1D scalar full waveform inversion inferring convergence properties with analytic and numerical examples

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Summary

Formulated as a least-squares form, Full waveform inversion (FWI) seeks to minimize the difference between the modeling data and the observed data and estimate the subsurface parameters. It has been widely studied in recent years, but some problems still remain to be addressed. In this research, we performed the analytic analysis of 1D scalar FWI. The analysis to this simplest condition can help us achieve some new ideas and discoveries in FWI. A simple two-interface model and a homogeneous background model are used as the true velocity model and initial velocity model respectively. And two iterations are performed for analysis based on some optimal assumptions. We found that: (1) after the first iteration, the placement error at the second interface is influenced by the velocity contrast and interfaces distance; (2) after the second iteration, the placement error at the second interface become smaller for small velocity contrast, but may become larger for large velocity contrast; (3) and the noises produced in the cross-correlation have a negative influence to the amplitude recovery of the second interface, which will decrease the convergence rate of FWI; (4) but the noises have no significant influence to the placement error of the second interface.

Analytic Analysis of FWI

As a least-squares local optimization, full waveform inversion seeks to minimize the difference between the synthetic data and observed data and update the model iteratively. The misfit function is given in a least-squares norm:

$$\phi\left(s_0^{(n)}(\mathbf{r})\right)=\frac{1}{2}\int d\omega\left(\sum_{\mathbf{r}_s}\sum_{\mathbf{r}_g}\left\|\delta P\left(\mathbf{r}_g,\mathbf{r}_s,\omega|s_0^{(n)}(\mathbf{r})\right)\right\|_2\right)$$

For analytic analysis of FWI, we build a very simple 1D velocity model. The true velocity model consists of two interfaces and the initial velocity model is a homogeneous velocity model, as indicated by Fig.1.

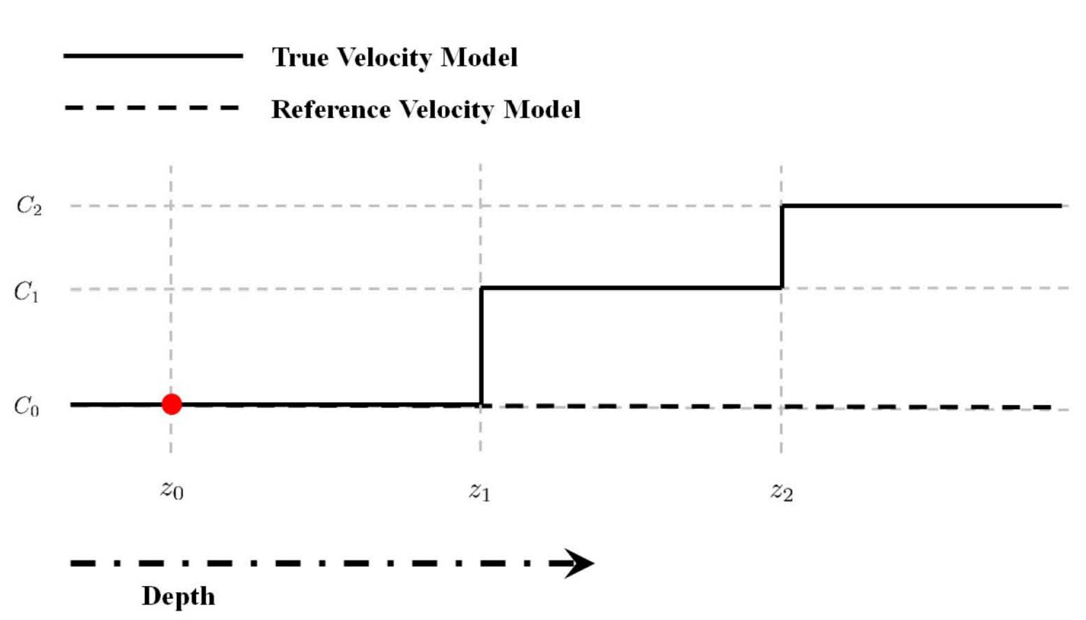

Fig.1 The solid line and dash line indicate the true velocity model and initial velocity model respectively.

1st Iteration

The gradient in FWI can be constructed by crosscorrelating the forward modeling wavefields and backpropagated wavefields:

$$g(z)=\sum d\omega\Re\left(G(z,0,\omega)G(z,0,\omega)\delta P^*(0,0,\omega)\right)$$

The Green's function can be expressed as:

$$G(z,0,\omega)=G(0,z,\omega)=\frac{e^{ik_0z}}{i2k_0},$$

Then we can calculate the analytic solution for the gradient in the first iteration, which consists of two Heaviside functions:

$$g_0(z)=H(z-z_1)+\frac{R_2'}{R_1}H(z-Z_2'),$$

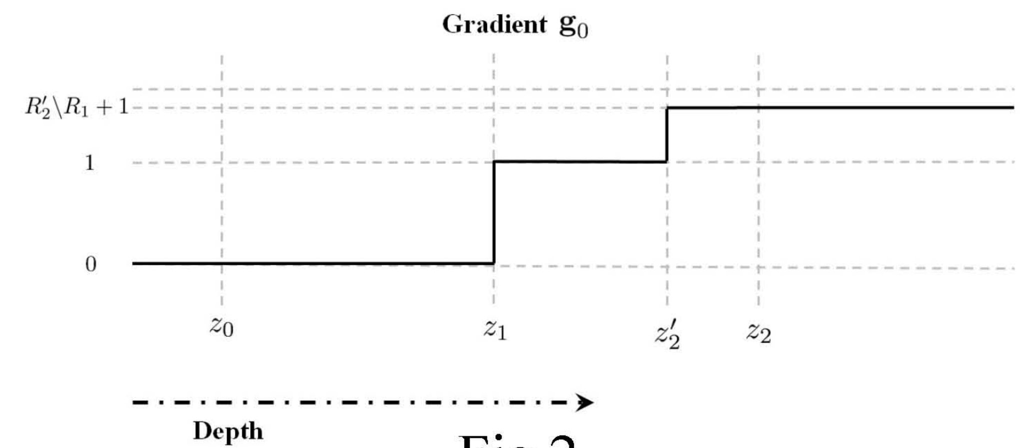

Fig.2

We can then update the initial velocity model using the gradient with an optimal step length.

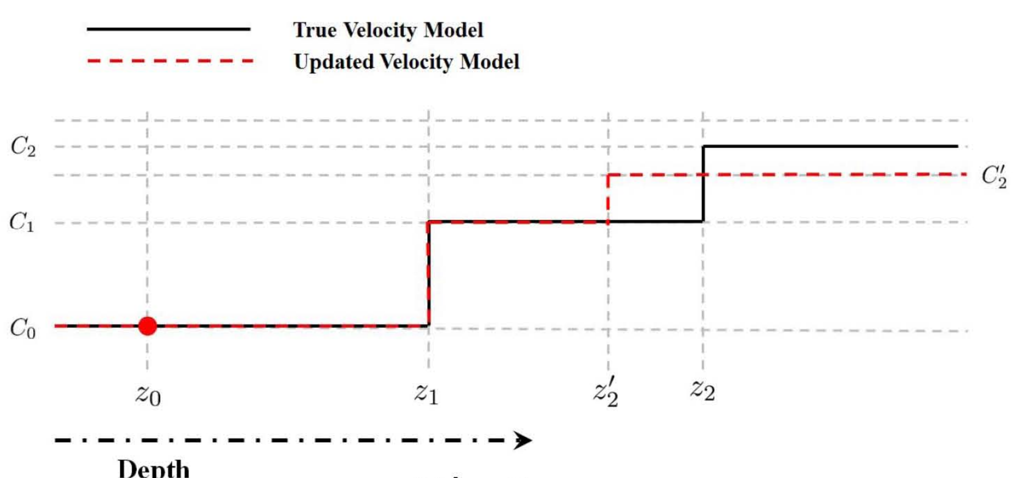

Fig.3

It can be seen that the first interface of updated velocity matches the real velocity model very well. While the location of the second interface is not right.

2nd Iteration

Similarly, we can calculate the analytic solution of the gradient in the second iteration. The explicit expression can be found in the research report. The following figure shows the form of the gradient in the second iteration.

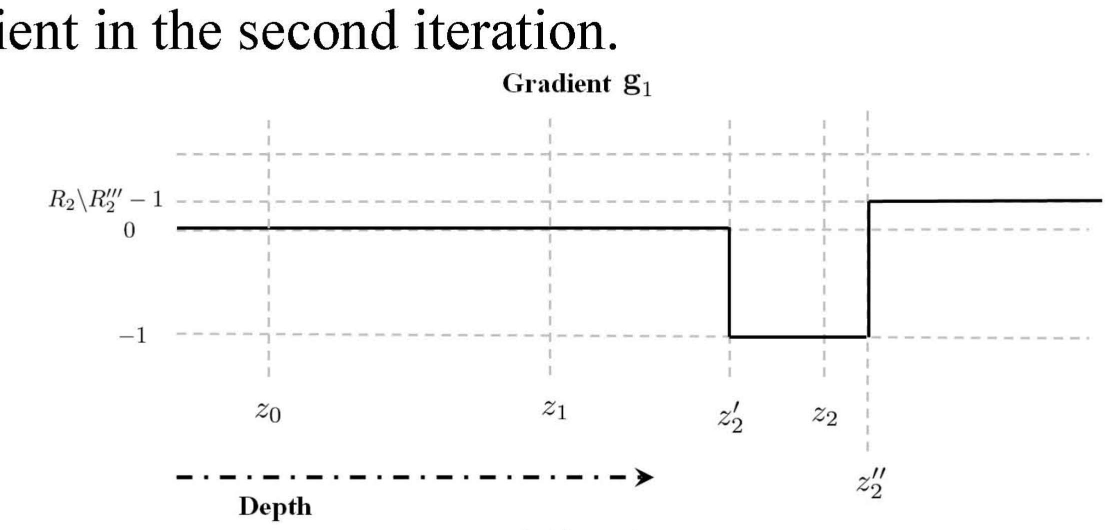

Fig.4

2nd Iteration

Then we can update the velocity model using the gradient in the second iteration. The red-dash line and green-dash line in the following figure indicate the updated velocity model in first iteration and second iteration respectively.

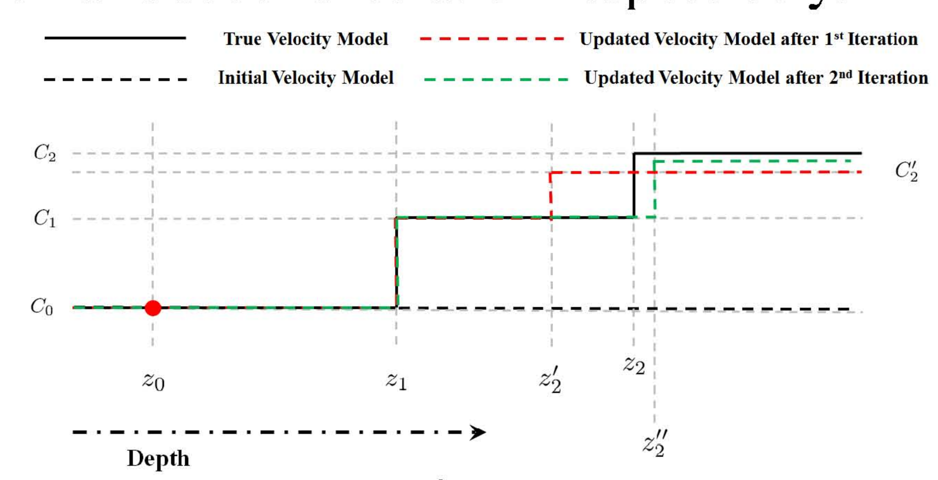

Fig.5

It can be seen that the location of the second interface of the velocity model is still not right while the location error becomes smaller.

Numerical Experiment

We performed a numerical example on a two interfaces velocity model with varying the velocity from 1500m/s, to 3000m/s and 4500m/s. The initial velocity model is a homogeneous velocity model with a velocity of 1500m/s.

We can see that the numerical results for the gradients in the first and second iterations match the analytic predictions very well.

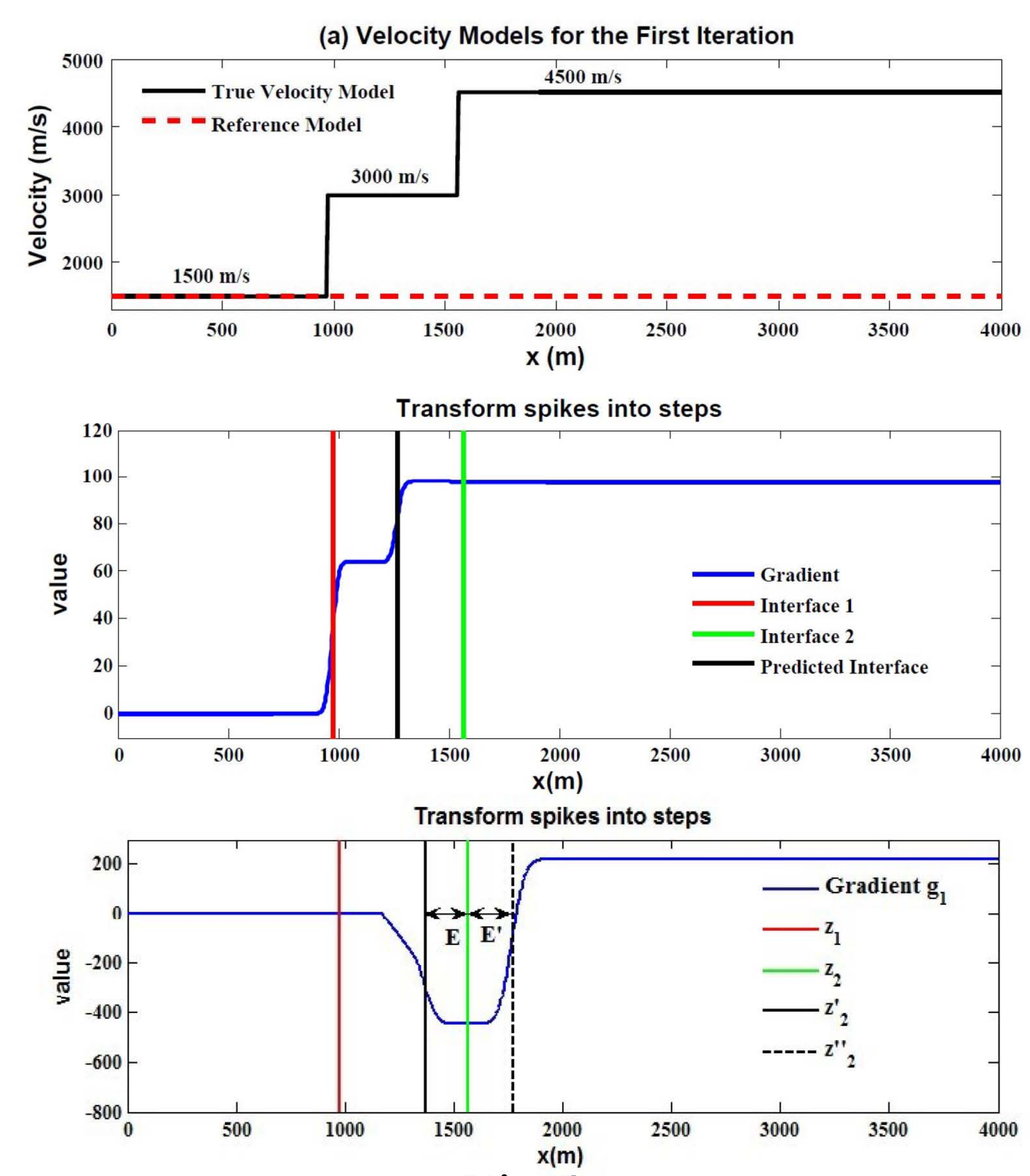

Fig.6

Acknowledgements

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