Multiparameter full waveform inversion of pre-critical reflection data

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Introduction

Two key full waveform inversion challenges (see Further Reading):

- **multiple parameters,**
- **pre-critical reflection data.**

We develop a hybrid discrete / continuous FWI formulation to optimize computational cost vs. convergence rate.

Quasi-Newton updates

The inverse Hessian $\mathbf{H}^{-1}$ moves the FWI update direction $\delta\mathbf{s}$ away from the one suggested by the gradient $\mathbf{g}$:

$$\delta\mathbf{s} = -\mathbf{H}^{-1}\mathbf{g}.$$

An update parallel to **g** suffers, amongst other problems, from **cross-talk**, the leakage of one parameter into others. $\mathbf{H}^{-1}$, it is thought, adjusts for this, increasing the convergence rate, but at a huge computational cost. Quasi-Newton updates, by approximating $\mathbf{H}^{-1}$, seek to balance convergence and compute cost.

—- A Cautionary Note —-

"Any old" Hessian approximation will not do. A diagonal Hessian, for instance,

$$\mathbf{H} \approx \begin{bmatrix} H_{11} & 0 & \dots & 0 \\ 0 & H_{22} & & \\ & & \dots & 0 \\ 0 & & 0 & H_{NN} \end{bmatrix},$$

leaves us just as prey to cross-talk as did gradient-based updates. An **analyzable** framework is needed to avoid this.

Multiparameter Newton updates

This poster contains a very brief summary. Please see the associated report for detailed derivations.

Two parameter acoustic case

In the two parameter acoustic case, with bulk modulus and density gradients $g_\kappa(\mathbf{r})$ and $g_\rho(\mathbf{r})$, the full Newton step in our continuous / discrete formulation is

$$\begin{bmatrix} \delta s_\kappa(\mathbf{r}) \\ \delta s_\rho(\mathbf{r}) \end{bmatrix} = -\int d\mathbf{r}' \mathcal{H}_2^{-1}(\mathbf{r},\mathbf{r}') \int d\mathbf{r}'' \begin{bmatrix} H_{\rho\rho} & -H_{\rho\kappa} \\ -H_{\kappa\rho} & H_{\kappa\kappa} \end{bmatrix} \begin{bmatrix} g_\kappa(\mathbf{r}'') \\ g_\rho(\mathbf{r}'') \end{bmatrix}$$

where $H_{AB} = H_{AB}(\mathbf{r}',\mathbf{r}'') = \partial^2\Phi/\partial s_A(\mathbf{r}')\partial s_B(\mathbf{r}'')$, and

$$\mathcal{H}_2(\mathbf{r},\mathbf{r}') = \int d\mathbf{r}''' \left[H_{\kappa\kappa}(\mathbf{r},\mathbf{r}''')H_{\rho\rho}(\mathbf{r}''',\mathbf{r}') - H_{\rho\kappa}(\mathbf{r},\mathbf{r}''')H_{\kappa\rho}(\mathbf{r}''',\mathbf{r}')\right].$$

Three parameter elastic case

The three-parameter elastic update, involving steps in V_P, V_S and ρ, is similarly

$$\begin{bmatrix} \delta s_P(\mathbf{r}) \\ \delta s_S(\mathbf{r}) \\ \delta s_\rho(\mathbf{r}) \end{bmatrix} = -\int d\mathbf{r}' \mathcal{H}_3^{-1}(\mathbf{r},\mathbf{r}') \int d\mathbf{r}'' \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{23} \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{bmatrix} \begin{bmatrix} g_P(\mathbf{r}'') \\ g_S(\mathbf{r}'') \\ g_\rho(\mathbf{r}'') \end{bmatrix}$$

where $\mathcal{H}_3^{-1}(\mathbf{r},\mathbf{r}')$ and Λ_{ij} both involve functions $H_{AB}(\mathbf{r}',\mathbf{r}'')$, in various combinations, for all of $\{A,B\} = \{P,S,\rho\}$.

Three quasi-Newton updates

The update formulas have continuous aspects (the integrals over space) and discrete aspects (the 2×2 and 3×3 matrices). There are, thus, three ways the Hessian operators in these formulas may be diagonally approximated.

Three quasi-Newton updates

For simplicity we will summarize the quasi-Newton updates for the two parameter acoustic case.

Parameter-type update

By keeping the spatially diagonal parts of the Hessian $H_{AB}(\mathbf{r}',\mathbf{r}'') \approx \Gamma_{AB}\delta(\mathbf{r}',\mathbf{r}'')$, we obtain

$$\begin{bmatrix} \delta s_\kappa(\mathbf{r}) \\ \delta s_\rho(\mathbf{r}) \end{bmatrix} \approx -\frac{1}{\Gamma_{\kappa\kappa}\Gamma_{\rho\rho} - \Gamma_{\rho\kappa}\Gamma_{\kappa\rho}} \begin{bmatrix} \Gamma_{\rho\rho}(\mathbf{r}) & -\Gamma_{\rho\kappa}(\mathbf{r}) \\ -\Gamma_{\kappa\rho}(\mathbf{r}) & \Gamma_{\kappa\kappa}(\mathbf{r}) \end{bmatrix} \begin{bmatrix} g_\kappa(\mathbf{r}) \\ g_\rho(\mathbf{r}) \end{bmatrix}.$$

Space-type update

Whereas, by keeping the diagonal parts of the 2×2 matrix and $\mathcal{H}_3^{-1}$, we obtain

$$\begin{bmatrix} \delta s_\kappa(\mathbf{r}) \\ \delta s_\rho(\mathbf{r}) \end{bmatrix} \approx -\int d\mathbf{r}' \begin{bmatrix} H_{\kappa\kappa}^{-1}(\mathbf{r},\mathbf{r}')g_\kappa(\mathbf{r}') \\ H_{\rho\rho}^{-1}(\mathbf{r},\mathbf{r}')g_\rho(\mathbf{r}') \end{bmatrix}.$$

Combined-type update

Finally, by keeping only the elements which are diagonal in both senses we obtain

$$\begin{bmatrix} \delta s_\kappa(\mathbf{r}) \\ \delta s_\rho(\mathbf{r}) \end{bmatrix} \approx -\begin{bmatrix} \Gamma_{\kappa\kappa}^{-1}(\mathbf{r})g_\kappa(\mathbf{r}) \\ \Gamma_{\rho\rho}^{-1}(\mathbf{r})g_\rho(\mathbf{r}) \end{bmatrix}.$$

Qualitatively, we expect the **parameter - type** update to maintain the capability of managing cross-talk. The proof of that appears in the companion report.

Further Reading

- FWI Workshop Proceedings, SEG Expanded Abstracts, Houston 2013.
- Special Section on FWI, The Leading Edge 32, 9, Sep 2013.
- Please see the accompanying report for full referencing.