Can we see a velocity ramp in reflection seismic

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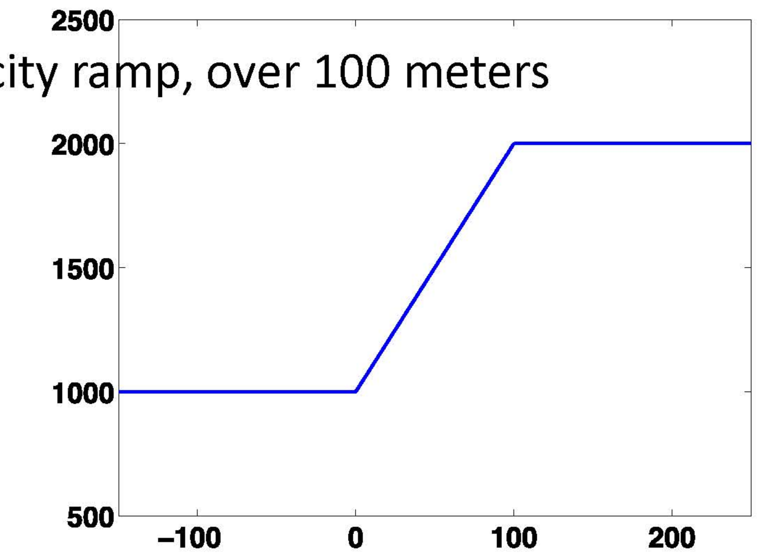
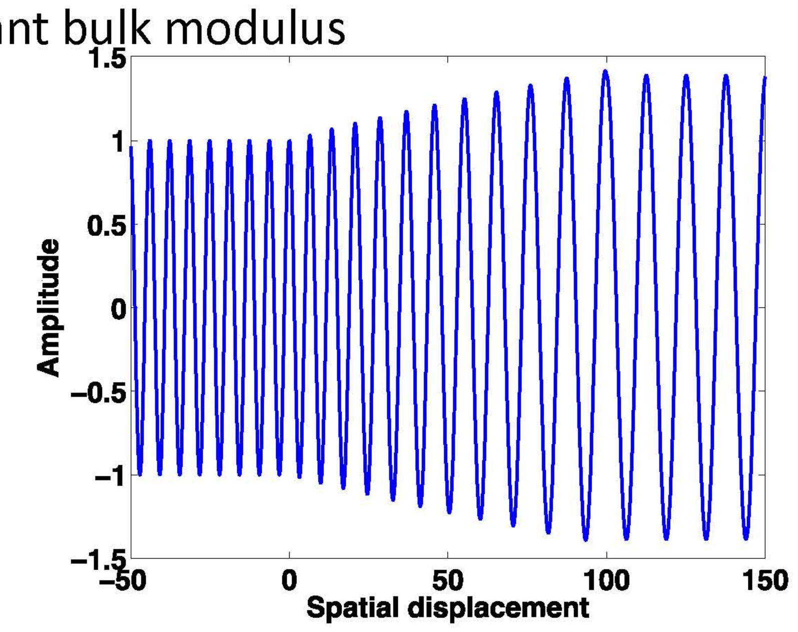
- When a seismic wave propagates from one region of constant velocity to another region of different velocity, through a smooth transition zone, the amplitude and wavelength both change smoothly across the zone.
- Despite the lack of sharp reflections, can we detect this transition with seismic reflection data?

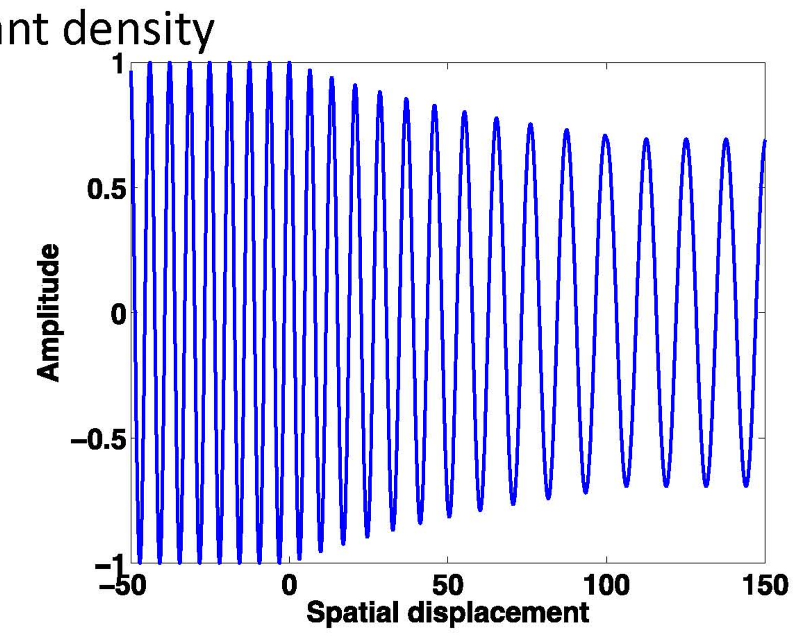
A velocity ramp, over 100 meters



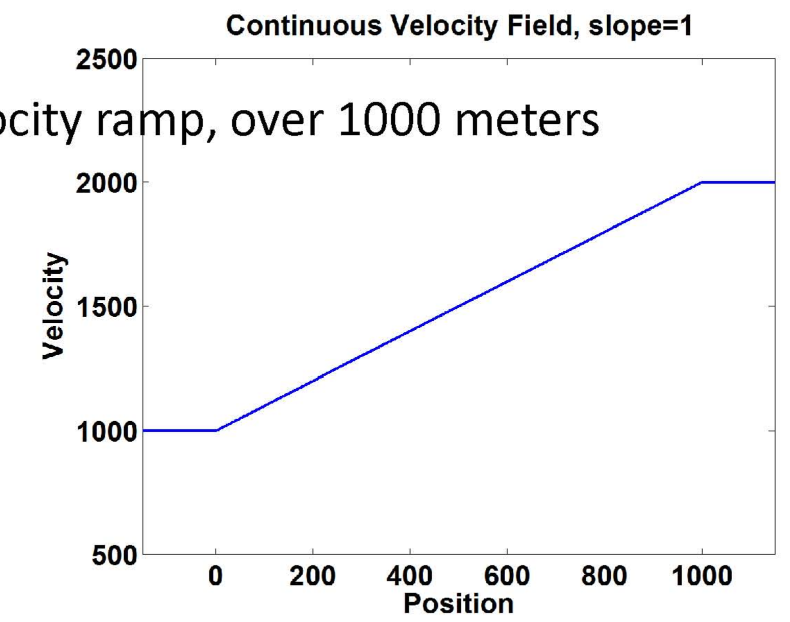
Wave propagation through the transition zone, constant bulk modulus



Wave propagation through the transition zone, constant density



A velocity ramp, over 1000 meters



- Beginning with a 1D elastic wave equation, we can produce exact analytic solutions for waves propagation through the velocity ramp. By applying continuity conditions at the interface, we compute the frequency dependent reflection and transmission coefficients

The elastic wave equation.

$$\rho(x)\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left(K(x)\frac{\partial u}{\partial x}\right)$$

The linear velocity ramp model

$$c^2(x) = \frac{K}{\rho} = (c_1 + mx)^2$$

Exact solutions, for constant bulk modulus

$$u_{left}(x,t) = e^{i\omega(\pm x/c_1 - t)}$$
$$u_{mid}(x,t) = (c_1 + mx)^{1/2 \pm \sqrt{1/4 - (\omega/m)^2}} e^{-i\omega t}$$
$$u_{right}(x,t) = e^{i\omega(\pm x/c_2 - t)}$$

Exact solutions, for constant density

$$u_{left}(x,t) = e^{i\omega(\pm x/c_1 - t)}$$
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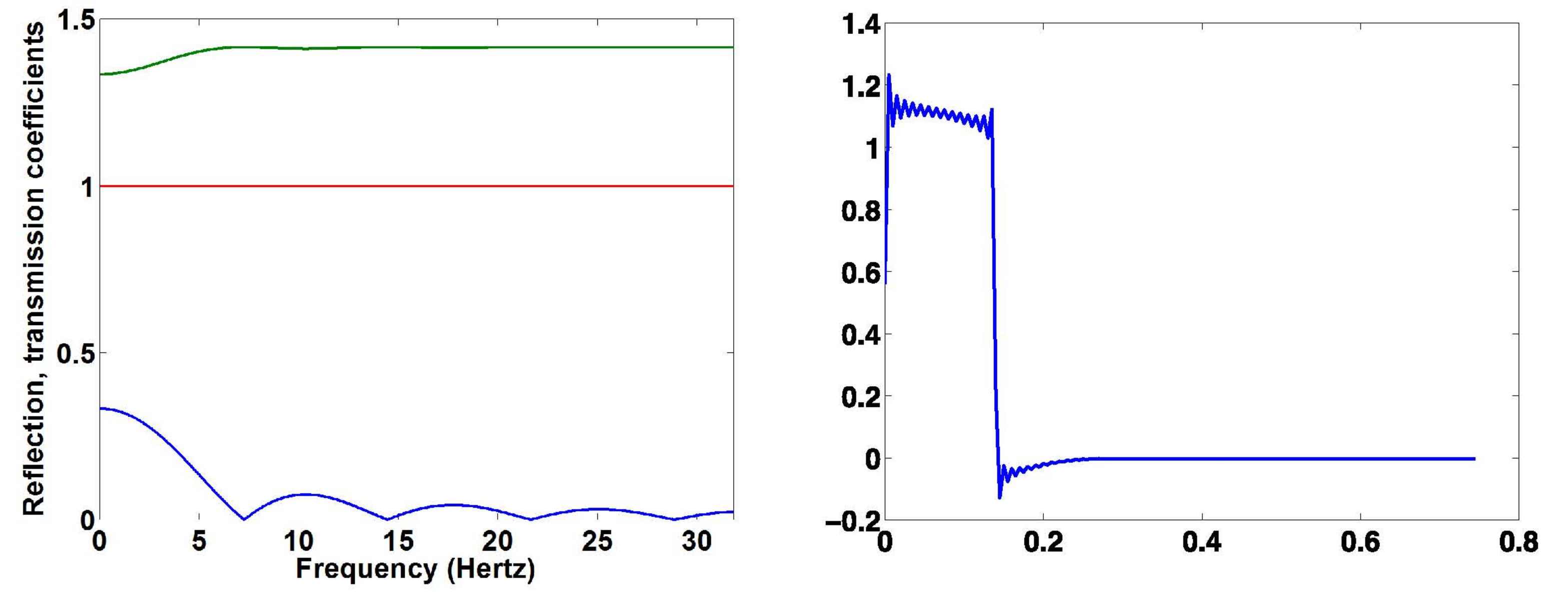
- An important question is how slow a transition is observable in the reflection data? The examples above show a velocity change of 1000 m/s in a distance of 100 m. In those cases, the reflection data shows an observable node (zero) at 7 Hz.
- For a more gentle transition over 1000m (on the left) more typical of a sedimentary basin, the first node (shown on the right) in the reflection data is at 0.7 Hz, which would be much harder to observe with current technology.

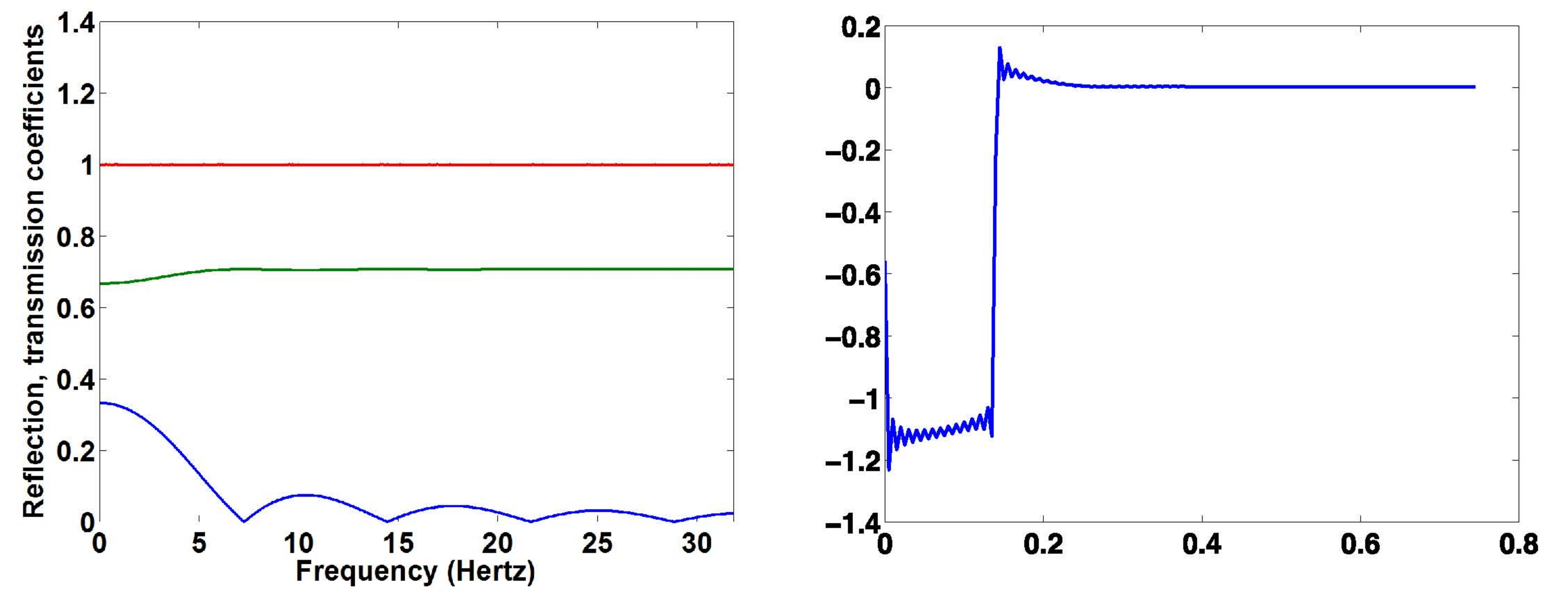
- For a given velocity ramp, we compute the exact form of the frequency dependent reflection coefficient. For a transistion from velocity c1 to velocity c2, over a ramp of slope m, the reflection coefficient as a function of angular frequency is:

$$R(\omega) = \frac{(c_2/c_1)^a - (c_2/c_1)^{-a}}{2(i\omega/m)\left[(c_2/c_1)^a - (c_2/c_1)^{-a}\right] + 2a\left[(c_2/c_1)^a + (c_2/c_1)^{-a}\right]}$$
where $a = \sqrt{1/4 - (\omega/m)^2}$

Frequency dependent reflection and transmission coefficients, and the corresponding impulse response, constant bulk modulus



Ditto, constant density case. Note the sign change in the reflection.



Ditto, for the slow transition zone of 1000m. Note the low frequency detail in the reflection response.