Linear and nonlinear poroelastic AVO

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ACHIEVING EXACT, LINEAR, AND NONLINEAR R_{PP}

A P-wave that strikes a planar interface between two elastic solids gives rise to four other plane waves: the reflected vertically polarized shear (SV) wave, the reflected P-wave, the transmitted P-wave, and the transmitted SV-wave (Keys, 1989). Since the horizontally polarized shear (SH) wave does not come into play in the Zoeppritz equations, we will refer to the SV wave as the S-wave for simplicity. The amplitudes of these plane waves are related to one another by the requirement that normal and tangential components of stress and displacement must be continuous across the reflecting interface (Keys, 1989). From these continuity conditions, a set of four equations for the four P-wave displacement amplitudes can be derived:

$$\begin{bmatrix} -X & -\sqrt{1-B^2X^2} & CX & -\sqrt{1-D^2X^2} \\ \sqrt{1-X^2} & -BX & \sqrt{1-C^2X^2} & DX \\ 2B^2X\sqrt{1-X^2} & B\left(1-2B^2X^2\right) & 2AD^2X\sqrt{1-C^2X^2} & -AD\left(1-2D^2X^2\right) \\ -\left(1-2B^2X^2\right) & 2B^2X\sqrt{1-B^2X^2} & AC\left(1-2D^2X^2\right) & 2AD^2X\sqrt{1-D^2X^2} \end{bmatrix} \begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = \begin{bmatrix} X \\ \sqrt{1-X^2} \\ 2B^2X\sqrt{1-X^2} \\ 1-2B^2X^2 \end{bmatrix}$$

$$\mathbf{P}\begin{bmatrix} R_{PP} \\ R_{PS} \\ T_{PP} \\ T_{PS} \end{bmatrix} = \mathbf{m}_P$$

where R_{PP} is the amplitude of the reflected P-wave, R_{PS} is the amplitude of the reflected S-wave, T_{PP} is the amplitude of the transmitted P-wave, and T_{PS} is the amplitude of the transmitted S-wave (Keys, 1989). The variable X is $\sin\theta_0$, where θ_0 is the angle of incidence. A, B, C, and D represent the ratios of the various elastic parameters: $A=\rho_1/\rho_0$, $B=V_{S_0}/V_{P_0}$, $C=V_{P_1}/V_{P_0}$, and $D=V_{S_1}/V_{P_0}$, where ρ_i, V_{P_i}, and V_{S_i} are the density, P-wave velocity and S-wave velocity of medium i, respectively.

TRANSITION FROM $V_P, V_S, \rho \rightarrow a_f, a_\mu, a_\rho$

Here is a sample of elements found in the first column of the Zoeppritz equations after transitioning from the elastic constants of V_P, V_S, ρ to poroelastic perturbations a_f, a_μ, a_ρ.

$$\tilde{A}_{11} = -\sin\theta_0$$

$$\tilde{A}_{12} = -\left[1-(\gamma_0)^2_{\text{sat}}\sin^2\theta_0\right]^{1/2}$$

$$\tilde{A}_{13} = \left(\frac{(\gamma_0)^2_{\text{dry}}}{(\gamma_0)^2_{\text{sat}}}\frac{(1-a_\rho)}{(1-a_\mu)} + \left(1-\frac{(\gamma_0)^2_{\text{dry}}}{(\gamma_0)^2_{\text{sat}}}\right)\frac{(1-a_\rho)}{(1-a_f)}\right)^{1/2}\sin\theta_0$$

$$\tilde{A}_{14} = -\left[1-\frac{(1-a_\mu)^{-1}(1-a_\rho)}{(\gamma_0)^2_{\text{sat}}}\sin^2\theta_0\right]^{1/2}$$

VALIDATION WITH RUSSELL AND GRAY

Russell and Gray have presented a linearized poroelastic approximation for PP reflection coefficients that resembles forms shown by Aki and Richards (2002), Shuey (1985), Wiggins et al. (1983), and Smith and Gidlow (1987). It is different from those mentioned by having the ability to detect fluid of the target of interest by predicting fluid directly from the amplitude data. Their equation takes the form

$$R_{PP}^{(RG)}(\theta) = \left[\left(1-\frac{\gamma^2_{\text{dry}}}{\gamma^2_{\text{sat}}}\right)\frac{\sec^2\theta}{4}\right]\frac{\Delta f}{f} + \left[\frac{\gamma^2_{\text{dry}}}{4\gamma^2_{\text{sat}}}\sec^2\theta - \frac{2}{\gamma^2_{\text{sat}}}\sin^2\theta\right]\frac{\Delta\mu}{\mu} + \left[\frac{1}{2}-\frac{\sec^2\theta}{4}\right]\frac{\Delta\rho}{\rho},$$

where θ is the average between the incidence and refraction angles, $\Delta f/f$, $\Delta\mu/\mu$, $\Delta\rho/\rho$ are the reflectivities, $\gamma_{\text{dry}} = (V_{P_0}+V_{P_1})_{\text{dry}}/(V_{S_0}+V_{S_1})_{\text{dry}}$ and $\gamma_{\text{sat}} = (V_{P_0}+V_{P_1})_{\text{sat}}/(V_{S_0}+V_{S_1})_{\text{sat}}$, where the subscripts 'dry' and 'sat' represent the skeleton framework of the geologic matrix of the material and the skeleton framework that has been saturated in fluid, respectively. By comparing the first order approximation of $\tilde{R}_{PP}$ in reflectivity with the Russell and Gray approximation, we argue that this would validate our method. Our result yielded a first order approximation such that

$$\tilde{R}_{PP}(\theta_0)_1 = \left[\left(1-\frac{(\gamma_0)^2_{\text{dry}}}{(\gamma_0)^2_{\text{sat}}}\right)\frac{\sec^2\theta_0}{4}\right]\frac{\Delta f}{f} + \left[\frac{(\gamma_0)^2_{\text{dry}}}{4(\gamma_0)^2_{\text{sat}}}\sec^2\theta_0 - \frac{2}{(\gamma_0)^2_{\text{sat}}}\sin^2\theta_0\right]\frac{\Delta\mu}{\mu} + \left[\frac{1}{2}-\frac{\sec^2\theta_0}{4}\right]\frac{\Delta\rho}{\rho}.$$

Russell and Gray's approximation and the first order approximation are relatively similar to each other in comparison. The only difference is the fact that the Russell and Gray approximation uses parameter averages instead of incident medium parameters used by the first order approximation. This is explained by the two different approaches that are used to derive each equation. The Russell and Gray approximation is derived through the Aki and Richards approximation which, in itself, is parameterized by averages of the elastic properties V_P, V_S, ρ and average angle θ. The first order approximation is derivative of the Zoeppritz equations that is written in terms of elastic property components (V_{P_0}, V_{P_1}, V_{S_0}, V_{S_1}, ρ_0, ρ_1). By ultimately transforming the property components into perturbations, it was by choice to leave the parameters in terms of medium 0. The same form of AVO expressions, using our derivation technique, would result in the same form if we chose to involve parameters in terms of medium 1. Thus, the first order approximation would contain instances of $(\gamma_1)_{\text{dry}}$ and $(\gamma_1)_{\text{sat}}$. These two equations are therefore equivalent in form.

NUMERICAL RESULTS

The numerical results are calculated such that fluid, shear modulus, and density are predetermined for medium 1. Depending on the level of contrast between medium 0 and medium 1 would be selected by the perturbation/reflectivity value. This value would then be used to calculate fluid, shear modulus, and density for medium 0.

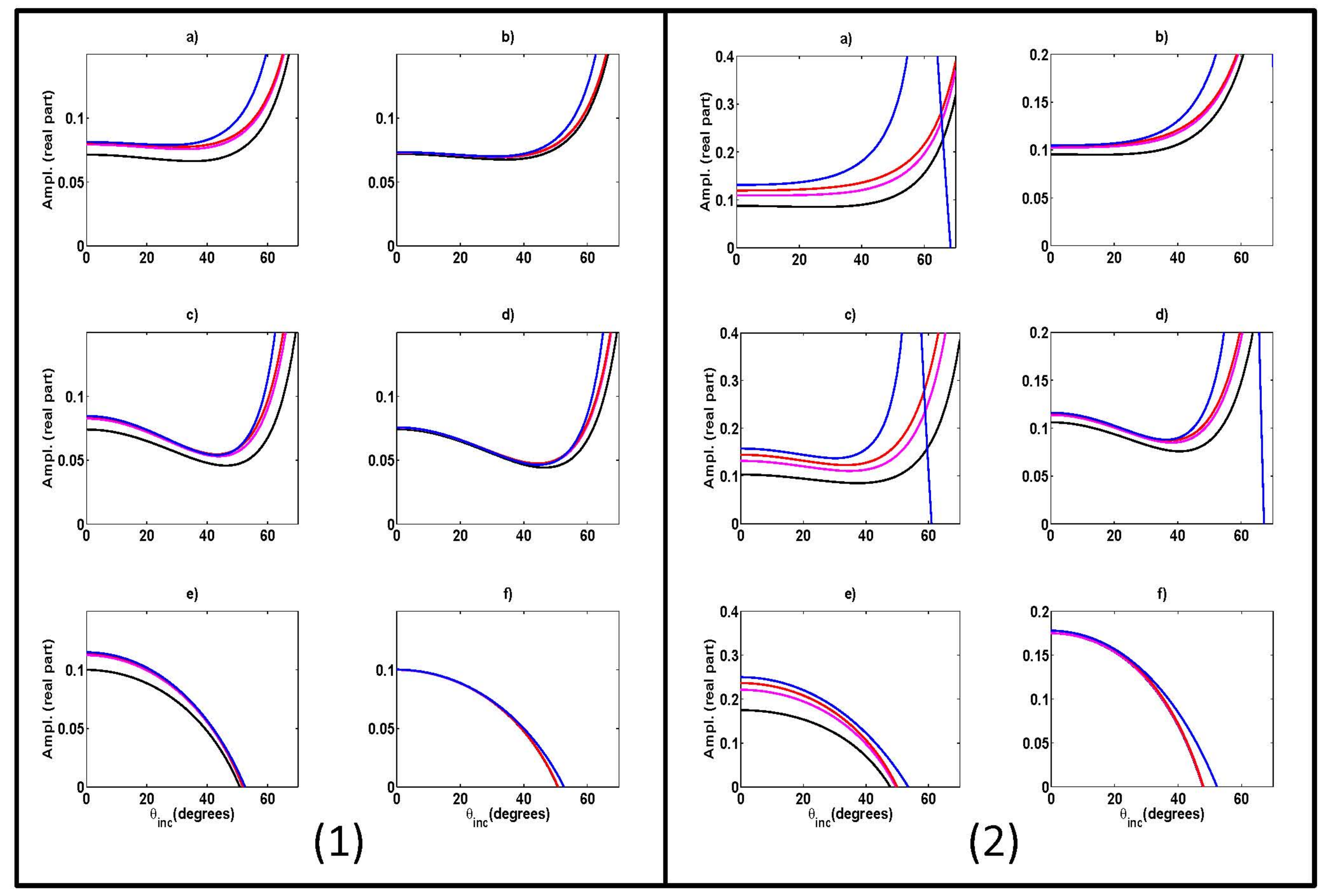

FIG. 1&2: (a), (c), and (e) represent the perturbation based poroelastic AVO approximations of 1st, 2nd, and 3rd order and are shown by the black, magenta, and red curves respectively. (b), (d), and (f) represent the reflectivity based poroelastic AVO approximations of 1st, 2nd, and 3rd order and are shown by the black, magenta, and red curves respectively. The exact amplitudes are shown by the blue curve.

Figure	a_f	a_μ	a_ρ	Figure	$\Delta f/f$	$\Delta\mu/\mu$	$\Delta\rho/\rho$	Figure	a_f	a_μ	a_ρ	Figure	$\Delta f/f$	$\Delta\mu/\mu$	$\Delta\rho/\rho$
1a	0.300	0.100	0.100	1b	0.300	0.100	0.100	2a	0.600	0.100	0.100	2b	0.600	0.100	0.100
1c	0.100	0.300	0.100	1d	0.100	0.300	0.100	2c	0.100	0.600	0.100	2d	0.100	0.600	0.100
1c	0.100	0.100	0.300	1f	0.100	[illegible]	[illegible]	[illegible]	[illegible]	0.100	0.600	2f	0.100	0.100	0.600

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