VSP modelling in 1D with attenuation

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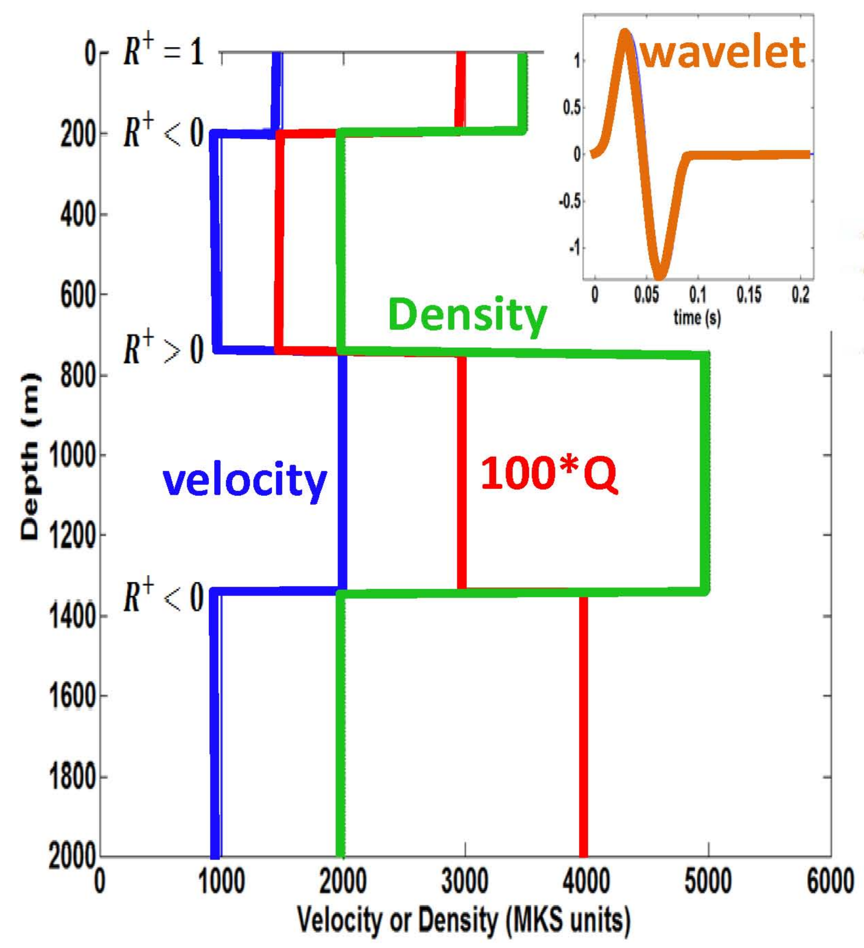
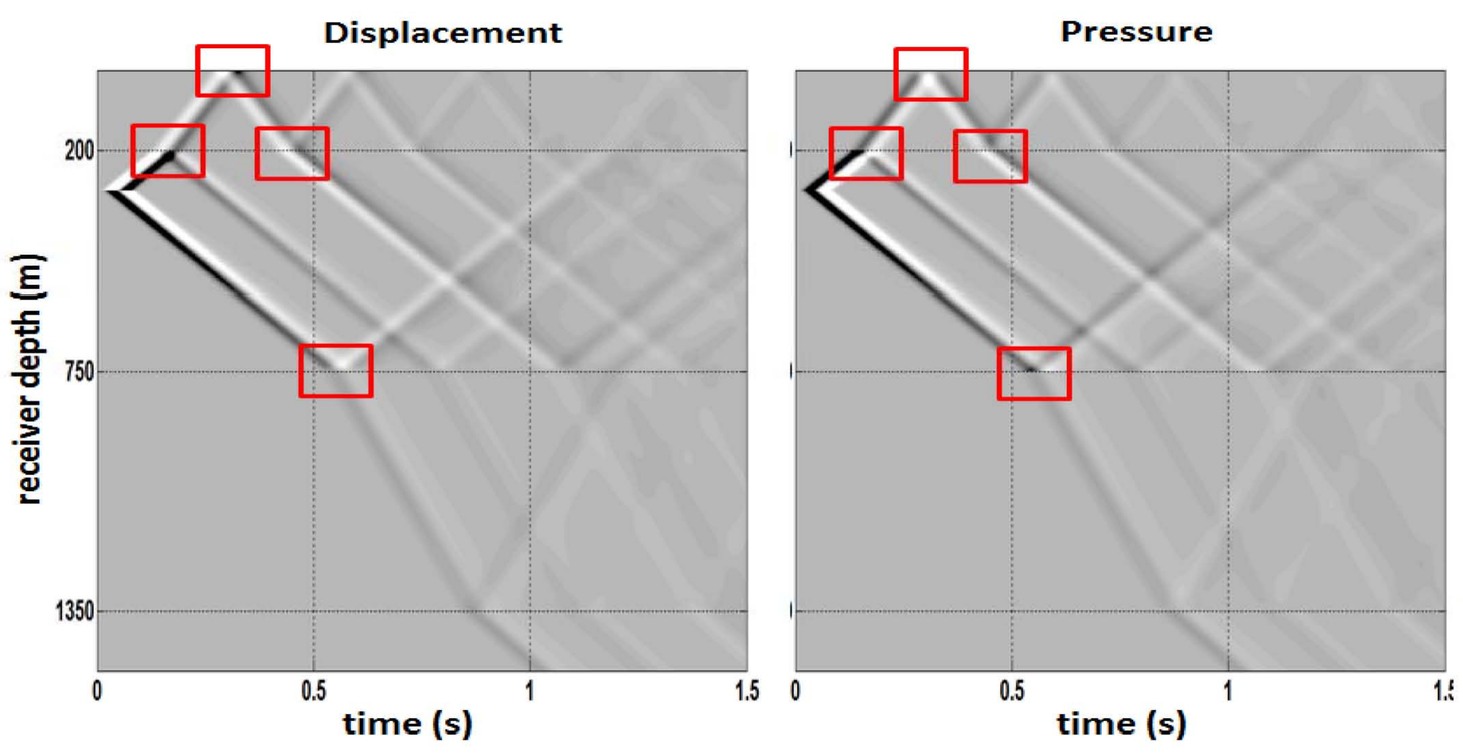
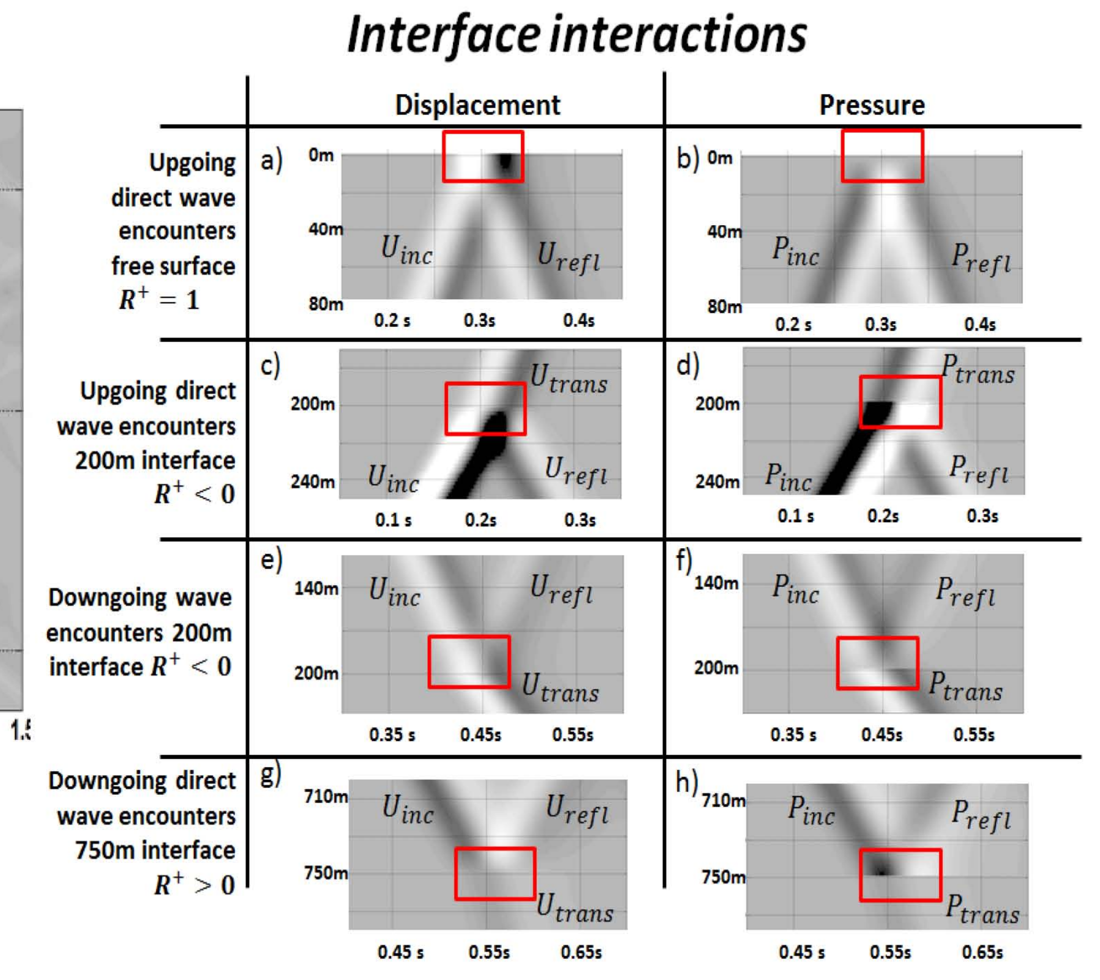
Summary: A method for the construction of a 1D VSP including the effects of all multiples and Q is described. The method uses propagator matrix theory and is capable of producing accurate results at high frequencies. Earth models from well logs with thousands of layers are easily accommodated. This source code is available to all sponsors.

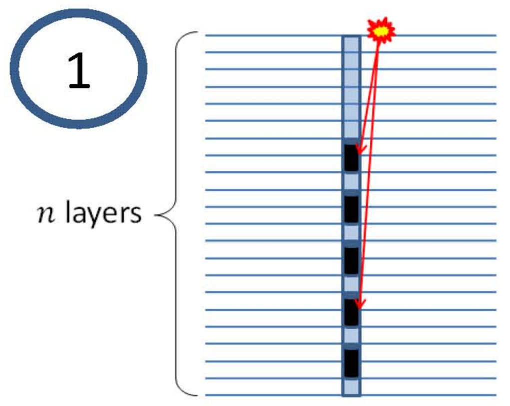






Construction of a Synthetic VSP after Ganley (1981, Geophysics)

1



1D layered earth, source at z=0, receivers at arbitrary depths.

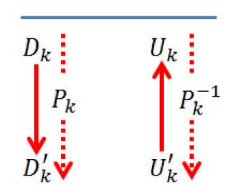
Include effects of transmission, reflection, multiples, and Q.

$n+1$ denotes half-space

2

Extrapolating across a Layer

frequency domain algorithm



D_k downgoing field at layer top

U_k upgoing field at layer top

D_k' downgoing field at layer bottom

U_k' upgoing field at layer bottom

Let P_k be the *layer propagator* that takes the downgoing wavefield from the top to the bottom of the layer as in

$$D_k' = P_k D_k.$$

Then P_k^{-1} takes the upgoing wavefield from the top to the bottom of the layer:

$$U_k' = P_k^{-1} U_k.$$

The layer propagator is given by

$$P_k = \underbrace{e^{-\pi f h_k/(Q_k v_k)}}_{\text{attenuation}} \underbrace{e^{-i2\pi f h_k/v_k}}_{\text{propagation}},$$

where **v_k is the frequency-dependent phase velocity**, h_k is the layer thickness, f is frequency, and Q_k is the layer attenuation parameter.

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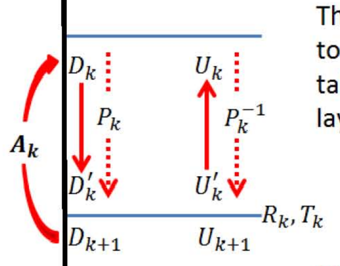
3

The layer matrix

displacement solution



The Propagator takes the solution at the top of a layer to the bottom of the same layer. The layer matrix $\boldsymbol{A}_k$ takes the solution at the top of layer k+1 to the top of layer k:

$$\begin{bmatrix} D_k \\ U_k \end{bmatrix} = \frac{1}{T_k} \begin{bmatrix} P_k^{-1} & -R_k P_k^{-1} \\ -R_k P_k & P_k \end{bmatrix} \begin{bmatrix} D_{k+1} \\ U_{k+1} \end{bmatrix}$$

or

$$\begin{bmatrix} D_k \\ U_k \end{bmatrix} = \boldsymbol{A}_k \begin{bmatrix} D_{k+1} \\ U_{k+1} \end{bmatrix}$$

where R_k is the frequency-dependent reflection coefficient for incidence from above

$$R_k = \frac{\rho_{k+1} v_{k+1} - \rho_k v_k}{\rho_{k+1} v_{k+1} + \rho_k v_k}$$

and

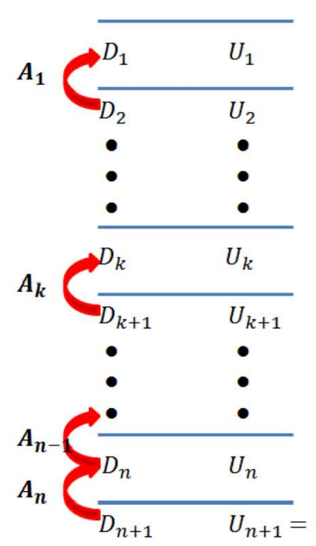
$$T_k = 1 - R_k$$

is the frequency-dependent transmission coefficient. 16

4

The VSP problem

Connecting the top and bottom



The solutions in the top layer and the bottom half-space are related by a product of layer matrices

$$\begin{bmatrix} D_1 \\ U_1 \end{bmatrix} = \boldsymbol{A}_1 \boldsymbol{A}_2 \cdots \boldsymbol{A}_{n-1} \boldsymbol{A}_n \begin{bmatrix} D_{n+1} \\ 0 \end{bmatrix}$$

Or

$$\begin{bmatrix} D_1 \\ U_1 \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} D_{n+1} \\ 0 \end{bmatrix}$$

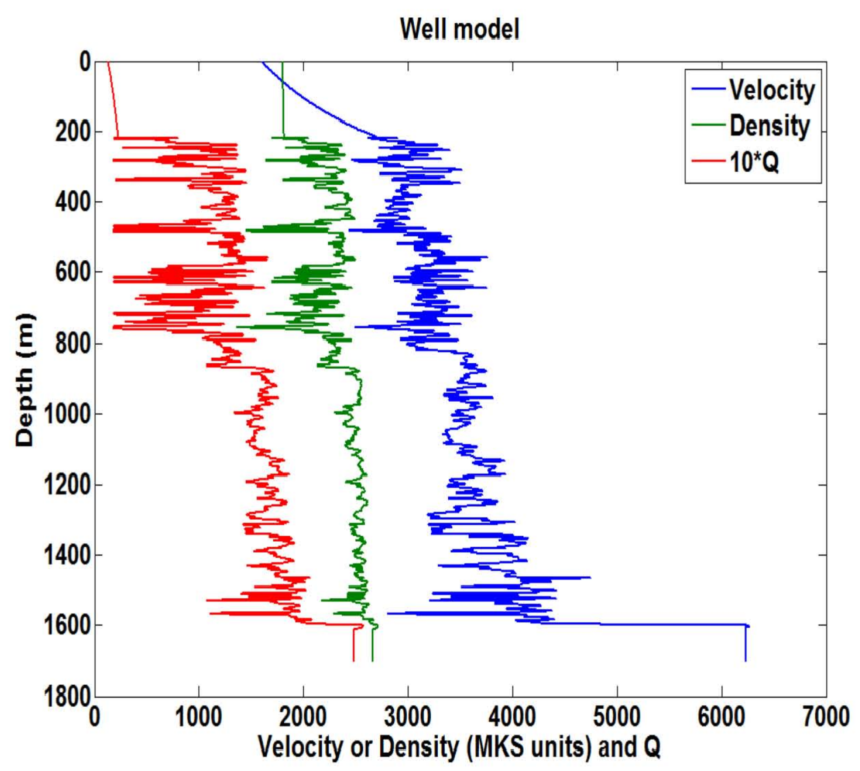
Where $\boldsymbol{A} = \boldsymbol{A}_1 \boldsymbol{A}_2 \cdots \boldsymbol{A}_{n-1} \boldsymbol{A}_n = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

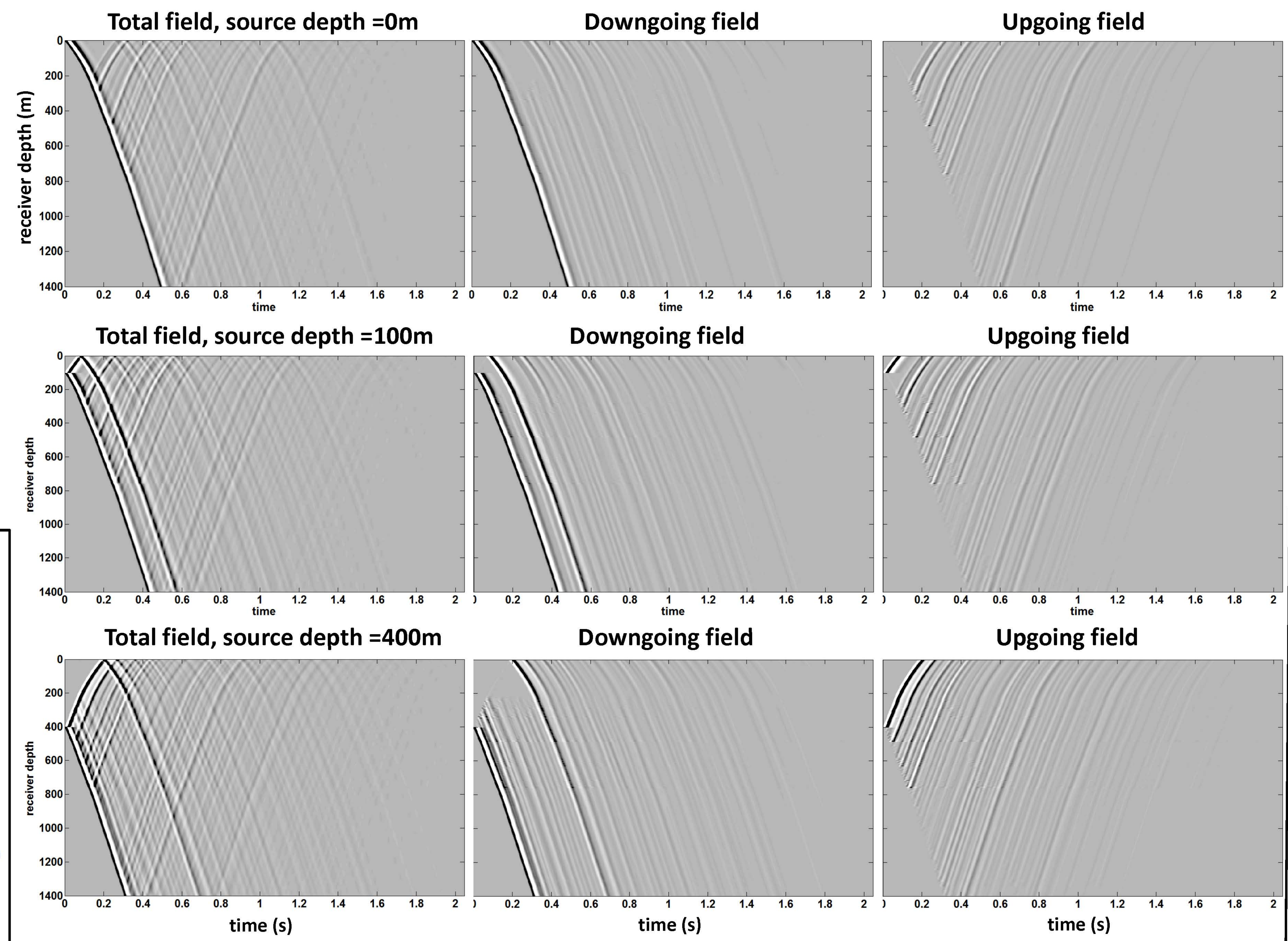
Let $D_1 = W + R_0 U_1$ where W is the source wavelet and R_0 is the surface reflection coefficient (from above). Then we can write two equations with two unknowns

$$W + R_0 U_1 = A_{11} D_{n+1}$$

and

$$U_1 = A_{21} D_{n+1}$$





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