# Recovering low frequencies for impedance inversion by frequency domain deconvolution

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## Introduction

- Acoustic impedance is a rock property that can be derived from seismic data and contains important information about subsurface properties.
- Acoustic impedance can be calculated from earth's reflectivity function and this function could be estimated from seismic data.
- Estimation of reflectivity from seismic data is always bandlimited.
- The acoustic impedance can not be estimated properly due to the bandlimited seismic data.
- In this study we investigate the performance of standard deconvolution and its ability to recover low frequency content directly from seismic data.

## Theory

In homogenous medium for normal incident wavelet the reflection coefficient can be written as:

$$r_n = \frac{I_{n+1} - I_n}{I_{n+1} + I_n} \qquad (1)$$

Acoustic Impedance can be written as (Lindseth, 1979):

$$I_{n+1} = I_n\left(\frac{1+r_n}{1-r_n}\right) = I_1\prod_{i=1}^{n}\left(\frac{1+r_i}{1-r_i}\right) \approx I_1 \exp\left(2\sum_{i=1}^{n} r_i\right) \qquad (2)$$

### Reflectivity estimation:

$$s(t) = r(t) \bullet w(t) \;\Rightarrow\; r(t) = s(t) \bullet d(t)$$

$d(t)$: deconvolution operator & $w(t) \bullet d(t) = w_d(t)$ Estimated wavelet

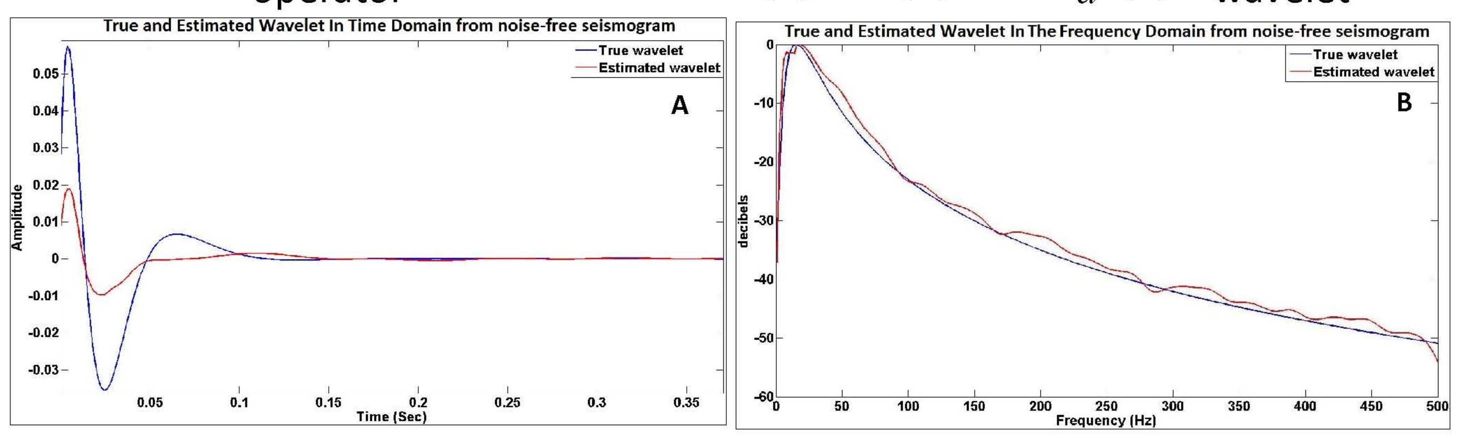
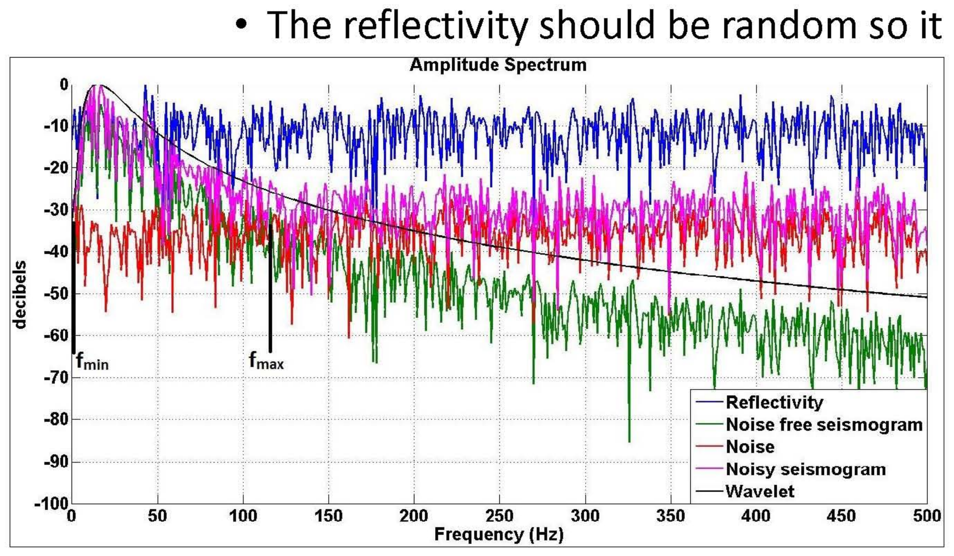


FIG. 1. Estimated wavelet in time domain (A) and in the frequency domain (B).

To construct a frequency domain deconvolution operator some assumptions are required:

- The wavelet should be minimum phase
- The wavelet spectrum should be smooth
- The wavelet should be stationary
- The reflectivity should be random so it has a white spectrum



$$f_{min} \le f \le f_{max}$$

$$\underbrace{|S(f)|}_{\text{Noisy Seismogram}} \approx \underbrace{|R(f)||W(f)|}_{\text{Noise free Seismogram}}$$

FIG. 2. Amplitude Spectrum of white reflectivity and synthetic seismogram

For white reflectivity: $\overline{|R(f)|} \approx 1 \;\Rightarrow\; \overline{|S(f)|} \approx |W(f)|$

which means that the amplitude spectrum of the estimated wavelet is equal to smoothing of the amplitude spectrum of the seismic data.

Deconvolution operator which is minimum phase can be defined as (Margrave, 2002):

$$D(f) = \frac{1}{|W(f)|_{est} + \mu A_{max}} e^{i\phi_D(f)} \quad \begin{cases} A_{max} = \max\left(|W(f)|_{est}\right) \\ \phi_D(f) = H\left(\ln\left(|D(f)|\right)\right) \\ 0.01 < \mu < 0.000001 \end{cases} \qquad (3)$$

$$\text{Estimated reflectivity} = S(f)D(f) = R(f)W(f)D(f) = R(f)W_D(f)$$

## Reflectivity estimation from real data

For creating synthetic seismic data P-wave velocity and density log from well 12-27 near hussar, Alberta and a minimum phase wavelet with 15Hz dominant frequency have been used.

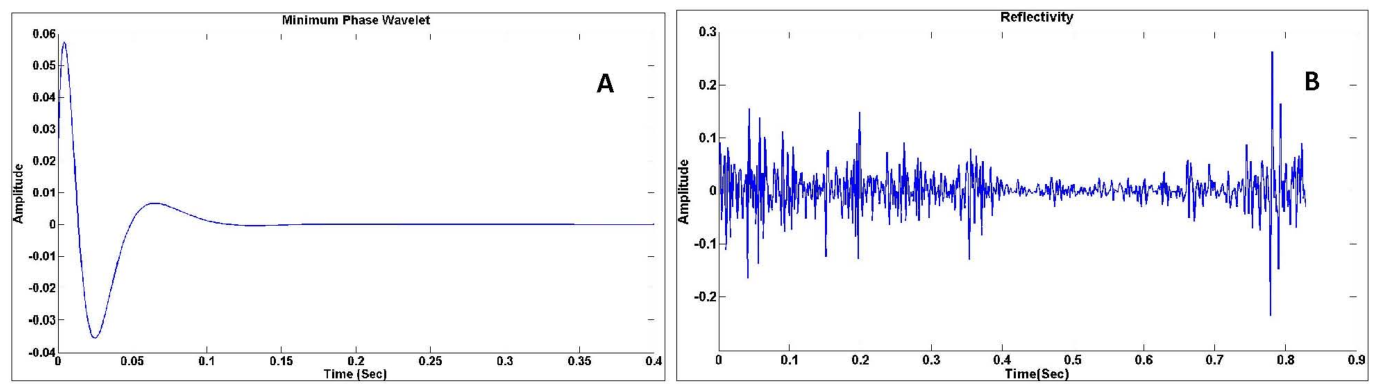


FIG. 3. Minimum phase wavelet with 15Hz dominant frequency (A) and well 12-27 reflectivity function (B)

### Spectral smoothing

It is possible to improve the wavelet estimation in order to get the better reflectivity approximation. Two different smoother have been used: boxcar smoother and Gaussian smoother.

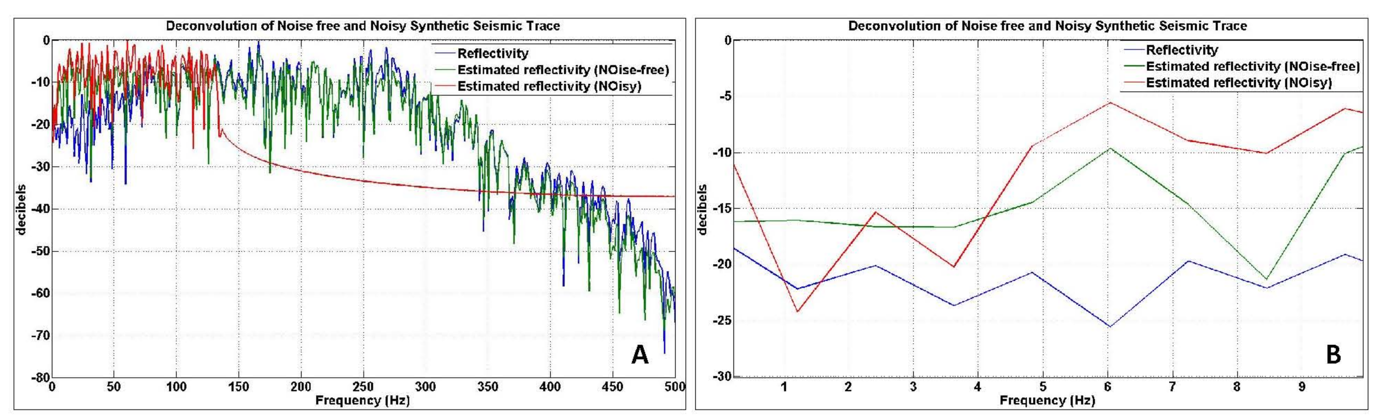


FIG. 4. Amplitude spectrum of deconvolved noise-free and noisy seismogram by using **boxcar smoother** (A) and zooming in low-frequency part (B).

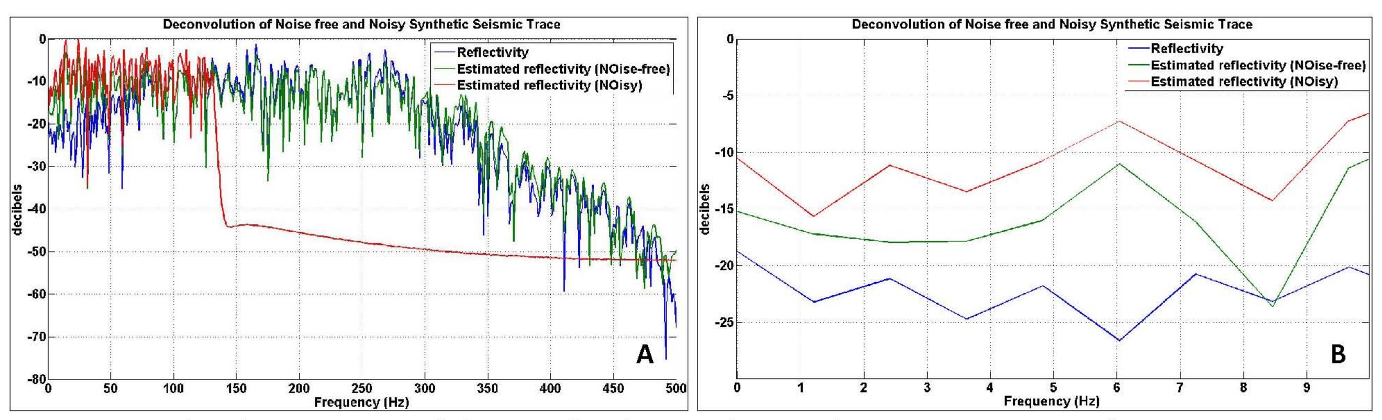


FIG. 5. Amplitude spectrum of deconvolved noise-free and noisy seismogram by using **Gaussian smoother** (A) and zooming in low-frequency part (B).

### Spectral color operator

The spectral color operator can be applied to the deconvolved seismogram, and improve white spectrum property of deconvolution operator.

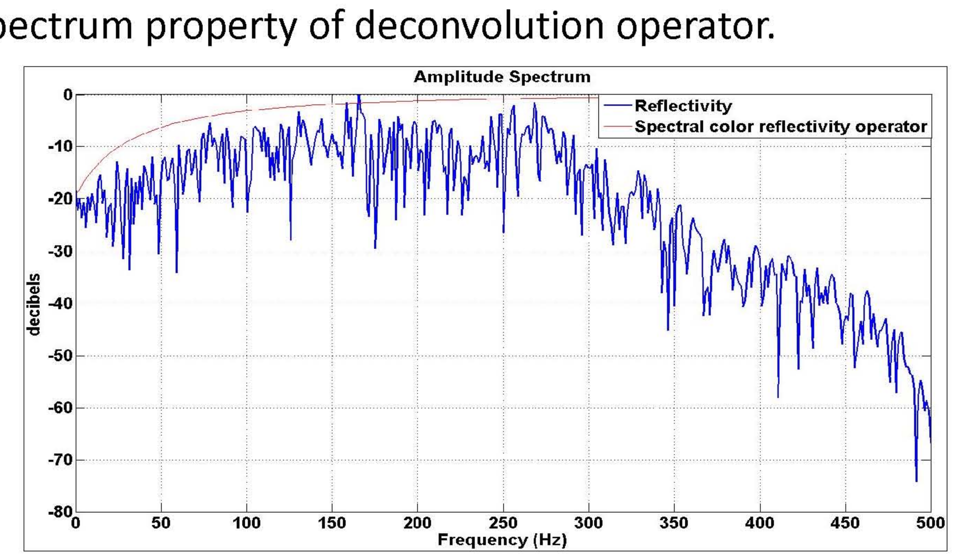


FIG. 5. Amplitude spectrum of colored spectrum reflectivity (blue) and color spectral operator (red).

The spectral color operator can be applied to the deconvolved seismogram and the amplitude spectrum of the deconvolved noise-free and noisy seismogram becomes:

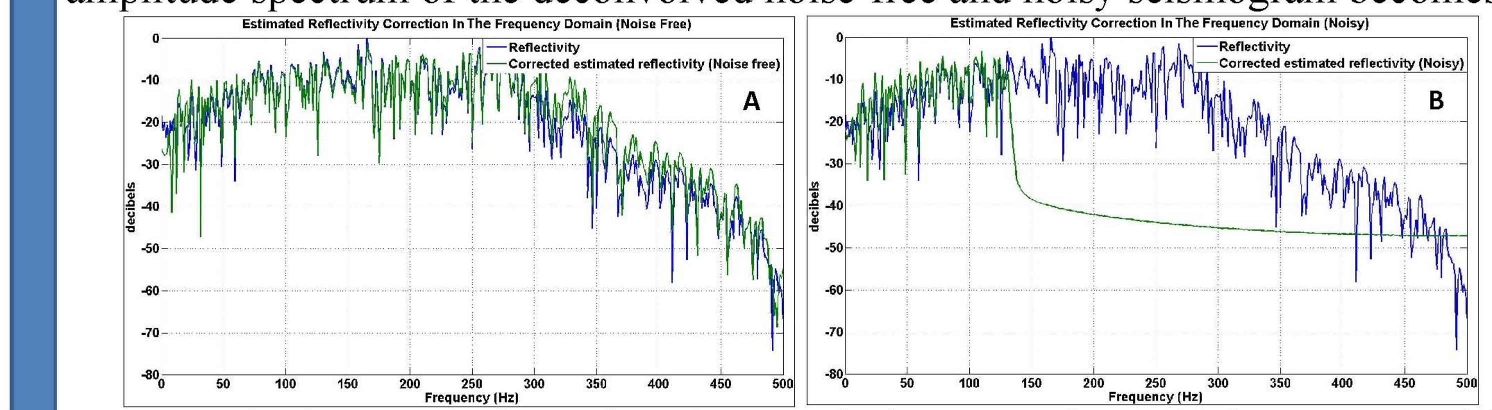


FIG. 5. Amplitude spectrum after applying the spectral color operator for a noise-free seismogram(A) and for a noisy seismogram (B).

## Acoustic impedance inversion

Given the impedance of the first layer and the estimated reflectivity function, acoustic impedance can be calculated from equation 2:

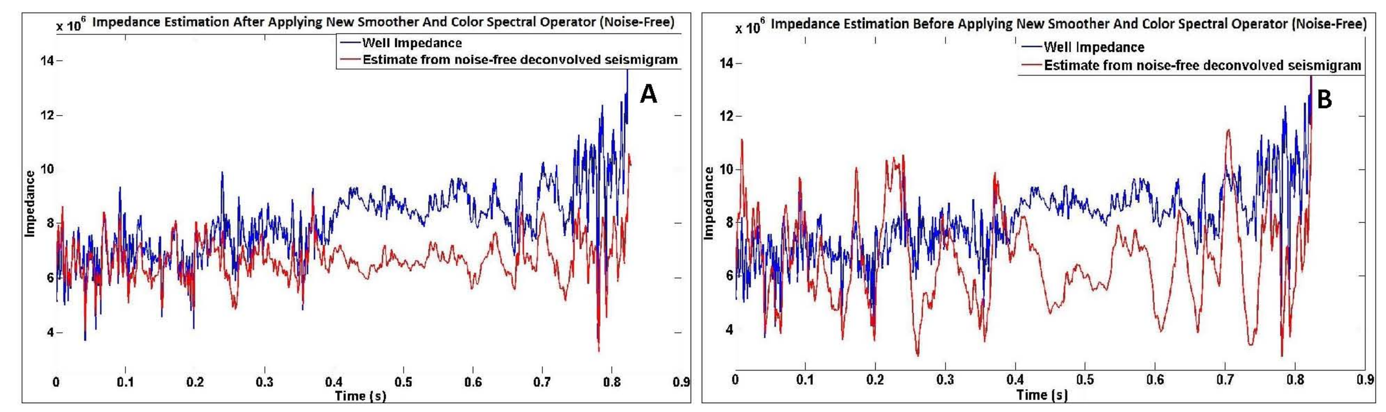


FIG. 5. Acoustic impedance estimation from the noise-free seismic trace and after applying new smoother and color spectral operator (A), and before applying these corrections (B).

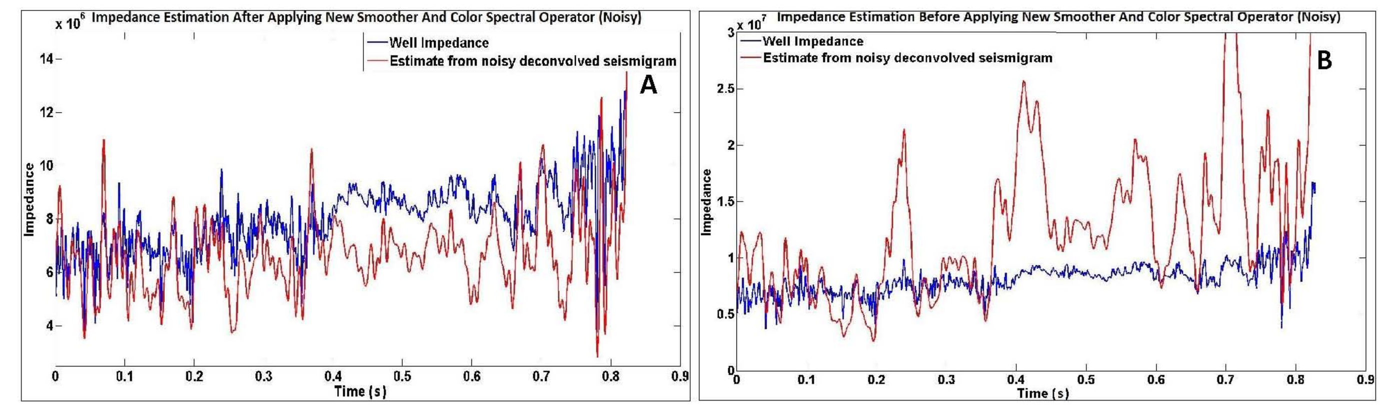


FIG. 5. Acoustic impedance estimation from the noisy seismic trace and after applying new smoother and color spectral operator (A), and before applying these corrections (B).

## Conclusion

- ✓ The bandlimited nature of wavelet and also noise contamination causes the missing low and high frequency in recorded data.
- ✓ The better seismic data smoothing, the more realistic reflectivity estimation we can reach.
- ✓ The colored spectrum of low frequencies data could be recovered by spectral color operator.

## References

Lindseth, R. O. (1979). Synthetic sonic logs-a process for stratigraphic interpretation. *Geophysics*, 3-26.
Margrave, G. F. (2002). *Methods of seismic data processing.* Calgary: Department of Geoscience, University of Calgary.

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