Seismic wavelet estimation

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ABSTRACT

The seismic wavelet is the important link between seismic data and stratigraphy as well as rock properties of the subsurface. Seismic wavelet estimation is done to deconvolve the seismic trace, tie the well log to the seismic data, design inversion operator and etc. to make seismic data better represent the geology. In this paper, four methods of seismic wavelet estimation are investigated as well as the influences of algorithm parameters and data types on it.

Stationary convolutional model

The seismic trace $s(t)$ can be modeled by the stationary convolution of the seismic wavelet $w(t)$ and the reflectivity $r(t)$ plus noise $n(t)$

$$s(t) = w(t) \bullet r(t) + n(t), \quad (1)$$

where all the nonstationary effects are ignored, such as wavefront spreading, transmission loss, multiples, attenuation, and etc.

Two kinds of idealized wavelets are common in geophysics, the minimum-phase wavelet and the constant-phase wavelet (Figure 1). A synthetic random time series and a real reflectivity calculated from the Hussar well 12-27 are created in Figure 2. Convolving a reflectivity with a wavelet, a seismic trace is created in Figure 3. We assume the event time of the well log is well calibrated to that of the seismic trace (aligned) or there is only a static time shift between them (misaligned).

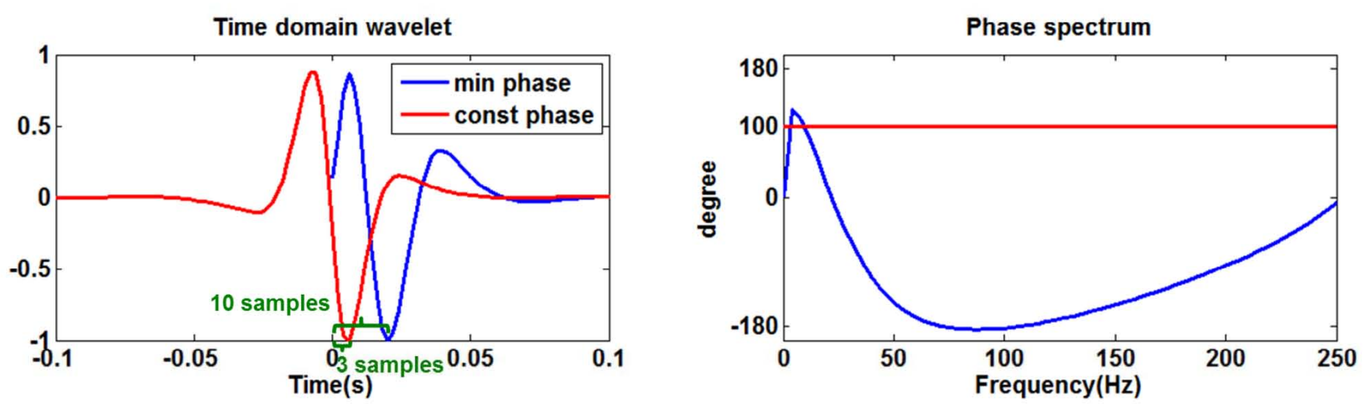

FIG 1: A minimum-phase wavelet and a 100-degree constant-phase wavelet with the same amplitude spectrum.

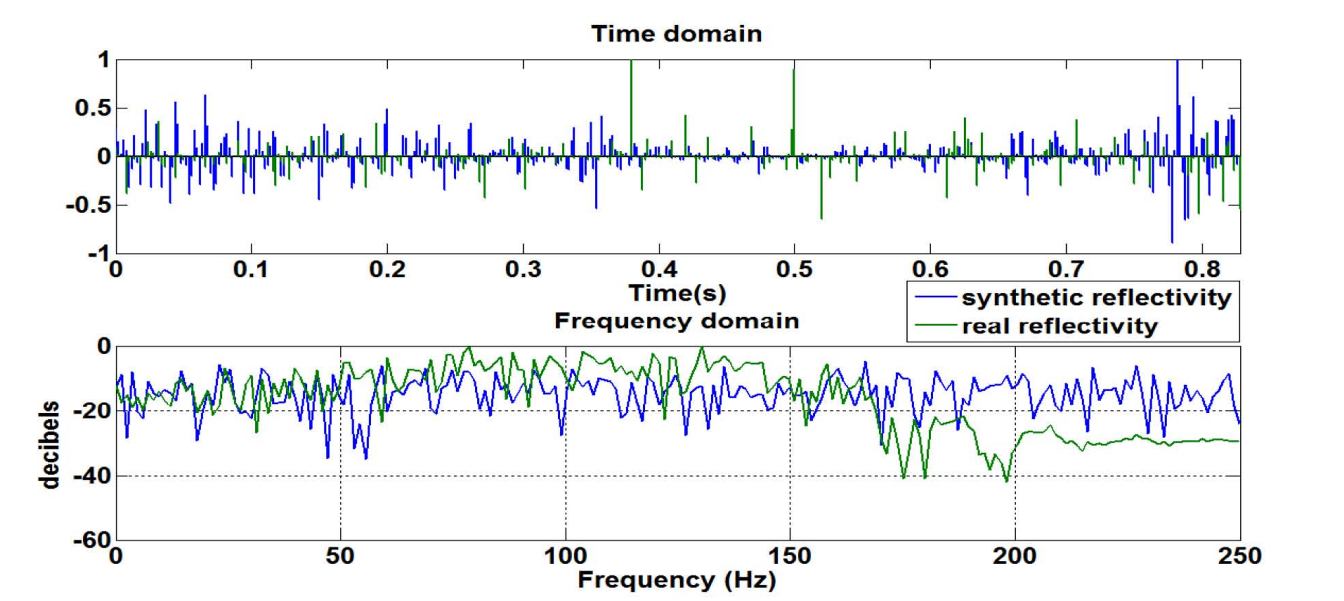

FIG 2: The synthetic reflectivity with a white spectrum and the real reflectivity with a colored spectrum.

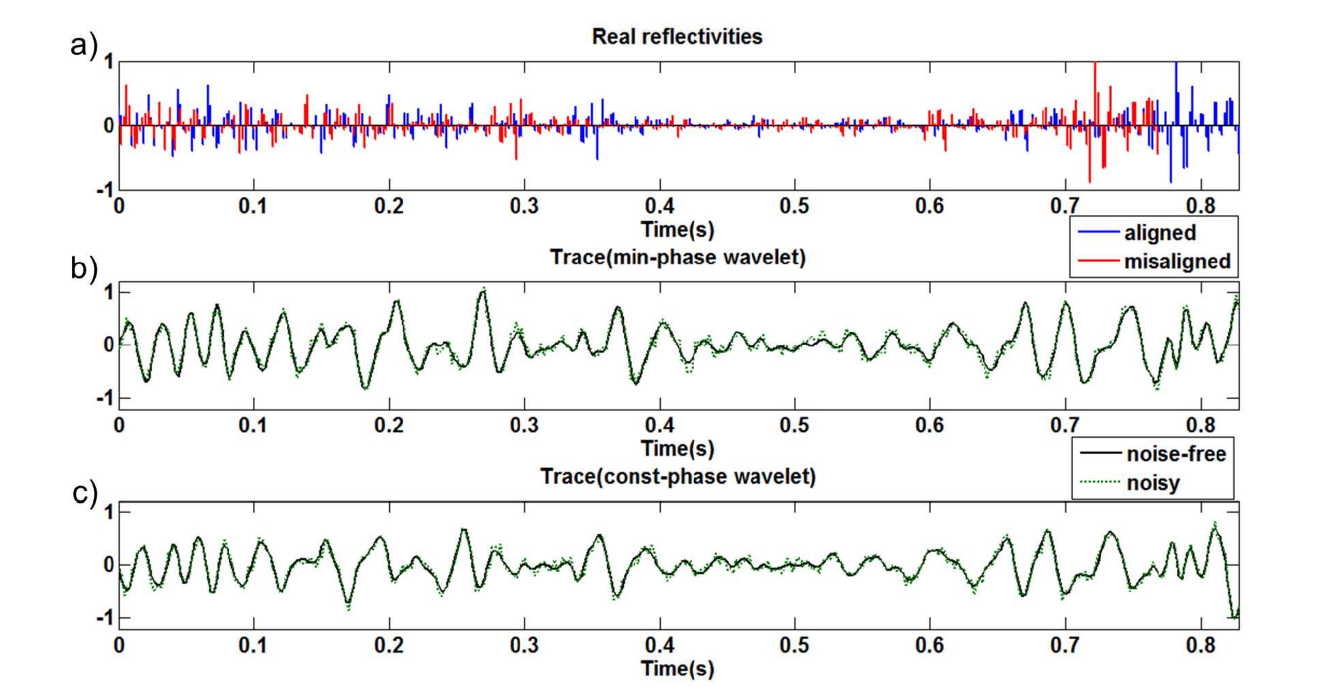

FIG 3: Reflectivities calculated from well 12-27. The red one is 30 samples (0.06 s) advanced relative to the blue one (a). Seismic traces from the convolution of the blue reflectivity with the minimum-phase wavelet (b). Seismic traces from the convolution of the blue reflectivity with the constant-phase wavelet. The signal to noise ratio is 3 in the noisy traces. Thus, the blue reflectivity is aligned with the seismic traces while the red one not.

The statistical method

The statistical method estimates the amplitude spectrum of the wavelet from the seismic trace alone. A phase spectrum has to be supplied. The procedure is:

1. **Extract $s(t)$ of a window length.**
2. **Calculate the autocorrelation of $s(t)$ for certain lags**

$$\phi_s(t) = \phi_w(t) \bullet \phi_r(t) \approx a\phi_w(t), \quad (2)$$

where $\phi_s(t), \phi_w(t), \phi_r(t)$ is the autocorrelation of $s(t), w(t), r(t)$; a is a constant.

3. **Apply a Gaussian or Bartlett window to $\phi_s(t)$ to smooth the amplitude spectrum.**
4. **In the frequency domain, equation 3 is: $|S(f)|^2 \approx a\,|W(f)|^2$ (3)**
5. **Supply a minimum-phase or constant-phase $\varphi_w(f)$: $W(f) = |W(f)|e^{i\varphi_w(f)}$ (4) where the minimum-phase is calculated by the Hilbert Transform H**

$$\varphi_w(f) = H[\ln(|W(f)|)] \quad (5)$$

6. **Calculate the inverse Fourier Transform: $w(t) = IFT[W(f)]$ (6)**

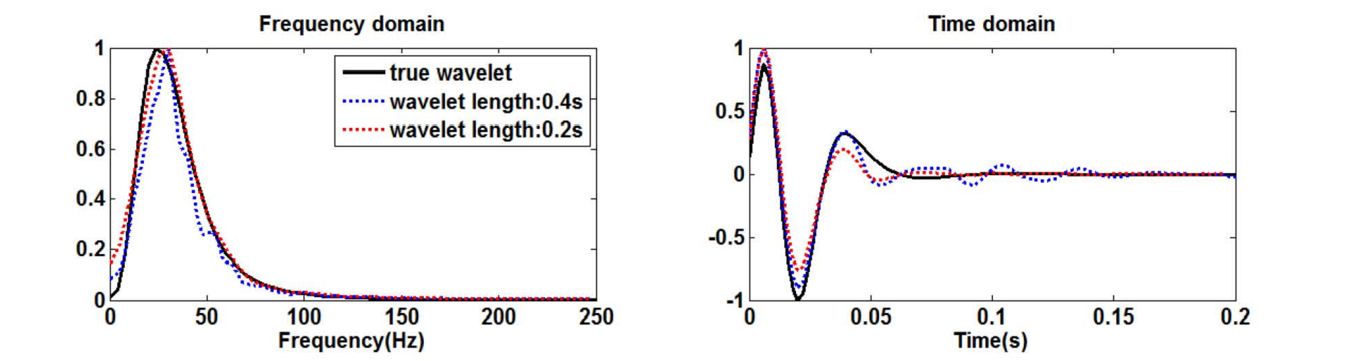

FIG 4: A shorter desired wavelet length, namely less correlation lags, produces a smoother amplitude spectrum and a less oscillatory waveform.

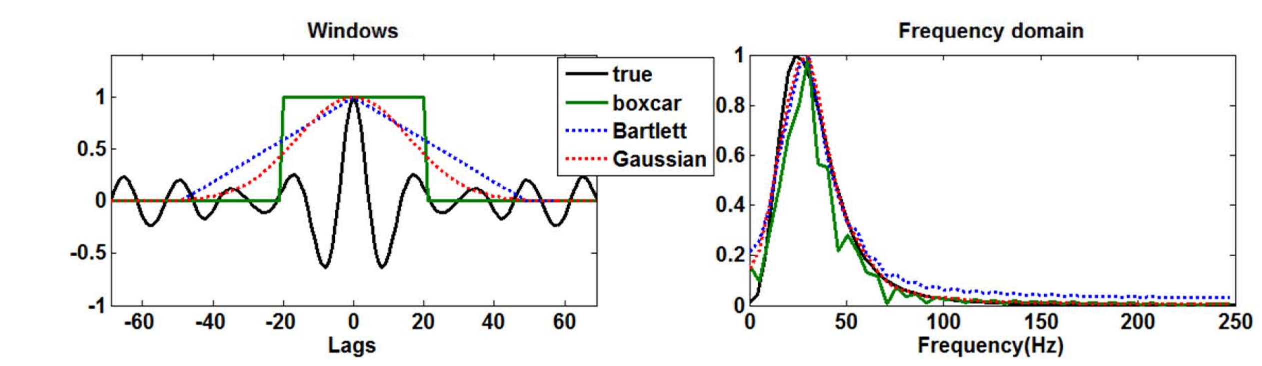

FIG 5: The Gaussian or Bartlett window can smooth the amplitude spectrum.

The full wavelet method

The full wavelet method uses both the seismic and reflectivity to determine the wavelet amplitude and phase spectra by applying a least-square filter which shapes the reflectivity to the seismic. The estimation procedure is:

1. **Extract $s(t)$ and $r(t)$ of a window length and choose a certain wavelet length.**
2. **Build the design equation 7 and derive the normal equation 8**

$$\begin{pmatrix} r(1) & r(0) & 0 \\ r(2) & r(1) & r(0) \\ r(3) & r(2) & r(1) \\ r(4) & r(3) & r(2) \\ 0 & r(4) & r(3) \end{pmatrix} \begin{pmatrix} w(-1) \\ w(0) \\ w(1) \end{pmatrix} = \begin{pmatrix} s(0) \\ s(1) \\ s(2) \\ s(3) \\ s(4) \end{pmatrix} \quad (7)$$

$$\begin{pmatrix} \phi_r(0) & \phi_r(1) & \phi_r(2) \\ \phi_r(1) & \phi_r(0) & \phi_r(1) \\ \phi_r(2) & \phi_r(1) & \phi_r(0) \end{pmatrix} \begin{pmatrix} w(-1) \\ w(0) \\ w(1) \end{pmatrix} = \begin{pmatrix} \phi_{rs}(-1) \\ \phi_{rs}(0) \\ \phi_{rs}(1) \end{pmatrix} \quad (8)$$

3. **Calculate $\phi_r(t)$ and $\phi_{rs}(t)$. Solve equation 8 to get the estimated wavelet.**

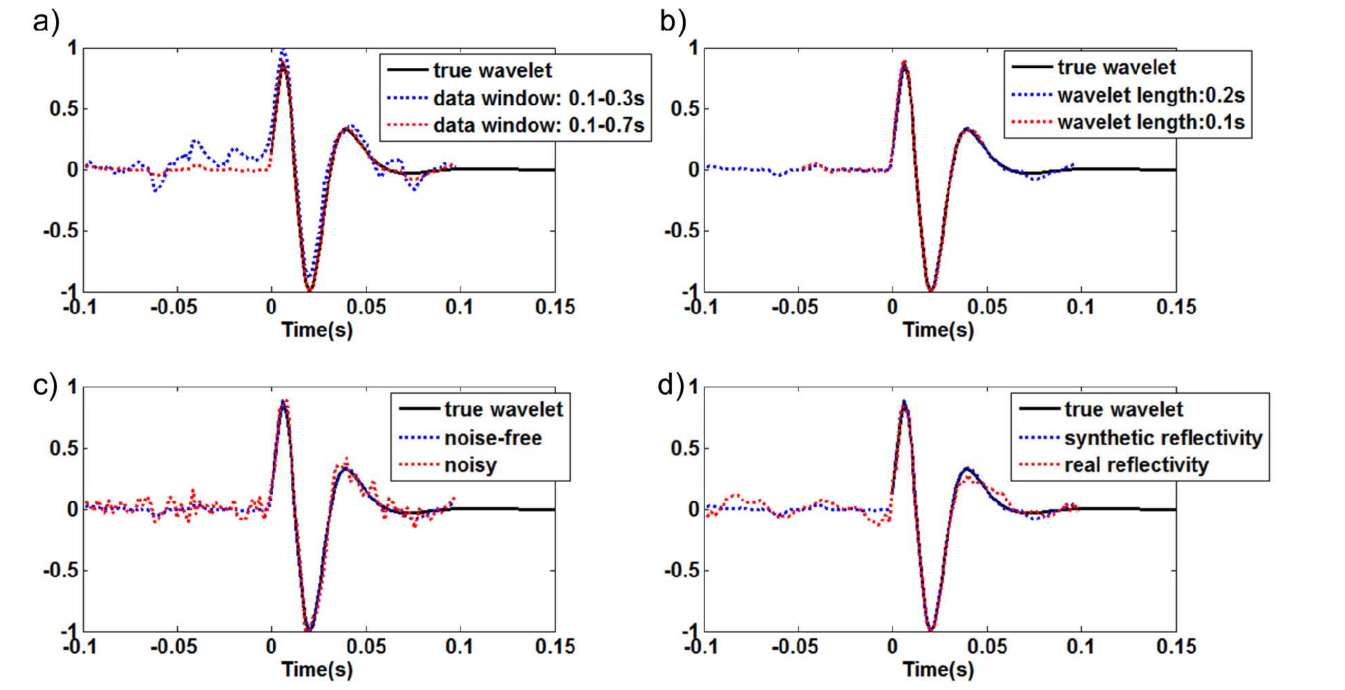

FIG 6: A longer data window gives a more accurate estimation (a). The desired wavelet length does not affect the waveform (b). The wavelet estimated from a noisy trace has unrealistic trembling (c). Wavelets estimated from the synthetic and real reflectivities are similar (d).

The constant phase method

The constant phase method uses the seismic trace and the reflectivity to estimate the wavelet by constraining its phase to an approximate constant. The procedure is:

1. **Estimate the amplitude spectrum of the wavelet by the statistical method.**
2. **Apply a series of constant-phase rotations ranging from -180 to 179 degrees.**
3. **For each rotation, calculate $w(t)$, convolved with $r(t)$ to get a synthetic trace. Calculate the maximum crosscorrelation coefficient between it and $s(t)$.**
4. **Choose the maximum coefficient among the 359 ones. The corresponding phase is the wavelet phase and the lag indicates a time shift between $s(t)$ and $r(t)$.**

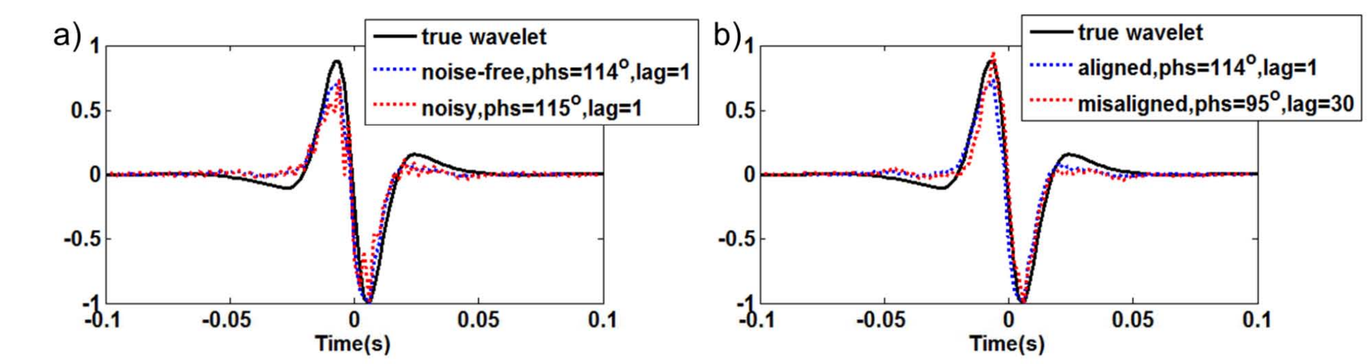

FIG 7: Wavelet estimation on $S(t)$ created by the real $r(t)$ and the 100-degree wavelet. The waveform is contaminated by noise but the phase is almost correct (a). Wavelet estimation from the misaligned reflectivity detects the correct time shift between $S(t)$ and $r(t)$.

The Roy White method

The Roy White method estimates the wavelet by correlating $r(t)$ and $s(t)$.

1. Search the best tie location using the coherence function

$$G(m) = \frac{\left|\sum_f R^*(m,f)S(f)\right|^2}{\sum_f [R^*(m,f)R(m,f)+stab]\sum_f S^*(f)S(f)} \quad (9)$$

where * is complex conjugate, $R(m,f)$ is the Fourier transform of $r(t)$ shifted by m samples relative to $s(t)$. $S(f)$ is the Fourier transform of $s(t)$.

We search for N to maximize the coherence function

$$G(N) = \max[G(m)], -M \le m \le M \quad (10)$$

2. Estimate the wavelet at the best tie location

$$W(f) = \frac{R^*(N,f)S(f)}{R^*(N,f)R(N,f)+stab} \quad (11)$$

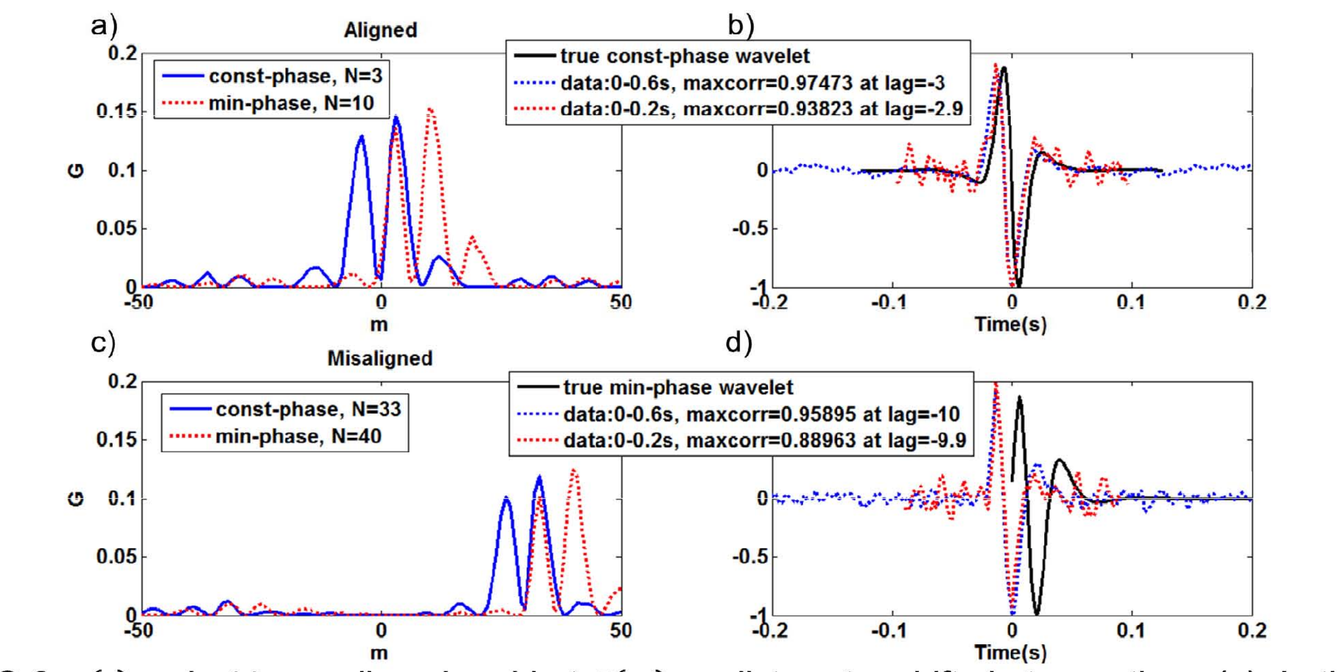

FIG 8: $s(t)$ and $r(t)$ are aligned and but $G(m)$ predicts extra shifts between them (a). In the misaligned case (c), the detected shifts are the sum of the real shifts (30) and the extra shifts in a), which are caused by the embedded waveform (FIG 1) since $G(m)$ gets maximum by matching the wavelet troughs. The extra shifts are compensated by the same shifts in the estimated wavelets in the opposite direction in (b and d).

CONCLUSIONS

Methods	Statistical	Full wavelet	Constant phase	Roy White
Data used	Seismic	Seismic and well log		
Data window length	Longer data length gets more accurate estimation			Data window length=wavelet length, longer length causes trembling side lobes
wavelet length	Shorter length produces smoother amplitude and less trembling waveform	No influence	Shorter length produces smoother amplitude and more stable waveform	
Noise	Robust	trembling waveform	Robust in estimating phase and shifts, trembling waveform	Robust in estimating shifts, trembling waveform
Real log reflectivity	Distorted	Robust	Robust	Robust
Misalignment	N/A	Robust	Robust	Robust

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