Non-linear Vibroseis models for generating harmonics

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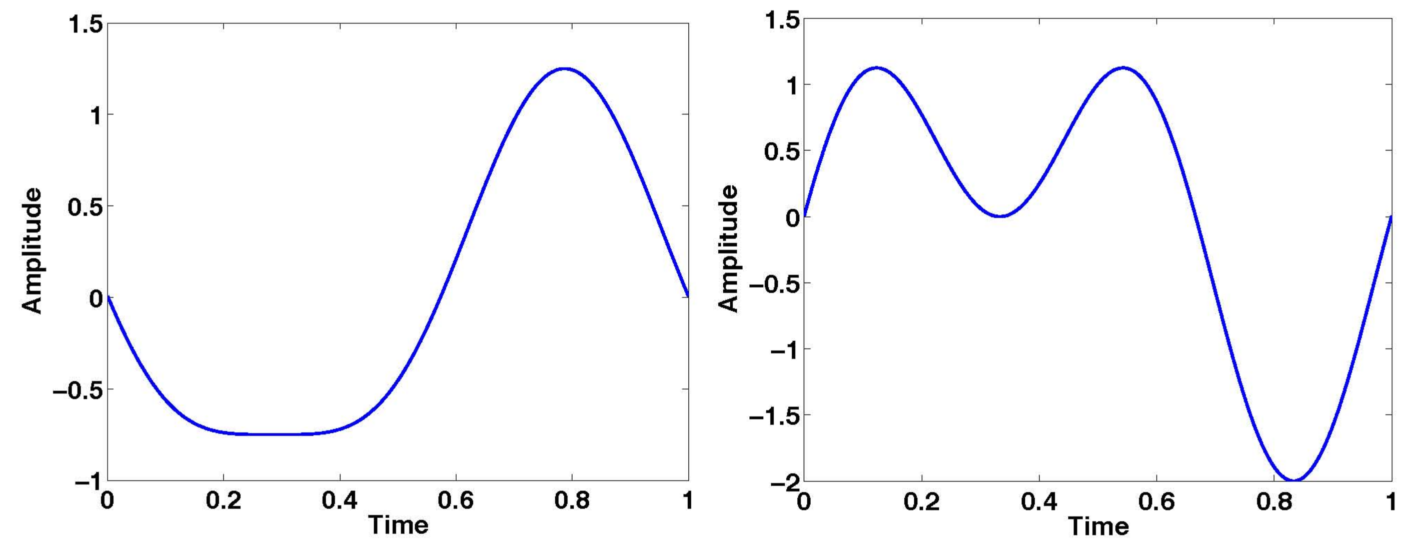
Overview

Vibroseis devices are a convenient source of seismic energy. For a variety of reasons, they produce harmonics even when driven by a pure sinusoid. These harmonics may be considered as noise, or as extra correlated data that might be used in the imaging algorithms. The goal of this project is to produce several simple mathematical models to understand where these harmonics come from.
In real seismic experiments, one often sees asymmetric waveforms in baseplate recordings, such as the following:



Observed waveforms: baseplate motion, acceleration.

These can arise as simple harmonic sums for the displacement and acceleration:

$$x(t) = -(\sin(2\pi t) + .25\sin(4\pi t + \pi/2))$$
$$x''(t) = (\sin(2\pi t) + \sin(4\pi t + \pi/2)) \cdot 4\pi^2$$

We would like to create a simple model in ordinary differential equations (ODE) that will reproduce such harmonics.

One-mass models

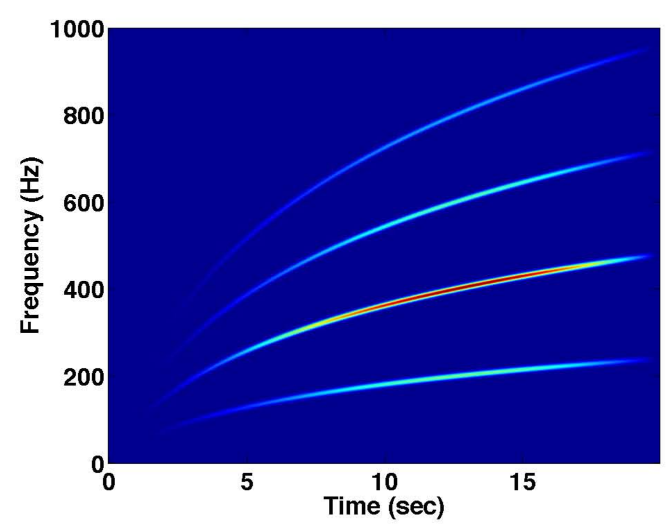
A simple linear oscillator, a mass on a spring , is given by the ODE

$$mx''(t) + bx'(t) + kx(t) = f(t),$$

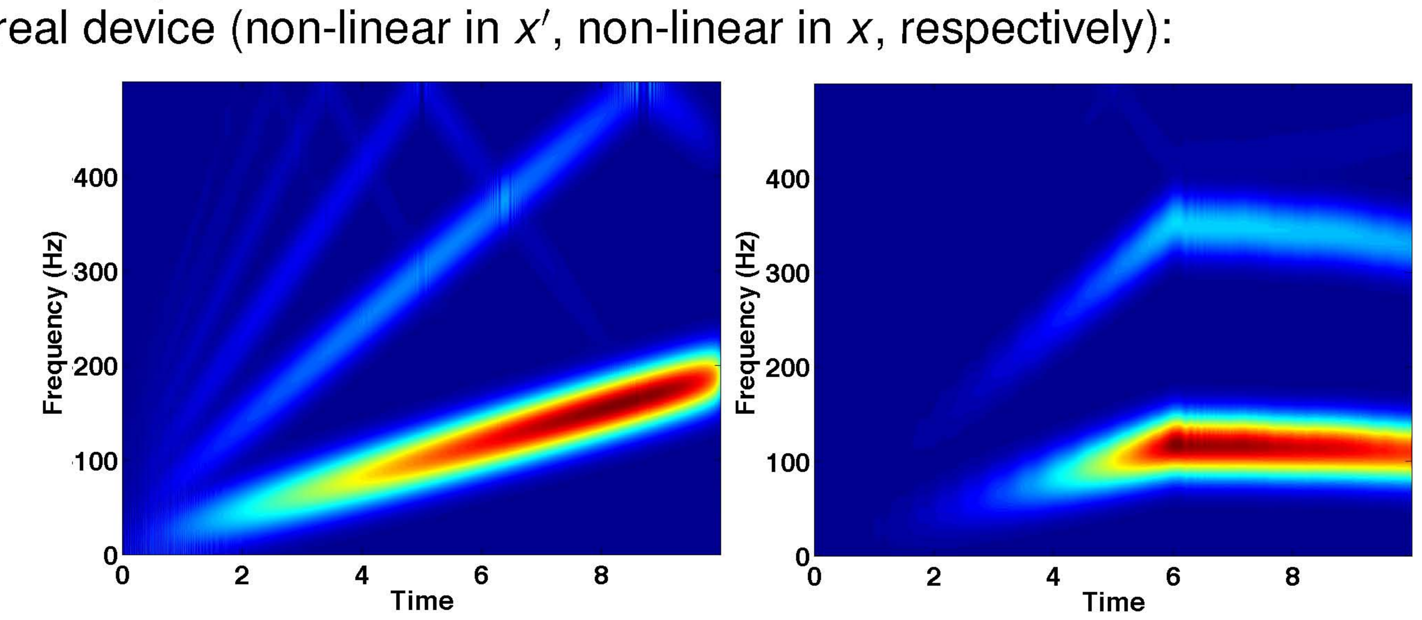
where the forcing term in vibroseis is a sweep of sinusoid,

$$f(t) = \sin(2\pi t \cdot freq(t)).$$

A real vibroseis device shows a non-linear response, with a range of harmonics appearing:

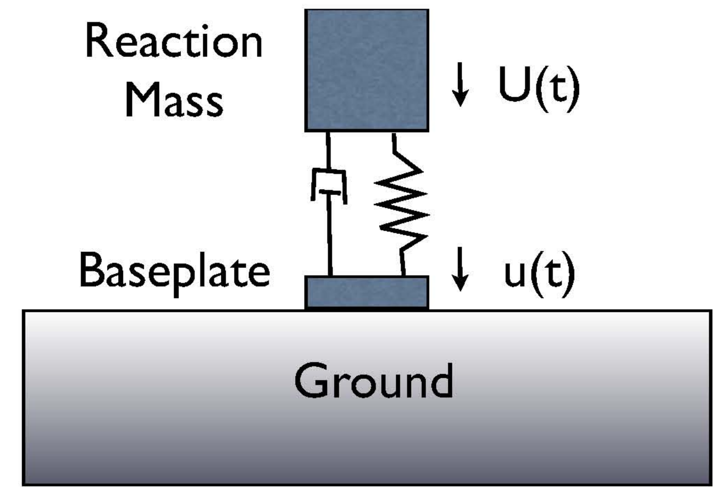


Our best attempts at modelling with one non-linear oscillator gives the following time-frequency responses, which are not representative of the real device (non-linear in x', non-linear in x, respectively):



Two-mass models

Following Easley (1995) we produce a simple two-mass model that includes the motion of both the baseplate and reaction mass.

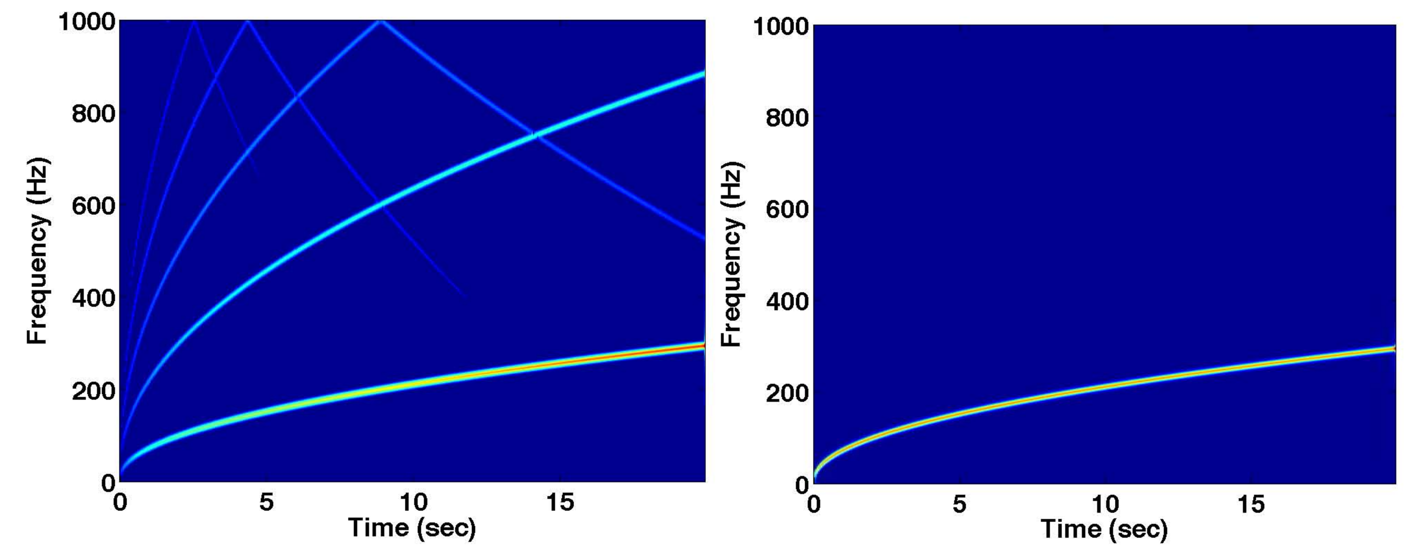


Vibroseis: reaction mass, baseplate, spring, dashpot.

The ODE system describing the motion includes their masses m, M of both, displacements u, U, a spring constant k and damping constant b connecting the two masses, and a force f that drives their motion. The pushback force from the earth is F and acts directly on the baseplate.

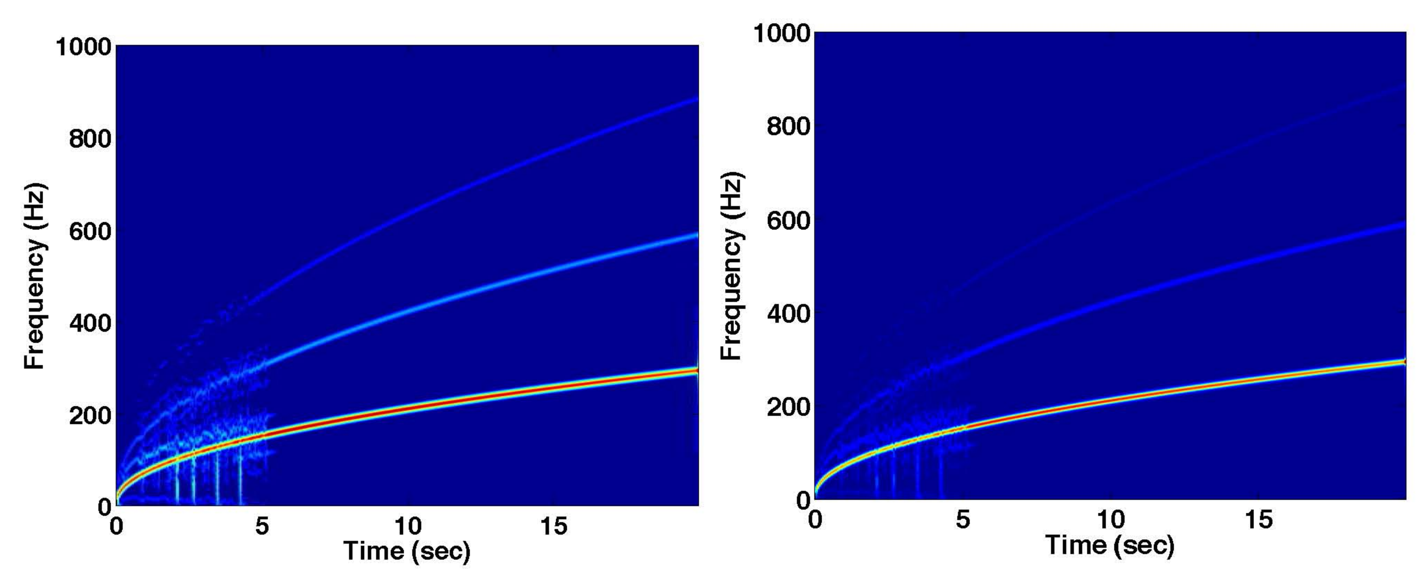
$$mu'' + b(u' - U') + k(u - U) = +f + F$$
$$MU'' - b(u' - U') - k(u - U) = -f$$

Including a sinusoidal driving force $f(t)$ with frequency range from 5 Hz to 200 Hz, and a nonlinear response F of the earth, we obtain time-frequency responses on the baseplate and reaction mass as follows:



Baseplate and reaction mass acceleration, non-linear earth.

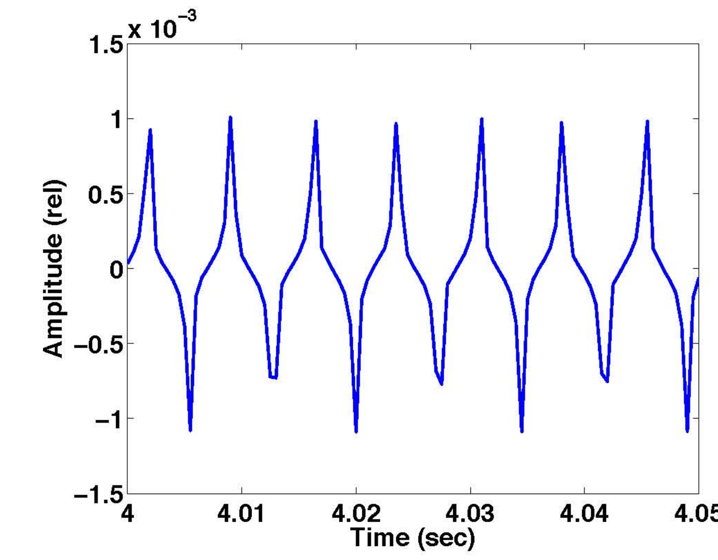
It is notable that the harmonics appear strongly in the baseplate, not so in the reaction mass. It is apparent that only the odd order harmonics are appearing in the baseplate, which is not quite what happens in a real baseplate recording where even and odd harmonics appear. The higher harmonics are clearly being aliased once they peak at the Nyquist frequency of 1000 Hz. This ia an artifact of the ODE solvers used in MATLAB.
It is also noteworthy that we did observe both odd and even order harmonics appear, when using low-order ODE solvers. These are less-accurate methods and the results were misleading. The harmonics persisted even when the ODE system was reduced to a linear system – for which there should be no harmonics. The following plot shows the strong presence of the 2nd harmonic, in both the baseplate and reaction mass data.



Baseplate, reaction mass acceleration, low order solver.

Waveform results

A plot of the waveforms produced by the non-linear two-mass model, demonstrates the strong presence of harmonics, as follows:



Baseplate acceleration, amplitude versus time.

Unfortunately, it is not the response we were hoping for. The waveform is very spiky, but from the real observed waveforms, we expect flattened, asymmetrical shapes.

Future work

The two-oscillator model is promising, and requires an approach to the earth's non-linearities that will give the full range of harmonics that is seen in real vibroseis data. Removing the welded-contact assumption is one approach, which allows for the possibility of a "pogo stick" effect where the truck loses contact with the ground. Another is to model an asymmetric non-linear earth response. There are also physical devices on the truck which constrain the motion of the reaction mass and baseplate – including these would represent another mechanism that could create harmonics. Some models in the literature include a third oscillating mass, as representing the effective ground mass.
More complex models as suggested by Wei (2010), include the flexing of the metal baseplate in order to accurately account for the harmonics. This would be an interesting model, but perhaps there are simpler models that will be successful.
Finally, there is the control machinery in the vibroseis device that could be included in the model.

Acknowledgements

We acknowledge support of NSERC and the CREWES sponsors through grant CRDPJ 379744-08, and an individual Discovery Grant. We also thank Art Siewert of StatOil and the PIMS/IMA Math Modeling workshop (2014) for introducing us to these vibroseis challenges.

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