A framework for linear and nonlinear S-wave and C-wave time-lapse difference AVO

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Introduction

The elastic properties of a rock change when the pressure and fluid flow is altered in a reservoir due to production (Greaves 1987). This raise the necessity of multicomponent 4D time-lapse analysis in a reservoir (Stewart et al. 2002 and 2003). A framework has been formulated to model linear and nonlinear elastic time-lapse difference AVO for P-P sections (Jabbari and Innanen 2013). The study described here focuses on applying the perturbation theory in a time-lapse amplitude variation with offset (Time-lapse AVO) method to model a framework for describing the difference data for the converted and shear waves (Figure 1).

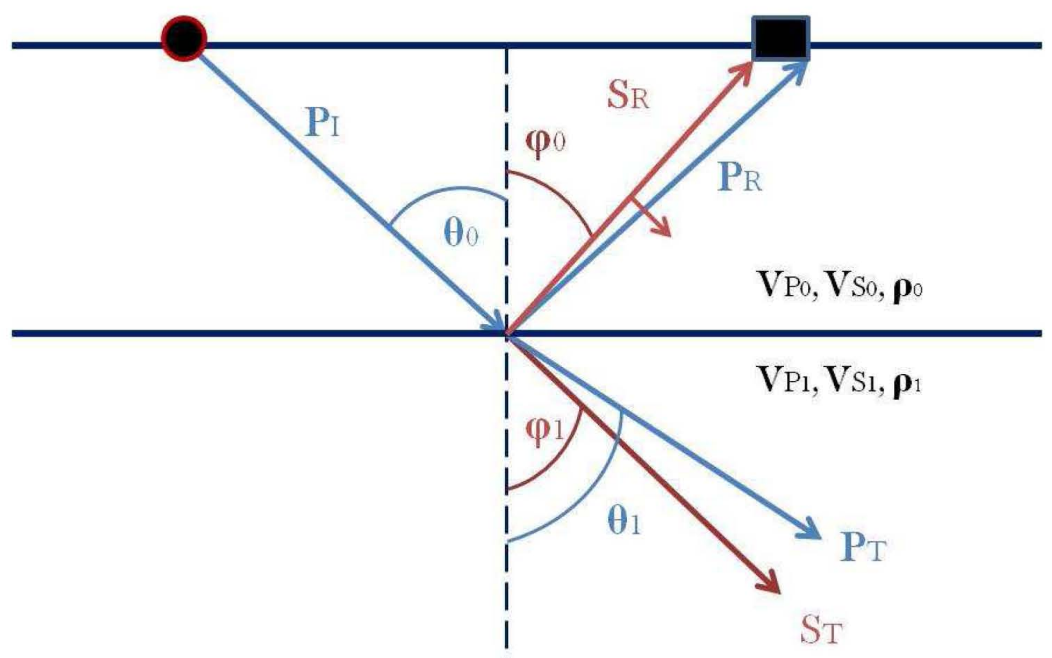

Figure 1: Displacement amplitude of an incident P-wave with related reflected and transmitted P and S waves.

Theory

The P-wave and S-wave velocities and the density change from the time of the baseline survey relative to the monitoring survey. Let V_{P_0}, V_{S_0}, ρ_0 and V_{P_b}, V_{S_b}, ρ_b be the rock properties of the cap rock and reservoir at the time of the baseline survey. Amplitudes of reflected and transmitted P and S waves impinging on the boundary of a planar interface between these two elastic media are calculated through setting the boundary conditions in the Zoeppritz equations in a matrix form (Aki and Richards 2002). Reflection coefficients are determined for the baseline and monitor surveys using the same method. Rock properties for the cap rock are the same, but the reservoir properties change to V_{Pm}, V_{Sm}, ρ_m for the monitor survey. The difference data reflection coefficients between the baseline and monitor survey is calculated as:

$$\begin{aligned}\Delta R_{PP}(\theta) &= R^m_{PP}(\theta) - R^b_{PP}(\theta)\\ \Delta R_{PS}(\theta) &= R^m_{PS}(\theta) - R^b_{PS}(\theta)\\ \Delta R_{SS}(\phi) &= R^m_{SS}(\phi) - R^b_{SS}(\phi)\\ \Delta R_{SP}(\phi) &= R^m_{SP}(\phi) - R^b_{SP}(\phi).\end{aligned} \quad (1)$$

In our time lapse study we have considered two groups of perturbation parameters (Innanen 2013 and Stolt 2012). The first group expresses the perturbation caused by propagating the wavefield from the first medium to the second medium in the baseline survey:

$$b_{VP} = 1 - \frac{V^2_{P_0}}{V^2_{P_b}},\ b_{VS} = 1 - \frac{V^2_{S_0}}{V^2_{S_b}},\ b_\rho = 1 - \frac{\rho_0}{\rho_b}. \quad (2)$$

The second group is to account for the change from the baseline survey relative to the monitor survey, the time lapse perturbation, we define:

$$a_{VP} = 1 - \frac{V^2_{P_b}}{V^2_{P_m}},\ a_{VS} = 1 - \frac{V^2_{S_b}}{V^2_{S_m}},\ a_\rho = 1 - \frac{\rho_b}{\rho_m}. \quad (3)$$

Elastic parameters in Zoeppritz matrixes may re-defined in terms of perturbations in the P- and S- waves velocities and the densities, and reflection coefficients for the shear and converted wave can be re-computed in terms of these perturbation parameters. Therefore, Equations (1) can be expanded in orders of all six perturbations as:

Theory continued

$$\begin{aligned}\Delta R_{PP}(\theta) &= \Delta R^{(1)}_{PP}(\theta) + \Delta R^{(2)}_{PP}(\theta) + \Delta R^{(3)}_{PP}(\theta) + \ldots\\ \Delta R_{PS}(\theta) &= \Delta R^{(1)}_{PS}(\theta) + \Delta R^{(2)}_{PS}(\theta) + \Delta R^{(3)}_{PS}(\theta) + \ldots\\ \Delta R_{SS}(\phi) &= \Delta R^{(1)}_{SS}(\phi) + \Delta R^{(2)}_{SS}(\phi) + \Delta R^{(3)}_{SS}(\phi) + \ldots\\ \Delta R_{SP}(\phi) &= \Delta R^{(1)}_{SP}(\phi) + \Delta R^{(2)}_{SP}(\phi) + \Delta R^{(3)}_{SP}(\phi) + \ldots\end{aligned} \quad (4)$$

Time-lapse difference data for converted wave and shear wave

$\Delta R_{PP}(\theta)$ for the first, second, and third orders can be found in the work done by Jabbari and Innanen (2013). Reflection coefficient difference data for converted and shear waves for the first, second, and third orders are:

$$\Delta R^{(1)}_{PS}(\theta) = k^1_1 a_{VS} + k^1_2 a_\rho$$

$$\Delta R^{(2)}_{PS}(\theta) = k^2_1 a^2_{VS} + k^2_2 a^2_\rho + k^2_3 b_\rho a_\rho + k^2_4 b_{VS} a_{VS} + k^2_5(a_{Vp} a_{VS} + a_{Vp} b_{VS} + b_{Vp} a_{VS}) + k^2_6(a_{Vp} a_\rho + b_\rho a_{Vp} + a_\rho b_{Vp} + a_\rho a_{VS} + a_\rho b_{VS} + b_\rho a_{VS})$$

$$\begin{aligned}\Delta R^{(3)}_{PS}(\theta) = {}& k^3_1 a^3_{VS} + k^3_2 a^3_\rho + k^3_3(b_{VS} a^2_{VS} + b^2_{VS} a_{VS}) + k^3_4(a^2_\rho a_{VS} + b^2_\rho a_{VS} + a^2_\rho b_{VS}) + k^3_5(a_\rho b^2_{VS} + a_\rho a^2_{VS} + b_\rho a^2_{VS}) + k^3_6(b_\rho a^2_{VP} + b^2_\rho a_{VP} + a_\rho b^2_{VP} +\\ & a^2_\rho b_{VP} + a^2_\rho a_{VP} + a_\rho a^2_{VP}) + k^3_7(b^2_{VP} a_{VS} + a^2_{VP} b_{VS} + a^2_{VP} a_{VS} + b_{VP} b_{VS} a_{VS} + a_{VP} b_{VS} a_{VS}) + k^3_8(b^2_\rho a_\rho + b_\rho a^2_\rho) + k^3_9(a_\rho b_{VP} a_{VS} +\\ & a_\rho b_{VP} b_{VS} + b_\rho a_{VP} a_{VS} + b_\rho a_{VP} b_{VS} + a_\rho a_{VP} a_{VS} + a_\rho a_{VP} b_{VS} + b_\rho b_{VP} a_{VS}) + k^3_{10}(b_\rho b_{VS} a_{VS} + a_\rho b_{VS} a_{VS}) + k^3_{11}(b_\rho a_\rho a_{VS} + b_\rho a_\rho b_{VS})\\ & + k^3_{12}(a_{VP} b^2_{VS} + a_{VP} a^2_{VS} + b_{VP} a^2_{VS})\end{aligned} \quad (5)$$

$$\Delta R^{(1)}_{SP}(\phi) = k^1_1 a_{VS} + k^1_2 a_\rho$$

$$\Delta R^{(2)}_{SP}(\phi) = k^2_1 a^2_{VS} + k^2_2 a^2_\rho + k^2_3 b_\rho a_\rho + k^2_4 b_{VS} a_{VS} + k^2_5(a_{VP} a_{VS} + a_{Vp} b_{VS} + b_{Vp} a_{VS}) + k^2_6(a_{Vp} a_\rho + b_\rho a_{Vp} + a_\rho b_{Vp} + a_\rho a_{VS} + a_\rho b_{VS} + b_\rho a_{VS})$$

$$\begin{aligned}\Delta R^{(3)}_{SP}(\phi) = {}& k^3_1 a^3_{VS} + k^3_2(a^3_\rho + b_{VS} b_\rho a_{VS} + b_{VS} a_\rho a_{VS} + b_\rho a^2_{VS} + a_\rho a^2_{VS} + a_\rho b^2_{VS}) + k^3_3(b_{VP} b_\rho a_{VS} + b_{VP} a_\rho a_{VS} + b_{VS} b_\rho a_{VP} + b_{VS} a_\rho a_{VP} + b_{VP} a_\rho b_{VS} +\\ & a_{VP} b_\rho a_{VS} + a_{VP} a_\rho a_{VS}) + k^3_4(a_\rho b_\rho a_{VS} + a_\rho b_\rho b_{VS}) + k^3_5(a_{VS} a^2_\rho + b_{VS} a^2_\rho + a_{VS} b^2_\rho) + k^3_6(b_\rho a^2_\rho + a_\rho b^2_\rho) + k^3_7(b_{VP} a^2_\rho + a_{VP} b^2_\rho + a_{VP} a^2_\rho + a^2_{VP} b_\rho + a^2_{VP} a_\rho +\\ & b^2_{VP} a_\rho) + k^3_8(a_{VS} b^2_{VS} + b_{VS} a^2_{VS}) + k^3_9(a_{VS} b^2_{VP} + b_{VS} a^2_{VP} + a_{VS} a^2_{VP} + a_{VS} b_{VS} a_{VP} + a_{VS} b_{VS} b_{VP}) + k^3_{10}(b_{VP} a^2_{VS} + a_{VP} b^2_{VS} + a_{VP} a^2_{VS})\end{aligned} \quad (6)$$

$$\Delta R^{(1)}_{SS}(\phi) = k^1_1 a_{VS} + k^1_1 a_\rho$$

$$\Delta R^{(2)}_{SS}(\phi) = k^2_1 a^2_{VS} + k^2_2 a^2_\rho + k^2_3(a_{VS} a_\rho + a_{VS} b_\rho + b_{VS} a_\rho) + k^2_4(a_{VS} b_{VS}) + k^2_5(a_\rho b_\rho)$$

$$\begin{aligned}\Delta R^{(3)}_{SS}(\phi) = {}& k^3_1 a^3_{VS} + k^3_2 a^3_\rho + k^3_3(a_{VP} a^2_{VS} + a_{VP} b^2_{VS} + b_{VP} a^2_{VS})\\ & + k^3_4(b_\rho a^2_{VS} + a_\rho a^2_{VS} + a_\rho b^2_{VS}) + k^3_5(b_{VS} a^2_{VS} + a_{VS} b^2_{VS})\\ & + k^3_6(b_{VS} b_\rho a_\rho + a_{VS} b_\rho a_\rho) + k^3_7(b_{VS} b_{VP} a_\rho + b_{VS} a_{VP} b_\rho +\\ & a_{VP} a_{VS} a_\rho + a_{VS} a_{VP} b_\rho + a_{VS} b_{VP} a_\rho + b_{VP} a_{VS} b_\rho + b_{VS} a_{VP} a_\rho) +\\ & k^3_8(b_{VP} a_\rho b_\rho + a_{VP} a_\rho b_\rho) + k^3_9(a_{VP} b^2_\rho + a_{VP} a^2_\rho + b_{VP} a^2_\rho) +\\ & k^3_{10}(b_{VS} a_{VP} a_{VS} + b_{VP} b_{VS} a_{VS}) + k^3_{11}(a_\rho b^2_\rho + b_\rho a^2_\rho) +\\ & k^3_{12}(b_{VS} a_{VS} a_\rho + b_{VS} a_{VS} b_\rho) + k^3_{13}(a_{VS} b^2_\rho + a_{VS} a^2_\rho + b_{VS} a^2_\rho)\end{aligned} \quad (7)$$

where all coefficients, k^n_m, are functions of V_{P_0}, V_{S_0}, $\sin^2\theta$, and $\sin^2\phi$. These coefficients are presented in details in the report.

Numerical example

In this section, we examine the derived linear and non linear difference time lapse AVO terms qualitatively with a numerical example for the PS and SP waves. The exact difference data are compared with our derived linear and higher order approximations in Figure 2. The second and third approximations are in better agreement with the exact difference data, especially for angles below the critical angle, which correspond to the range of study in this project.

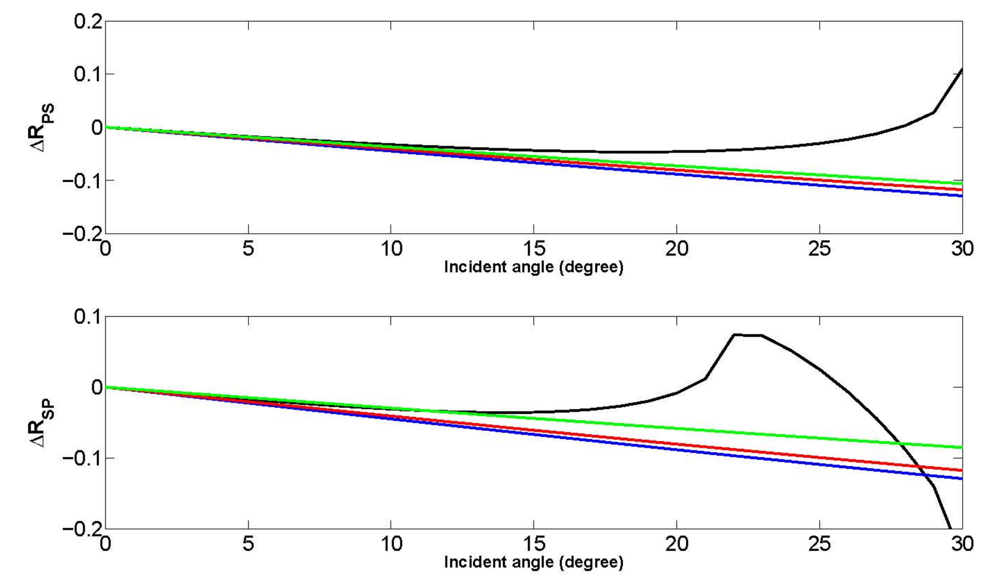

Figure 2: ΔR_{PS}, and ΔR_{SP} for the exact, linear, second order, and third order approximation. Elastic incidence parameters: $V_{P0} = 2000 m/s$, $V_{S0} = 1500 m/s$ and $\rho_0 = 2.0 g/cc$; Baseline parameters: $V_{P_{BL}} = 3000 m/s$, $V_{S_{BL}} = 1700 m/s$ and $\rho_{BL} = 2.1 g/cc$; Monitor parameters: $V_{P_{BL}} = 4000 m/s$, $V_{S_{BL}} = 1900 m/s$ and $\rho_{BL} = 2.3 g/cc$.
Black: Exact difference data, Blue: Linear approximation, Red: Second order approximation, and Green: Third order approximation.

Conclusions

Changes in the fluid saturation and pressure will have an impact in elastic parameters of subsurface, such as P, and S wave velocities and density, which can be approximated by applying time-lapse AVO analysis methods. Jabbari and Innanen (2013) have already investigated P-wave time-lapse AVO and showed that adding the higher order terms in ΔR_{PP} to the linear approximation for difference time-lapse data increases the accuracy of the ΔR_{PP} and corrects the error due to linearizing ΔR_{PP}. In the current research, we extended this work by formulating a framework for the difference reflection data in ΔR_{SS}, ΔR_{PS}, and ΔR_{SP}. The results showed that, including higher order terms in ΔR for shear and converted waves improves the accuracy of approximating time lapse difference reflection data, particularly for large contrast cases. Comparing linear, second , and third order terms for ΔR_{PS} and ΔR_{SP} indicates that, as we are moving toward higher order approximations; ΔR_{PS} and ΔR_{SP} are different. This confirms the difference between exact ΔR_{PS} and ΔR_{SP} which does not show up in the linear approximation case.

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