A review of converted wave AVO analysis

Jian Sun*, Kris Innanen
sun1@ucalgary.ca

Introduction

Due to less attenuation of S-wave than P-wave when propagating in unsaturated porosity of rocks, converted P-SV wave might have a greater signal to noise ratio. P-SV wave AVO analysis might provide more information than that obtained from P-wave AVO analysis. we review many of the milestones in the development of converted wave AVO methodology and classify those approximations of P-SV wave reflection coefficient in the light of the way of derivation and their characteristics. A glossary of symbols and references to equations using these symbols are shown in the report.

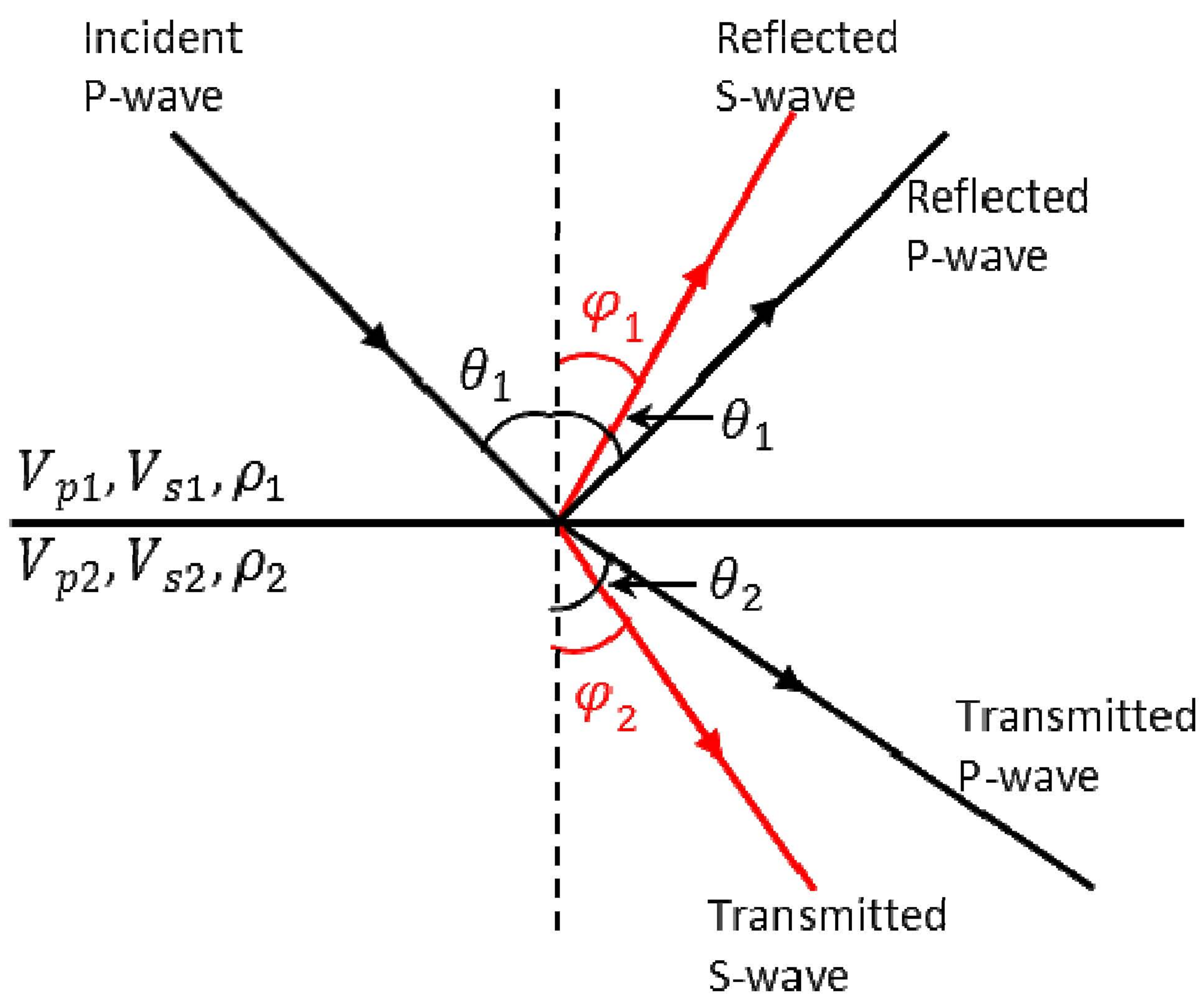

Figure 1: Reflections and transmissions between two elastic media for an incident P-wave

Zoeppritz equations

Reflection coefficients of the reflected and transmitted waves for both PP-wave and P-SV wave can be calculated by solving the exact following matrix equation (Zoeppritz, 1919):

$$\begin{bmatrix} -\sin\theta_1 & -\cos\varphi_1 & \sin\theta_2 & \cos\varphi_2 \\ \cos\theta_1 & -\sin\varphi_1 & \cos\theta_2 & -\sin\varphi_2 \\ \sin 2\theta_1 & \frac{V_{p1}}{V_{s1}}\cos 2\varphi_1 & \frac{\rho_2}{\rho_1}\frac{V_{p1}}{V_{p2}}\frac{V_{s2}^2}{V_{s1}^2}\sin 2\theta_2 & \frac{\rho_2}{\rho_1}\frac{V_{p1}V_{s2}}{V_{s1}^2}\cos 2\varphi_2 \\ -\cos 2\varphi_1 & \frac{V_{s1}}{V_{p1}}\sin 2\varphi_1 & \frac{\rho_2}{\rho_1}\frac{V_{p2}}{V_{p1}}\cos 2\varphi_2 & -\frac{\rho_2}{\rho_1}\frac{V_{s2}}{V_{p1}}\sin 2\varphi_2 \end{bmatrix} \begin{bmatrix} R_{pp} \\ R_{ps} \\ T_{pp} \\ T_{ps} \end{bmatrix} = \begin{bmatrix} \sin\theta_1 \\ \cos\theta_1 \\ \sin 2\theta_1 \\ \cos 2\varphi_1 \end{bmatrix} \quad (1)$$

Precise values of the amplitudes of the reflected and transmitted waves can be achieved using Zoeppritz equations, but it doesn't provide an intuitive understanding of the effects of the parameters changes on the amplitudes. Also, underlying elastic parameters, which caused amplitude changes, are difficult to be inverted using equation (1).

Approximations - 1

Aki-Richards approximation (1981)

$$R_{ps}(\theta) = -\frac{\sin\theta}{2\cos\varphi}[(1-2\frac{V_s^2}{V_p^2}\sin^2\theta + 2\frac{V_s}{V_p}\cos\theta\cos\varphi)\frac{\Delta\rho}{\rho} - (4\frac{V_s^2}{V_p^2}\sin^2\theta - 4\frac{V_s}{V_p}\cos\theta\cos\varphi)\frac{\Delta V_s}{V_s}]. \quad (2)$$

Equation (2) was proposed with the assumption of small changes of elastic parameter across the interface and it doesn't works for 90 degrees. It has indicated that all the conversions between P and S waves are insensitive to first-order changes in the P-wave velocity.

Power series approximations (1991-2011)

1) Zhou's approximation (1993)

$$R_{ps}(\theta) = S\sin\theta + C\sin^3\theta \quad (3)$$

Zhou (1993) demonstrated that the seismic property of SH wave can be derived from P-SV wave. And seismic sections of reflection coefficient, S-wave velocity, and density can be inverted by P-SV wave AVO analysis using equation (3).

2) Gonzalez's approximation (2000)

$$R_{ps}(\theta) = E\sin\theta \quad (4)$$

Equation (4) is only valid for small incident angles case. P-SV gradient (E) is defined and can be calculated from the converted wave CDP gathers, in the same way as the P-P AVO analysis, which is very helpful for simultaneous PP and PS wave inversion.

3) High-contrast approximation (1991 and 2001)

$$R_{ps}(\theta) = A\sin\theta + B\sin^3\theta + C\sin^5\theta \quad (5)$$

Equation (5) derived by Zheng (1991) and further analyzed by Romas et al. (2001), shows the approximate relationship between reflection coefficient and incidence angle without any constraints. But its' complicated form and the uncertainly relationship of lithological parameters make it difficult to be applied in converted AVO inversion.

4) Wang's approximation (2011)

$$R_{ps}(\theta) = A\sin\theta + B\sin^3\theta + C_2\sin^5\theta \quad (7)$$

Compared to Ramos's approximation, equation (6) has more accuracy without increasing its complexity and works well for large incidence angles. And a simplified equation can be acquired when incidence angle is less than 30 degrees.

Approximations - 2

Approximation with Shear-modulus (2006)

$$R_{ps}(\theta) = \frac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta v_p}{v_p}\right) + \frac{1}{2}\tan^2\theta\frac{\Delta v_p}{v_p} - 2\sin^2\theta\frac{\Delta\mu}{\lambda+2\mu} \quad (8)$$

Sun et al. (2006) simplified Aki-Richards approximation into a new approximation in terms of shear modulus, ratios change of density and P-wave velocity.

Approximation using Impedances (1999)

$$R_{ps}(\theta) = \frac{-V_p\tan\varphi}{2V_s}\left[\left(1+2\sin^2\varphi - 2\frac{V_s}{V_p}\cos\theta\cos\varphi\right)\frac{\Delta\rho}{\rho} - \left(4\sin^2\varphi - 4\frac{V_s}{V_p}\cos\theta\cos\varphi\right)\frac{\Delta J}{J}\right]. \quad (9)$$

Larsen（1999）reformulated Aki-Richards approximation into the function of P-wave and S-wave Impedances.

Normalized approximations (1998, 2003)

A normalized expression was derived by Donati (1998) and Sun (2003) demonstrated the intercept gradient attribute of P-SV wave reflectivity, which is very useful for P-SV wave AVO analysis.

$$R'_{ps} = R_{ps}/\sin\theta = P_{ps} + G_{ps}\sin^2\theta \quad (10)$$

Conclusions

Converted P-SV wave AVO analysis plays an increasingly important part in lithological exploration and might provide additional information because of better signal-to-noise ratio of S-wave at times. Beyond that, improved prediction can be achieved by combining P-P and P-SV wave inversions. The key features of those approximations and the most current achievements in converted P-SV wave AVO analysis are summarized, but we make no claim that the review is exhaustive. However, as an introduction, this review could be helpful for understanding the theoretical developments of converted P-SV wave AVO analysis.

Acknowledgments

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Bibliography

See all references from the report.