perturbation parameters, reflection coefficient are calculated as:

$$R_{PP}(\theta_0) = R^{(1)}PP(\theta_0)+R^{(2)}PP(\theta_0)+\ldots$$

$$R_{PP}^{(1)}(\theta_0) = \frac{1}{4}(1+X^2)a_{VP}-2(BX)^2a_{VS}+(\frac{1}{2}-2(BX)^2)a_\rho$$

$$R_{PP}^{(2)}(\theta_0) = \frac{1}{4}(\frac{1}{2}+X^2)a_{VP}^2+(B^3X^2-2(BX)^2)a_{VS}^2 + \left(\frac{1}{4}-\frac{1}{4}BX^2-(BX)^2+B^3X^2\right)a_\rho^2+(2B^3X^2-(BX)^2)a_\rho a_{VS}$$

First order is equivalent to the Aki-Richards approximation.

A framework for time-lapse AVO

We will consider two seismic experiments involved in a time-lapse survey, the baseline survey, followed by a monitoring survey.

In a time-lapse study, there are two groups of perturbation parameters; from the firs medium to the second medium in the baseline survey and from baseline to monitor survey:

$$a_{VP} = 1-\frac{V_{P_0}^2}{V_{P_{BL}}^2},\quad a_{VS} = 1-\frac{V_{S_0}^2}{V_{S_{BL}}^2},\quad a_\rho = 1-\frac{\rho_0}{\rho_{BL}},$$

$$b_{VP} = 1-\frac{V_{P_{BL}}^2}{V_{P_M}^2},\quad b_{VS} = 1-\frac{V_{S_{BL}}^2}{V_{S_M}^2},\quad b_\rho = 1-\frac{\rho_{BL}}{\rho_M},$$

The elastic parameters are:

$$A \equiv \left(1-b_\rho\right)^{-1}\times\left(1-a_\rho\right)^{-1}, C \equiv \left(1-a_{VP}\right)^{-\frac{1}{2}}\times\left(1-b_{VP}\right)^{-\frac{1}{2}},\quad D \equiv B\times\left(1-a_{VS}\right)^{-\frac{1}{2}}\times\left(1-b_{VS}\right)^{-\frac{1}{2}}$$

The same AVO method is used to calculate the reflection coefficient for the baseline and monitor survey. Difference data is then calculated as:

$$\Delta R_{PP}(\theta) = R_{PP}^{M}(\theta)-R_{PP}^{BL}(\theta)$$

$$\Delta R_{PP}^{(1)}(\theta_0) = \frac{1}{4}(1+X^2)\, b_{VP}-2(BX)^2\, b_{VS}+(\frac{1}{2}-2(BX)^2)\, b_\rho$$

$$\Delta R_{PP}^{(2)}(\theta_0) = \frac{1}{4}(\frac{1}{2}+X^2)\, b_{VP}^2+(B^3X^2-2(BX)^2)\, b_{VS}^2+\left(\frac{1}{4}-\frac{1}{4}BX^2-(BX)^2+B^3X^2\right)b_\rho^2$$
$$+(2B^3X^2-(BX)^2)\, b_\rho b_{VS}+(2B^3X^2-2(BX)^2)\, a_{VS}b_{VS}+(\frac{1}{4}X^2)\, a_{VP}b_{VP}$$
$$+(2B^3X^2-(BX)^2)\, a_\rho b_{VS}+(2B^3X^2-(BX)^2)\, b_\rho a_{VS}+(2B^3X^2-\frac{1}{2}BX^2)\, a_\rho b_\rho$$

Time-lapse AVO in terms of relative parameters

More recognizable and possibly numerically more accurate way is computing time-lapse difference data in terms of relative changes:

$$\frac{\Delta V_P}{V_P} = 2\frac{V_{Pb}-V_{P0}}{V_{Pb}+V_{P0}},\quad \frac{\Delta V_S}{V_S} = 2\frac{V_{Sb}-V_{S0}}{V_{Sb}+V_{S0}},\quad \frac{\Delta\rho}{\rho} = 2\frac{\rho_b-\rho_0}{\rho_b+\rho_0}$$

$$\frac{\delta V_P}{V_P} = 2\frac{V_{Pm}-V_{Pb}}{V_{Pm}+V_{Pb}},\quad \frac{\delta V_S}{V_S} = 2\frac{V_{Sm}-V_{Sb}}{V_{Sm}+V_{Sb}},\quad \frac{\delta\rho}{\rho} = 2\frac{\rho_m-\rho_b}{\rho_m+\rho_b}$$

Linear and second order difference data can be recalculated in terms of relative changes using the relations below:

$$a_{VP} = 2\left(\frac{\Delta V_P}{V_P}\right)-2\left(\frac{\Delta V_P}{V_P}\right)^2+\frac{3}{2}\left(\frac{\Delta V_P}{V_P}\right)^3-\ldots \qquad b_{VP} = 2\left(\frac{\delta V_P}{V_P}\right)-2\left(\frac{\delta V_P}{V_P}\right)^2+\frac{3}{2}\left(\frac{\delta V_P}{V_P}\right)^3-\ldots$$

$$a_{VS} = 2\left(\frac{\Delta V_S}{V_S}\right)-2\left(\frac{\Delta V_S}{V_S}\right)^2+\frac{3}{2}\left(\frac{\Delta V_S}{V_S}\right)^3-\ldots \qquad b_{VS} = 2\left(\frac{\delta V_S}{V_S}\right)-2\left(\frac{\delta V_S}{V_S}\right)^2+\frac{3}{2}\left(\frac{\delta V_S}{V_S}\right)^3-\ldots$$

$$a_\rho = \left(\frac{\Delta\rho}{\rho}\right)-\frac{1}{2}\left(\frac{\Delta\rho}{\rho}\right)^2+\frac{1}{4}\left(\frac{\Delta\rho}{\rho}\right)^3+\ldots \qquad b_\rho = \left(\frac{\delta\rho}{\rho}\right)-\frac{1}{2}\left(\frac{\delta\rho}{\rho}\right)^2+\frac{1}{4}\left(\frac{\delta\rho}{\rho}\right)^3+\ldots$$

The linear term is in agreement with Landrø's work (Landrø et al., 2001):

$$\Delta R_{PP}^{(1)}(\theta) = \frac{1}{2}\left(\frac{\delta\rho}{\rho}+\frac{\delta V_P}{V_P}\right)-2\frac{V_S^2}{V_P^2}\left(\frac{\delta\rho}{\rho}+2\frac{\delta V_S}{V_S}\right)\sin^2\theta+\frac{\delta V_P}{2V_P}\sin^2\theta$$

Also second, and third order terms are calculated and can be found in the reference (Jabbari and Innanen, 2012).

Conclusions

Time-lapse AVO measurements provide a tool to monitor the dynamic changes in subsurface properties during the time of the exploitation of a reservoir. Results showed that in plausible large contract time-lapse scenarios higher order term corrections is necessary.

ACKNOWLEDGEMENT

The authors wish to thank the sponsors, faculty, and staff of the Consortium for Research in Elastic Wave Exploration Seismology (CREWES) for their support of this work.

References

Landro, M., Discrimination between pressure and fluid saturation changes from time-lapse seismic data, GEOPHYSICS, 2001, **66**, No. 3, 836-844.

Jabbari, S. and Innanen, K. A., A framework for accurate approximation of difference reflection data from baseline to monitor survey in a time-lapse problem using AVO analysis, 2012, CREWES Research Report.