Full Waveform Inversion with Phase Encoded Pseudo-Hessian

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Summary

Full waveform inversion (FWI) is a very important method for estimating the subsurface parameters. In our implementation, a linear source encoding strategy is used for the gradient calculation in time-ray parameter domain. The plane wave encoding approach forms super-gathers by summing densely distributed individual shots, and can reduce the computational burden considerably. We also construct the diagonal part of the pseudo-Hessian using phase encoding method. The diagonal part of the encoded pseudo-Hessian is a good approximation to the full Hessian matrix, which preconditions the gradient. The preconditioning is equivalent applying a deconvolution imaging condition in prestack reverse time migration. To avoid the local minimum problem, a multi-scale approach in the time domain is employed, by (1) applying a low-pass filtering to the data residuals and (2) increasing the frequency bands step by step. This has been proved to be effective against cycle skipping. We assemble this suite of tools and carry out a numerical experiment with a modified Marmousi model, analyzing the effectiveness of this combination of strategies.

General Principle of Full Waveform Inversion

Full waveform inversion is iterative method to estimate the subsurface parameters by minimizing a least-squares misfit function:

$$\phi = \frac{1}{2}\int d\omega \left(\sum_{\mathbf{r}_s,\mathbf{r}_g} \|\delta P\left(\mathbf{r}_g,\mathbf{r}_s,\omega | s_0^{(n)}(\mathbf{r})\right)\|_2 \right)$$

The traditional shot-profile gradient can be expressed as:

$$g^{(n)}(\mathbf{r}) = -\sum_{\mathbf{r}_s,\mathbf{r}_g}\int d\omega \Re\left(\omega^2 \mathcal{F}_s(\omega) G(\mathbf{r},\mathbf{r}_s) G(\mathbf{r}_g,\mathbf{r}) \delta P^*\right)$$

Plane-wave Encoded Gradient

While it is extremely expensive to construct the gradient using the traditional shot by shot method. The plane-wave source method has been introduced to construct the gradient:

$$g^{(n)}(\mathbf{r}) = -\int d\omega \Re\left\{\omega^2 \mathcal{F}_s(\omega) \tilde{G}(\mathbf{r},\mathbf{r}_s) G(\mathbf{r}_g,\mathbf{r}) \delta P^*\right\}$$

where $\tilde{G}(\mathbf{r},\mathbf{r}_s) = G(\mathbf{r},\mathbf{r}_s) e^{i\omega p_s (x_s' - x_s)}$ and p_s is the ray parameter.

For plane-wave encoded gradient, when the sources are densely distributed, few crosstalk noise can be observed.

Phase Encoded Pseudo-Hessian

The poorly scaled gradient can be enhanced by the pseudo-Hessian, which can be constructed using phase encoding method. And Preconditioning the gradient using the diagonal part of the phase encoded pseudo-Hessian can be considered as an approximation to the deconvolution imaging condition in reverse time migration. The phase encoded Hessian can be expressed as:

$$H_{en\_p}^{(n)} = \sum_{\mathbf{p}_s,\mathbf{r}_s}\int d\omega \Re\left\{\omega^4 G(\mathbf{r}'',\mathbf{r}_s) G^*(\mathbf{r}',\mathbf{r}_s) e^{i\omega(p_s+\varepsilon\triangle p)(x_s'-x_s)}\right\}$$

when $\mathbf{r}'' = \mathbf{r}'$, we can get the diagonal part of the phase encoded pseudo-Hessian.

Numerical Example

Firstly, we applied the proposed strategies on a modified Marmousi model. The true velocity model and initial velocity model are shown in Fig.1a and b respectively.

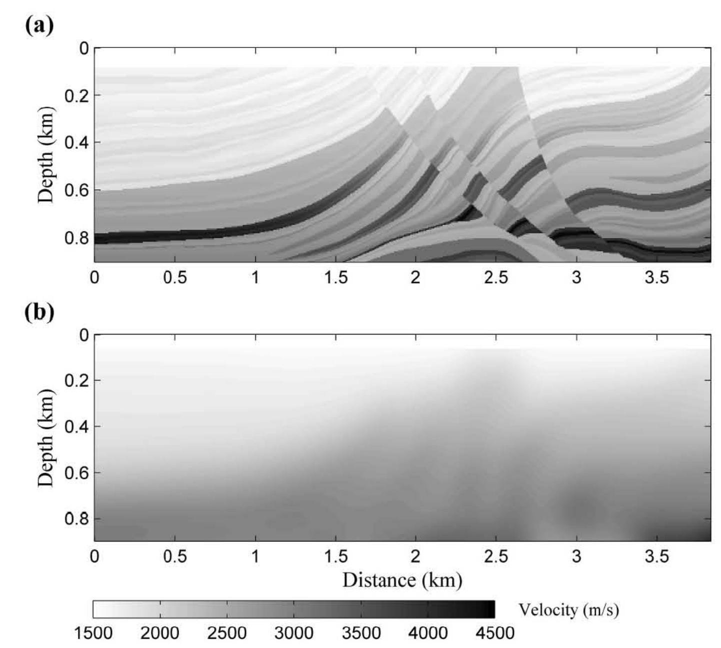

Fig.1. Velocity model

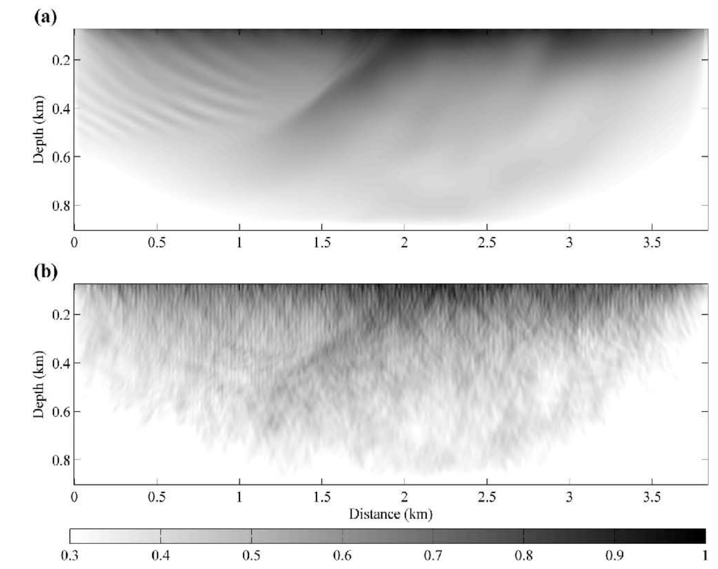

Fig.2. Diagonal parts of the pseudo-Hessian

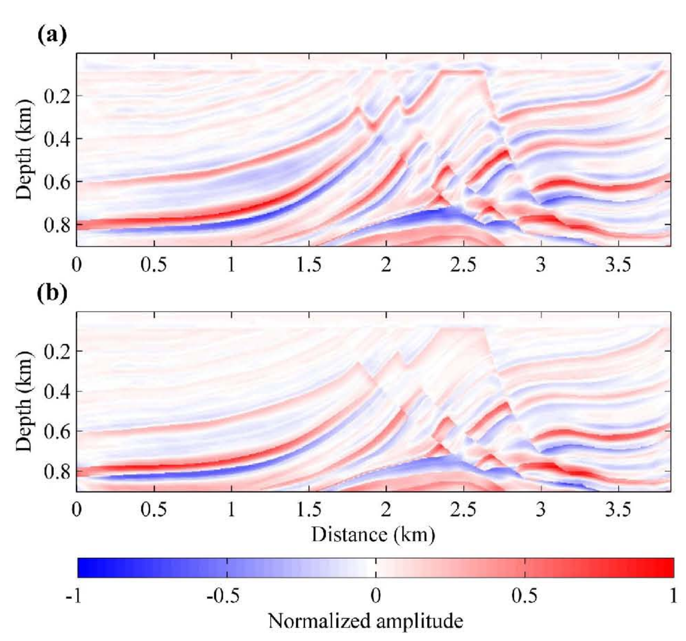

Fig.3. Gradient Errors

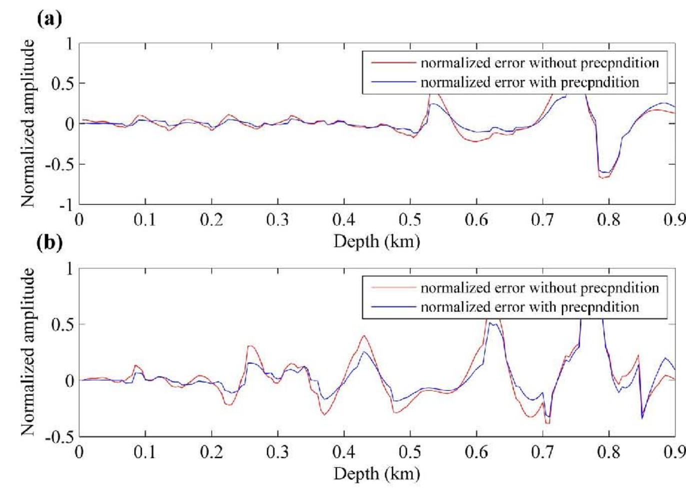

Fig.4. Gradient Errors at 1km and 3km respectively

Fig.2a and b show the diagonal parts of pseudo pseudo-Hessian and the phase encoded pseudo-Hessian. Fig.3a and b show the errors for the gradients without precondition and with precondition respectively. Fig.4 shows the errors for the gradients without precondition and with precondition at 1km and 3km respectively. We can see that the errors for the gradient with precondition are more close to zero.

Numerical Example

Fig.5a and b show the FWI results without precondition and with precondition respectively. We can recognize that the deep parts of the inversion result has been enhanced with the pseudo-Hessian precondition.

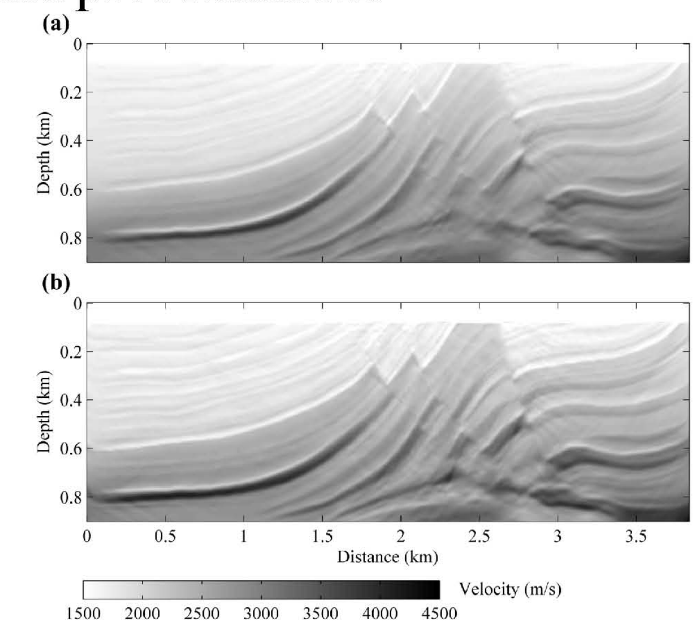

Fig.5. Inverted results without precondition (a) and with precondition (b).

The inversion results in Fig.5 were obtained using the iteration-dependent sets of ray parameters. As we discussed in the report, different ray parameters are responsible to update the subsurface layers with different steep angles. We tested the idea using another numerical example which consists of the subsurface layers with similar steep angles.

Fig.6a, b and c show true velocity model, initial velocity model and inverted velocity model using one fixed ray parameter p=-0.1s/km. We can see that the model was inverted very well.

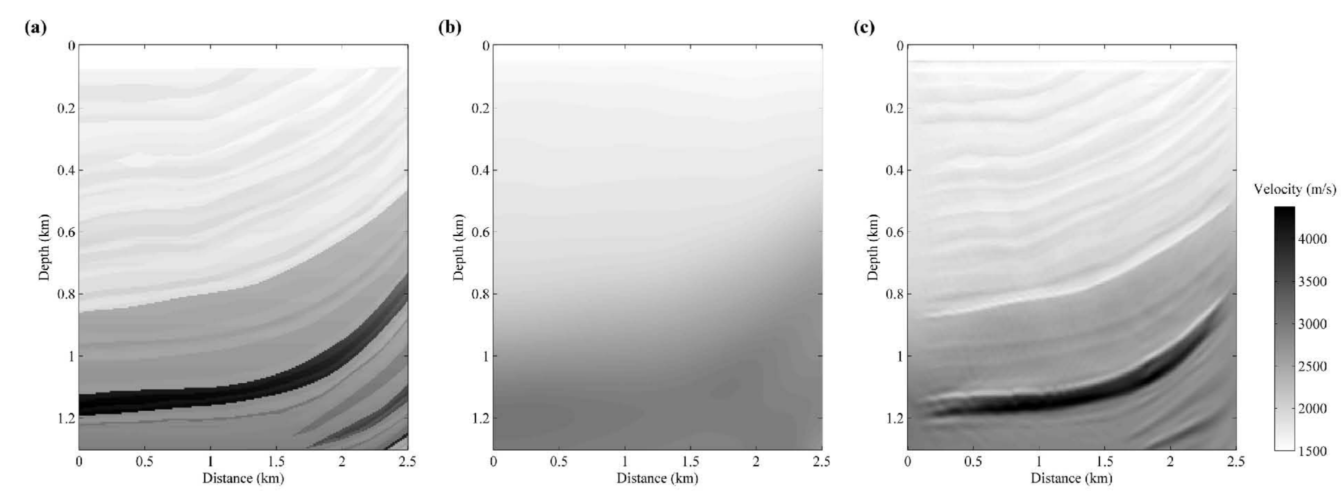

Fig.6 Inversion result with one ray parameter p=-0.1s/km.

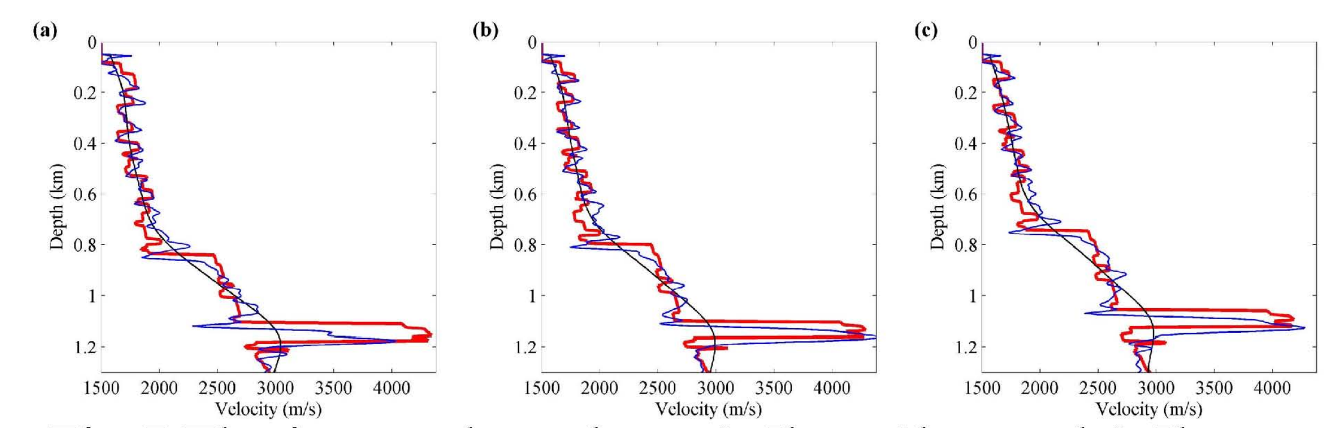

Fig.7 The inverted results at 0.5km, 1km and 1.5km.

Acknowledgements

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