A Turning-ray algorithm using Slotnick's equation

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Abstract

Turning-ray tomography is a good tool for estimating near surface velocity structure, especially in areas where conventional refraction statics fail such as in the case of a hidden layer. In a previous paper (Arenrin et al, 2014), we already demonstrated this by applying a tomostatics solution to Hussar 2D seismic line using Landmark's PROMAX software. In this paper we have developed a turning-ray tracing algorithm that uses Slotnick's equation. The algorithm traces turning rays through a linear velocity v(z) medium, and has the option of specifying the top layer velocity, takeoff angles, and the subsurface velocity gradient. However not in its final product, the turning-ray traveltimes from the algorithm are in agreement with the traveltimes generated using an acoustic finite-difference forward-modelling code.

Method

The turning-ray tracing algorithm is designed using Slotnick's equation for a linear v(z) medium. The receiver locations are specified just as is the case of a finite-difference forward-modelling code. Our method solves for z in Equation (1), calculates the lengths of raypaths from shots to receivers and builds the matrix of lengths of raypaths. The travel times are then computed using Equation (2),

$$x = \frac{1}{pc}\left\{\sqrt{1-p^2 v_0^{\,2}} + \sqrt{1-p^2\left(v_0+cz\right)^2}\right\} \qquad (1)$$

$$\mathbf{t} = \mathbf{Dn}, \qquad (2)$$

where x is the receiver location, p is the ray parameter, v_0 is the top layer velocity, c is the velocity gradient, z is the depth a ray has penetrated the subsurface at the receiver location, $\mathbf{D}$ is the matrix of the lengths of raypaths, $\mathbf{n}$ is the slowness vector and $\mathbf{t}$ is the computed traveltime vector.

Examples

Four synthetic shot records were generated using the acoustic finite-difference forward-modelling code from the CREWES toolbox. The shot spacing is 20 meters and the receiver spacing is 2 meters. The velocity model for forward-modelling has a cell size of 2 meters by 2 meters and a dimension of 800X800 cells. For ray tracing, the cell size is up scaled to 20 meters by 20 meters, given a velocity model with a dimension of 80X80cells. The minimum and maximum takeoff angles for turning rays are 40 and 75 degrees respectively.

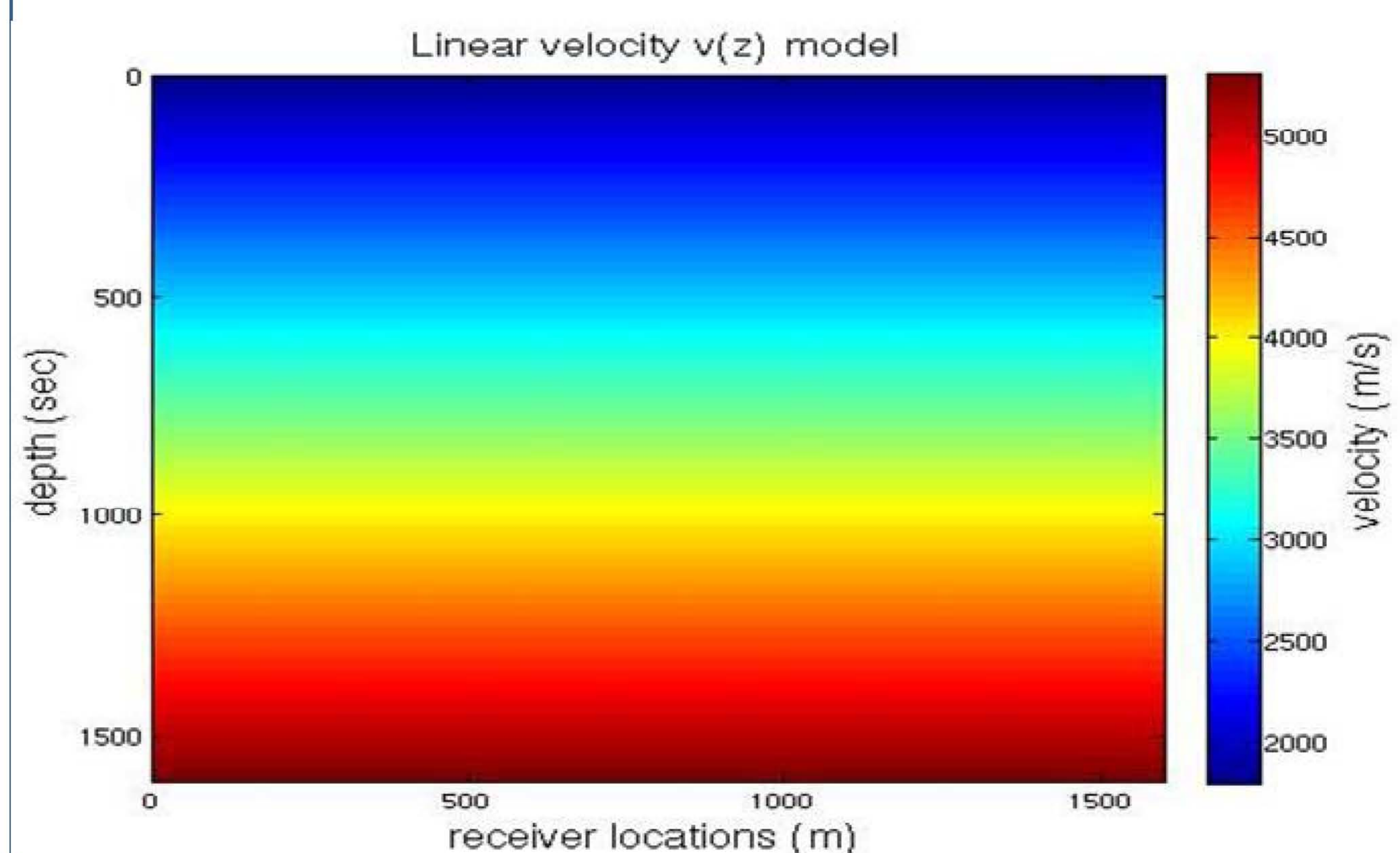

FIG. 1. Linear velocity v(z) model for ray tracing and for the synthetic shot records. The velocity model is 1598 meters wide and 1598 meters deep..

Finite-difference shot records and Turning ray tracing

The left panels in the figure below show direct arrivals and turning rays from the four shot records. Superimposed on the shot records are the calculated traveltimes-using our algorithm (magenta) to build the matrices of lengths of raypaths and applying Equation 2. On the right panels are the turning rays for the shots using our algorithm. The figure shows that there is a good match between the traveltimes from finite-difference modelling and the calculated traveltimes using the ray tracing algorithm.

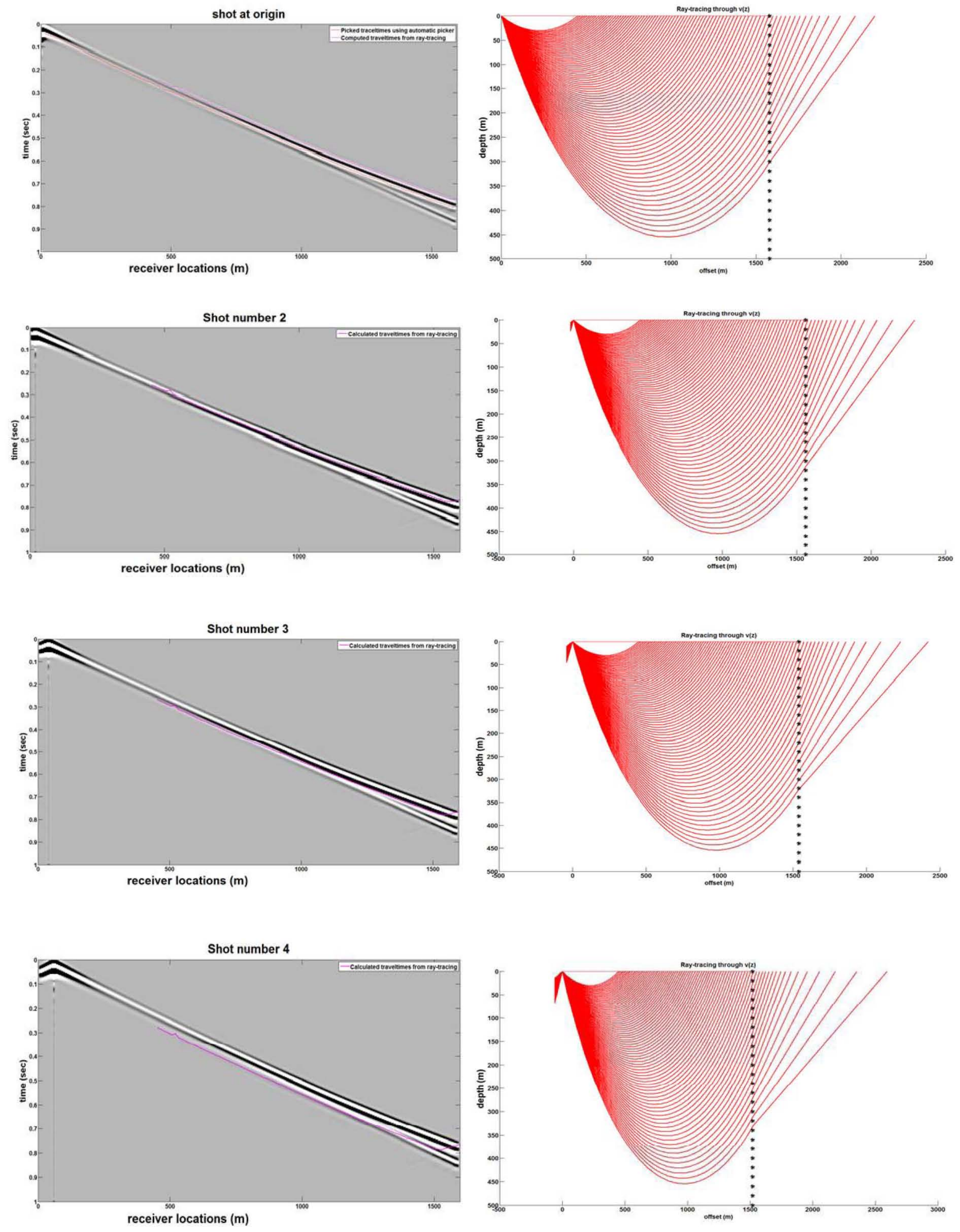

FIG. 2. Finite-difference shot records for the four shots showing tuning rays (left panels). Traveltimes from our algorithm superimposed on the shots (magenta). Modelling turning rays for the four shots using our algorithm (right panels). The stared lines are the positions of the maximum offsets for each shot.

Matrix of lengths of ray paths for the shot at the origin

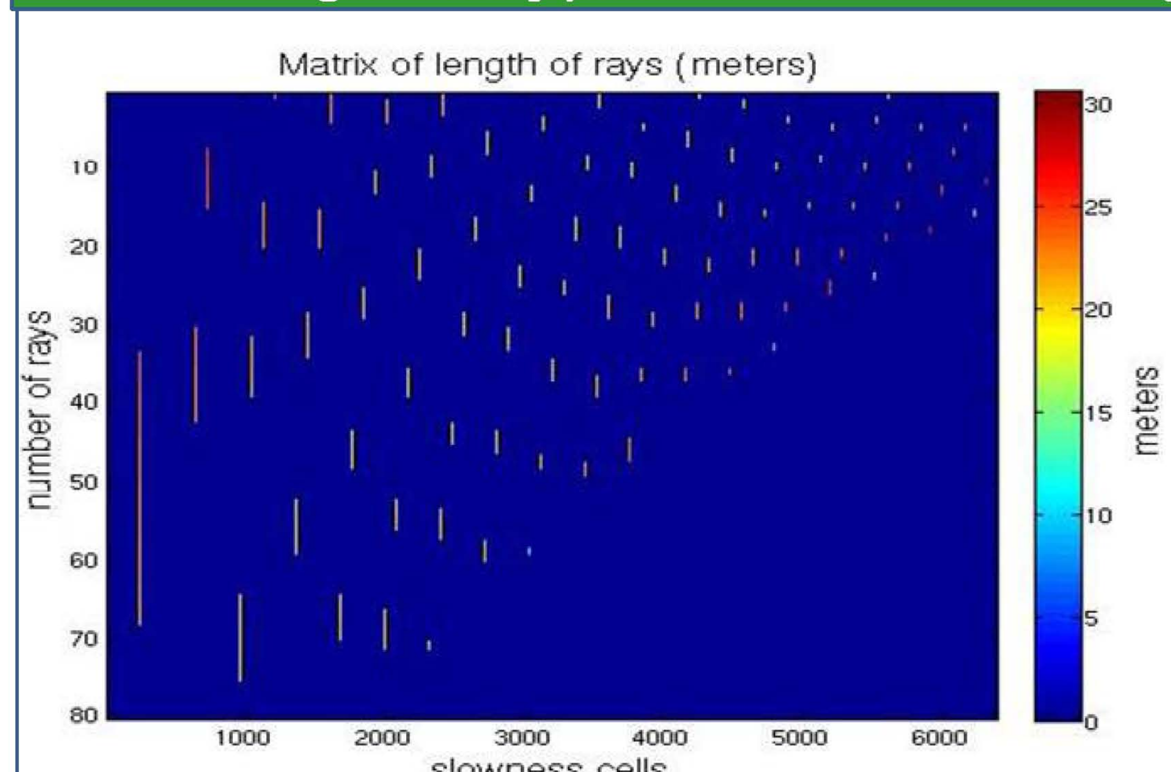

FIG. 3. The matrix of lengths of raypaths for the shot at the origin.

Matrices of lengths of raypaths for shots number 2,3 and 4

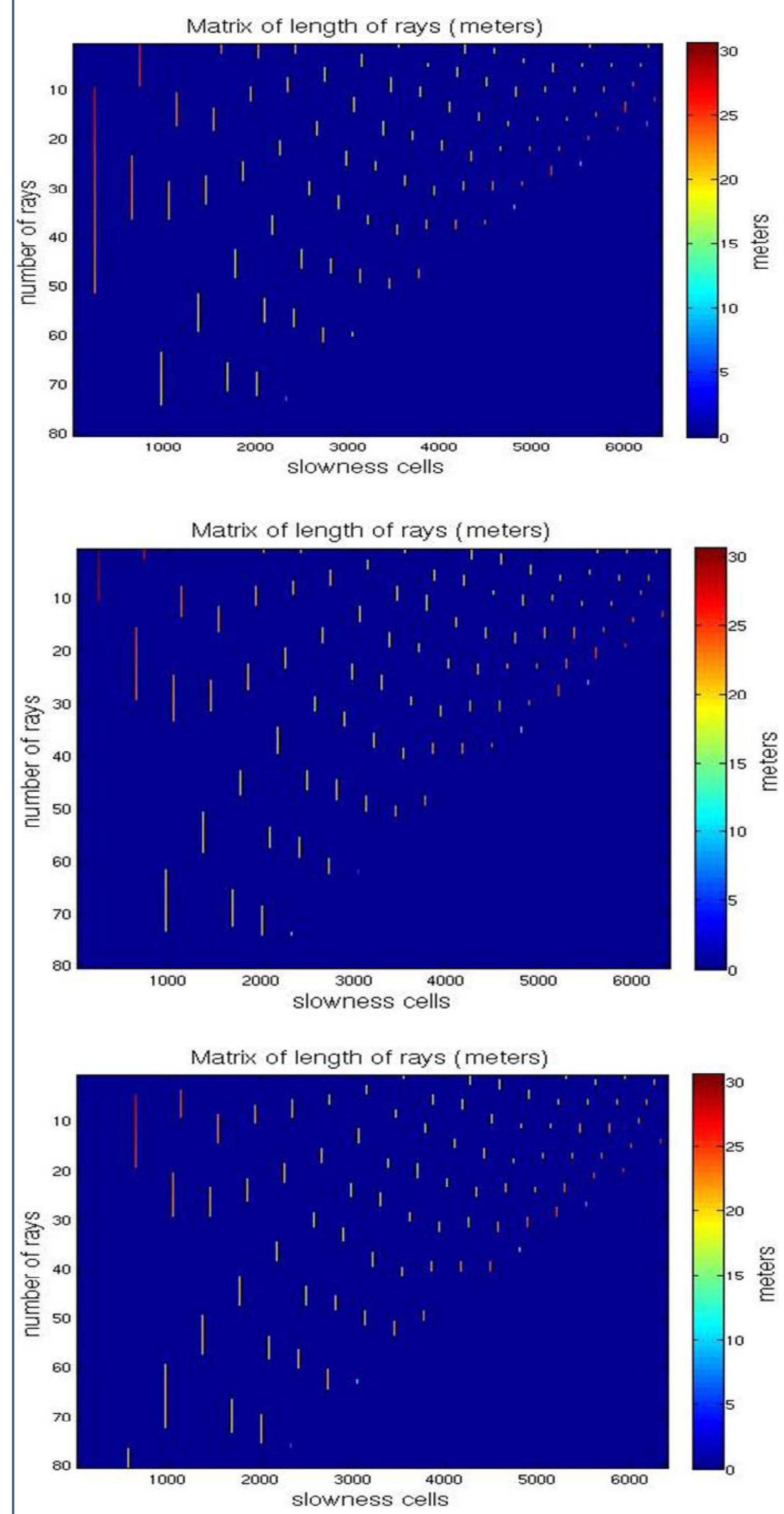

FIG. 4. The matrix of lengths of raypaths for; shot number 2 (top), shot number 3 (middle) and shot number 4 (bottom).

Conclusions

We have developed a turning-ray tracing algorithm for a linear velocity v(z) medium that uses Slotnick's equation. For the four shots we tested, the calculated traveltimes from turning-ray tracing match the traveltimes generated using finite-difference modelling.

Acknowledgements

We thank the sponsors of CREWES for their support. We also gratefully acknowledge support from NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 379744-08.