Series analysis of anisotropic reflection coefficients for inversion

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Abstract

Azimuthal AVO analysis is typically performed using linearizations of the exact formula for anisotropic reflection coefficients. These approximations often make simplifying assumptions about the types of media on each side of an interface and fail at large angles, especially when there is a large contrast in elastic parameters across the interface. Since the larger angles of incidence are more sensitive to azimuthal anisotropy, this failure can cause poor estimates of azimuthal anisotropy. In order to better understand and reduce the nonlinearity that can adversely affect inversions using linearizations, we analyze higher-order terms of the reflection coefficients. We show that the nonlinearity for large contrasts and long offsets is significant, indicating the need to use exact reflection coefficients in many situations.

R_{PP} along individual azimuths

Shuey (1985) showed that the linearized reflection coefficient for PP reflections at small incidence angles in isotropic media could be written in the form

$$R_{PP}^{iso}(\theta) = A + B\sin^2\theta + C\tan^2\theta\sin^2\theta, \quad (1)$$

where A is the AVO intercept, B is the AVO gradient, and C is the AVO curvature. Vavryčuk and Pšenčík (1998) calculated linearized reflection coefficients in this form for an interface between weak, generally anisotropic media using perturbations from background P-wave velocities, α, and S-wave velocities, β:

$$\begin{aligned} R_{PP}(\theta_P) =& \frac{\rho\Delta A'_{33} + 2\alpha^2\Delta\rho}{4\rho\alpha^2} \\ &+ \frac{1}{2}\left[\frac{\Delta A'_{33}}{2\alpha^2} - \frac{4(\rho\Delta A'_{55} + \beta^2\Delta\rho)}{\rho\alpha^2} + \Delta\delta^{*\prime}\right]\sin^2\theta_P \\ &+ \frac{1}{2}\left(\frac{\Delta A'_{33}}{2\alpha^2} + \Delta\epsilon^{*\prime}\right)\sin^2\theta_P\tan^2\theta_P, \end{aligned} \quad (2)$$

where

$$\delta^{*\prime} = \frac{A'_{13} + 2A'_{55} - A'_{33}}{\alpha^2}, \epsilon^{*\prime} = \frac{A'_{11} - A'_{33}}{2\alpha^2}, \quad (3)$$

and $A_{\alpha\beta}$ is Voigt notation for the density-normalized elastic parameters. The ' symbol denotes that the parameters are in the coordinate system which is rotated to be aligned with the vertical plane containing the source and receiver.

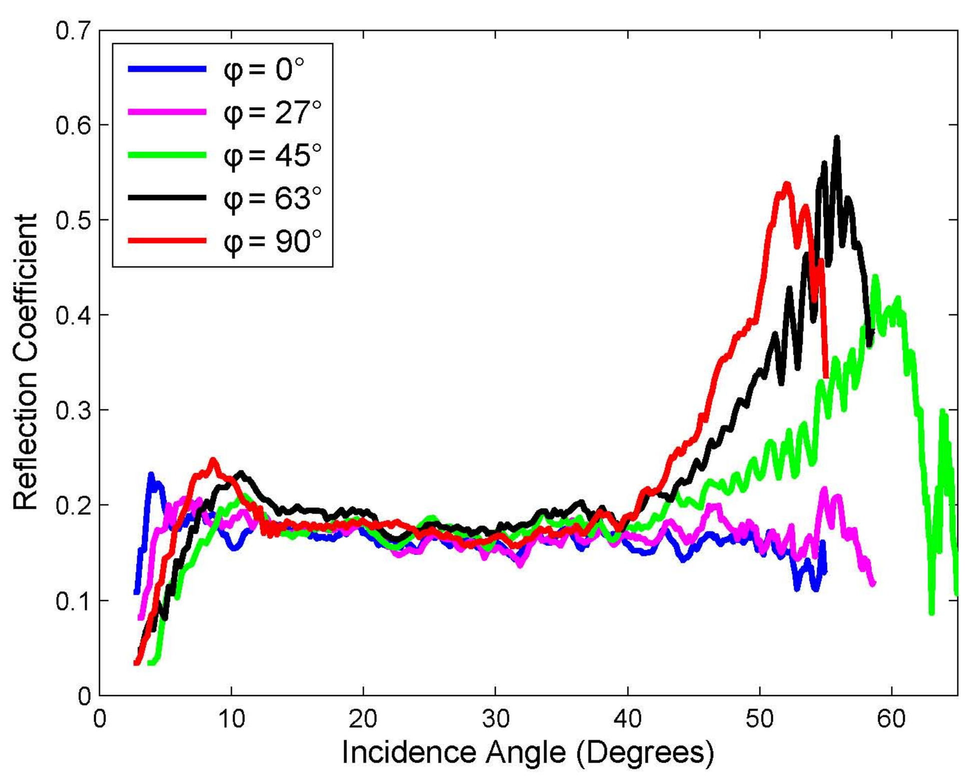

Figure : 1. Azimuthal reflection data from physical modeling from Mahmoudian (2013). The azimuthal anisotropy is most noticeable at large angles where the waves are traveling increasingly horizontally. Unfortunately, as the incidence angle increases, the waves become closer to critical, the reflection coefficients become progressively nonlinear and the error in equation 2 grows larger (see Figure 3).

Error in linearization

Figures 2 & 3 demonstrate the error in the estimated AVO intercept, gradient, and curvature, as a function of azimuth, due to the linearization from Vavryčuk and Pšenčík (1998) given in equation 2.

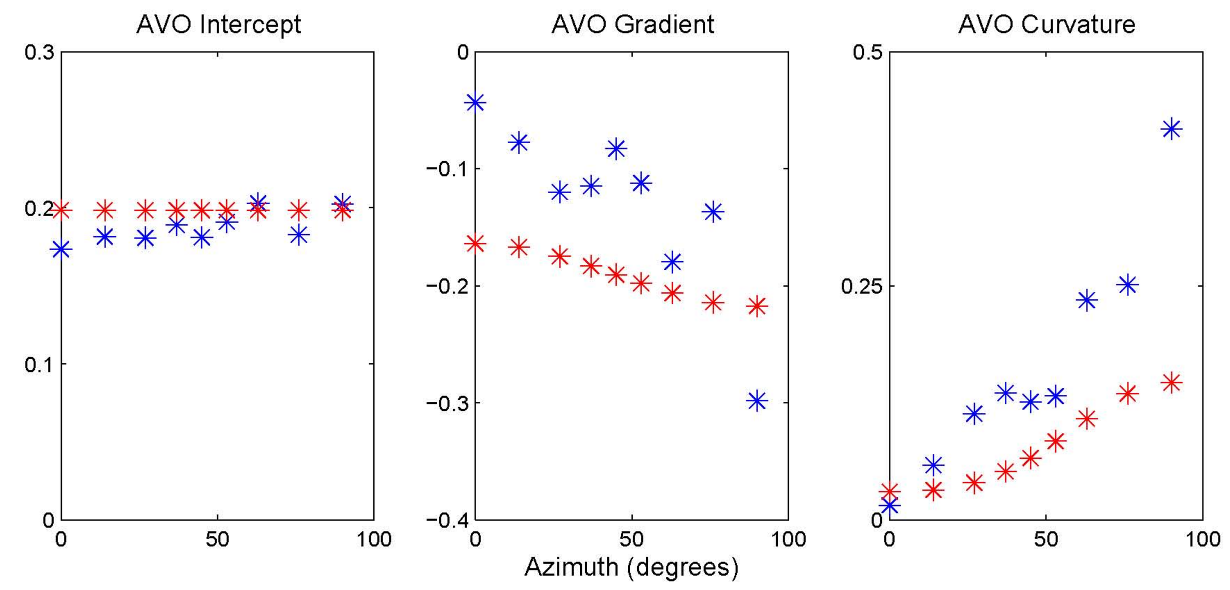

Figure : 2. AVO intercept, gradient, and curvature estimations vs. linearization along individual azimuths. (blue) AVO quantities estimated by solving for best-fitting coefficients A, B, and C from equation 1 for physical modeling data shown in Figure 1. (red) AVO quantities calculated using estimated elastic parameters from Mahmoudian (2013) with the linearization from equation 2.

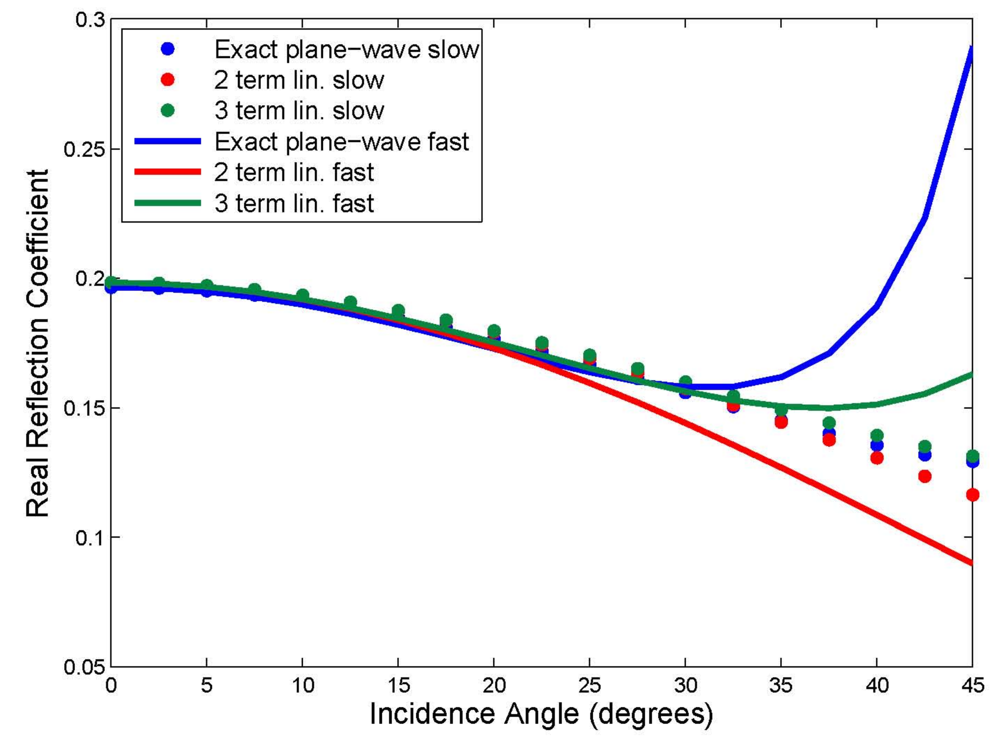

Figure : 3. Comparison of anisotropic reflection coefficient linearization to exact plane-wave coefficient for model with elastic parameters from Mahmoudian (2013). (blue) exact plane-wave anisotropic reflection coefficients. (green) linearizations from Vavryčuk and Pšenčík (1998) (eq. 2). (red) linearizations using only the first two angle terms in eq. 1. Solid lines indicate an azimuth of 90 which is the fast direction of the lower medium, representing the direction parallel to a fractured medium. Dotted lines indicate an azimuth of 0 which is the slow direction of the lower medium, representing the direction perpendicular to a fractured medium.

Parameterization

We perform the expansion by parameterizing the reflection coefficient in terms of the horizontal slowness, p, squared (p^2) and perturbations that measure contrasts across the layers at the interface. The first step we perform is parameterizing all the elastic parameters in terms of the parameters of the top layer. We do this by defining 5 weak contrast parameters:

$$\delta\rho = 1 - \frac{\rho^{(1)}}{\rho^{(2)}}, \delta a_{11} = 1 - \frac{a_{11}^{(1)}}{a_{11}^{(2)}}, \delta a_{13} = 1 - \frac{a_{13}^{(1)}}{a_{13}^{(2)}} \cdots, \quad (4)$$

where a_{11}, a_{13}, a_{33}, and a_{55} are density-normalized stiffness coefficients and the superscripts (1) and (2) refer to the upper and lower layers. This allows us to define the second layer parameters in terms of the top layer parameters:

$$\rho^{(2)} = \frac{\rho^{(1)}}{1-\delta\rho}, a_{11}^{(2)} = \frac{a_{11}^{(1)}}{1-\delta a_{11}}, a_{13}^{(2)} = \frac{a_{13}^{(1)}}{1-\delta a_{13}} \cdots. \quad (5)$$

Series expansion

Expanding the formula for exact VTI reflection coefficients from Graebner (1992) in the weak contrast parameters from equation 4 and p^2 results in

$$\begin{aligned} R_{PP} =& \frac{1}{4}(\delta a_{33} + 2\delta\rho) + \frac{a_{13}^{(1)}(a_{13}^{(1)} + a_{55}^{(1)})}{2(a_{33}^{(1)} - a_{55}^{(1)})}p^2\delta a_{13} - \frac{a_{33}^{(1)}(a_{13}^{(1)} + a_{55}^{(1)})^2}{4(a_{33}^{(1)} - a_{55}^{(1)})^2}p^2\delta a_{33} \\ &+ \frac{1}{8}(\delta a_{33})^2 - \frac{(a_{13}^{(1)} + a_{33}^{(1)})^2 a_{55}^{(1)}}{4(a_{33}^{(1)} - a_{55}^{(1)})^2}p^2\delta a_{55} - \frac{a_{55}^{(1)}(a_{13}^{(1)} + a_{33}^{(1)})(a_{13}^{(1)} + a_{55}^{(1)})}{(a_{33}^{(1)} - a_{55}^{(1)})^2}p^2\delta\rho \\ &+ \frac{1}{4}(\delta\rho)^2 + \cdots. \end{aligned} \quad (6)$$

Equation 6 is written to 2nd order in the perturbation parameters. The comparison of equation 6 to the linearization using background medium properties is shown in Figure 4. Average medium properties can be formed as a series expansion in our small contrast parameters (as shown in Innanen (2013) for isotropy):

$$\begin{aligned} \frac{\Delta V_{P_V}}{V_{P_V}} &= \frac{1}{2}\delta a_{33} + \frac{1}{4}(\delta a_{33})^2 + \frac{5}{32}(\delta a_{33})^3 + \cdots, \\ \frac{\Delta V_{P_H}}{V_{P_H}} &= \frac{1}{2}\delta a_{11} + \frac{1}{4}(\delta a_{11})^2 + \frac{5}{32}(\delta a_{11})^3 + \cdots, \end{aligned} \quad (7)$$

etc. V_{P_V} refers to the vertical P-velocity and V_{P_H} refers to the horizontal P-velocity (there is only one since this formulation is for a single vertical plane).

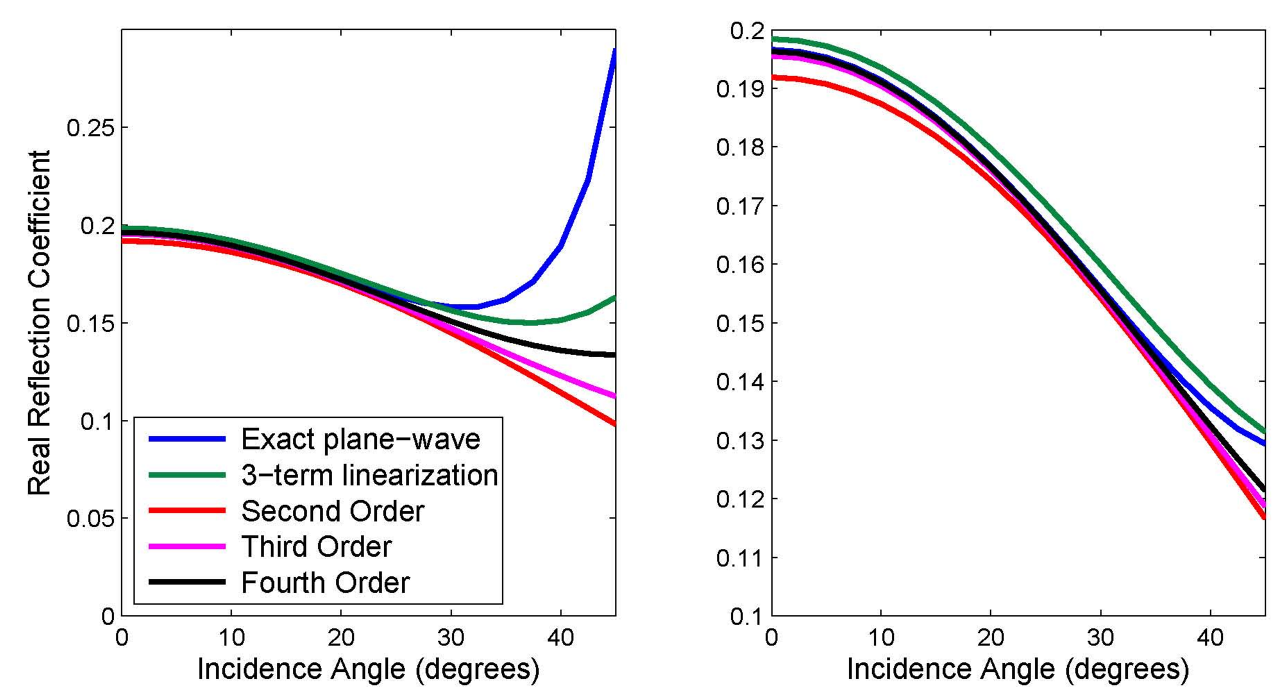

Figure : 4. Comparison of series expansion from eq. 6 using upper layer parameters to equation 2 from Vavryčuk and Pšenčík (1998) using background medium terms. (left) Fast direction (of the bottom layer) of the synthetic model. (right) Slow direction (of the bottom layer) of the synthetic model. The 3-term linearization using average medium properties approximates the plane-wave reflection coefficient better than our fourth order (lower order when not counting theta terms) at large angles of incidence for large contrasts.

Conclusions

- There is a large amount of nonlinearity hidden in the linearization's parameterization using background media.
- Our result is in agreement with isotropic AVO theory in the precritical region for which there is also a large degree of increasing nonlinearity.
- In practice, using exact reflection coefficients may be necessary to model nonlinearity at large angles where azimuthal anisotropy has its largest influence.

References

Please see the report for a full reference list.