Phase correction of Gabor deconvolution

Tianci Cui* and Gary F. Margrave
cuit@ucalgary.ca

Abstract

Seismic data is always nonstationary due to ubiquitous anelastic attenuation modeled by the constant-Q theory. The stationary spiking deconvolution of stationary traces is extended to Gabor deconvolution of nonstationary traces, in which a seismic trace is decomposed into a time-frequency spectrum by the windowed Fourier transform and a nonstationary wavelet is estimated within each window. The amplitude spectrum of the nonstationary wavelet is accurately estimated by a smoothing process while its phase spectrum is calculated by the discrete Hilbert transform integrating within the seismic frequency band only. The Gabor deconvolved seismic trace ties the well reflectivity in amplitude and spectral content, but has phase being corrected respect to the seismic Nyquist frequency only. The phase error is the phase difference of the nonstationary wavelet with respect to the well logging frequency and the seismic Nyquist frequency. It can be calculated by knowing the Q values and the well logging frequency, to serve as a phase correction operator in the Gabor domain, which is equivalent to a time-variant residual drift time correction operator in the time domain. Without knowledge of Q or the well logging frequency, the residual drift time can be estimated by smooth dynamic time warping. The Gabor deconvolved nonstationary trace with phase or residual drift time correction ties the well reflectivity with little amplitude or phase errors.

Gabor deconvolution

Margrave and Lamoureux (2001) extend the stationary deconvolution theory to the nonstationary case using the Gabor transform, which is essentially a windowed Fourier transform. The forward Gabor transform decomposes a 1-D nonstationary seismic trace $s(t)$ onto a 2-D time-frequency spectrum by windowing the signal with a set of Gaussian functions summing to unity and Fourier transforming:

$$\hat{s}_g(\tau,f)=\int_{-\infty}^{\infty}s(t)g_\sigma(t-\tau)e^{-2\pi ift}dt \qquad (1)$$

where $g_\sigma(t-\tau)$ is a Gaussian function of standard width 2σ centered at time τ and $\hat{s}_g(\tau,f)$ is the complex-valued Gabor spectrum of $s(t)$. The Gabor transform of the nonstationary convolutional model can be approximated as

$$\hat{s}_g(\tau,f)\approx\widehat{w_Q}(\tau,f)\hat{r}_g(\tau,f) \qquad (2)$$

where $\widehat{w_Q}(\tau,f)$ is the Fourier transform of the propagating wavelet to time τ and $\hat{r}_g(\tau,f)$ is the Gabor transform of the reflectivity. The simplest Gabor deconvolution algorithm estimates $|\widehat{w_Q}(\tau,f)|$ by smoothing $|\hat{s}_g(\tau,f)|$ via convolving it with a 2-D boxcar over τ and f

$$|\widehat{w_Q}(\tau,f)|_{est}=\overline{|\hat{s}_g(\tau,f)|}. \qquad (3)$$

Since both the source wavelet and the attenuation function are assumed to be minimum-phase, the phase of the nonstationary wavelet $\varphi_{w_Q}(\tau,f)$ at a constant time τ is also minimum and is calculated by the Hilbert transform

$$\varphi_{w_Q}(\tau,f)=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{ln|\widehat{w_Q}(\tau,\tilde{f})|_{est}}{f-\tilde{f}}d\tilde{f}. \qquad (4)$$

In the digital implementation, the integral must be calculated within the seismic frequency band only

$$\varphi_{w_Q}(\tau,f)=-\frac{1}{\pi}\int_{-f_{NYQ}}^{f_{NYQ}}\frac{ln|\widehat{w_Q}(\tau,\tilde{f})|_{est}}{f-\tilde{f}}d\tilde{f}. \qquad (5)$$

Next $\hat{r}_g(\tau,f)$ is estimated by dividing $\widehat{w_Q}(\tau,f)_{est}$ from $\hat{s}_g(\tau,f)$ and $r_{est}(t)$ is got by inverse Gabor transforming $\widehat{r_g}(\tau,f)_{est}$.

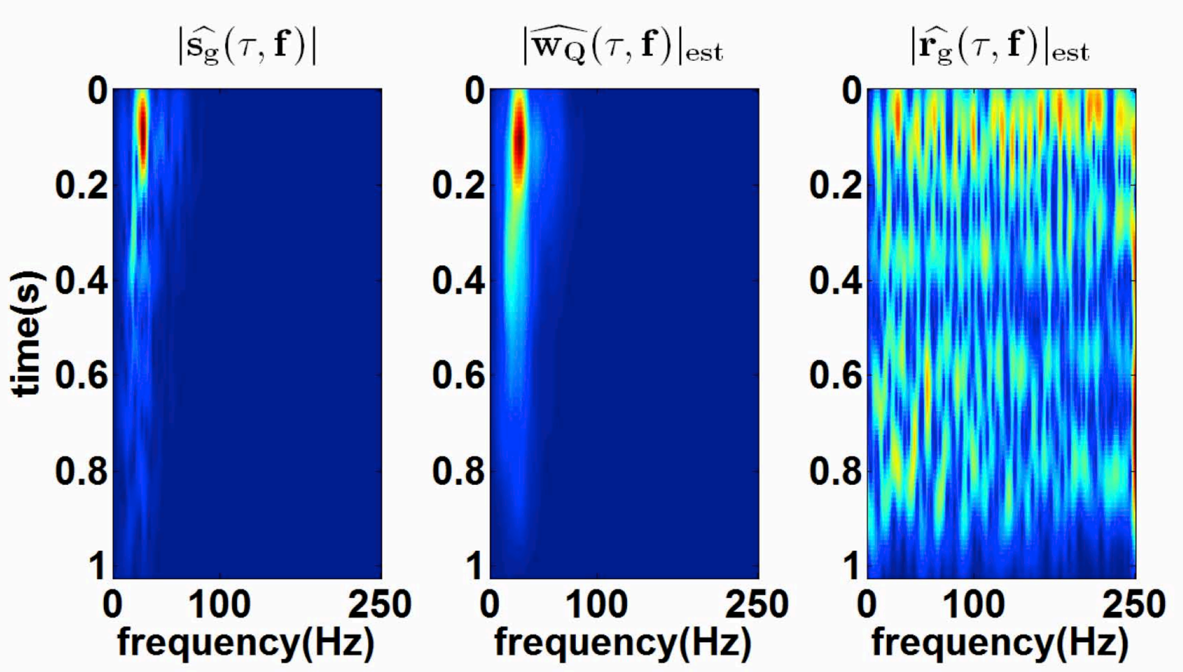

Figure 1: The Gabor magnitude spectra of the nonstationary seismic trace (left), the estimated propagating wavelet (middle) and the estimated reflectivity (right) by Gabor deconvolution.

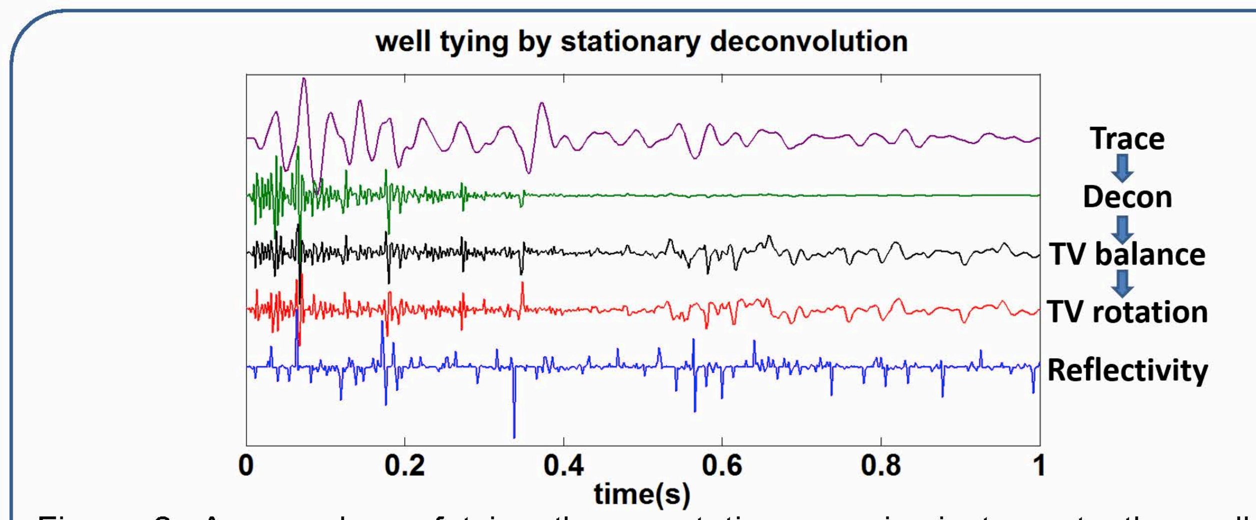

Figure 2: A procedure of tying the nonstationary seismic trace to the well reflectivity by stationary deconvolution, time-variant amplitude balancing and time-variant constant-phase rotation, leading to a "catastrophic" estimate with large amplitude and phase errors.

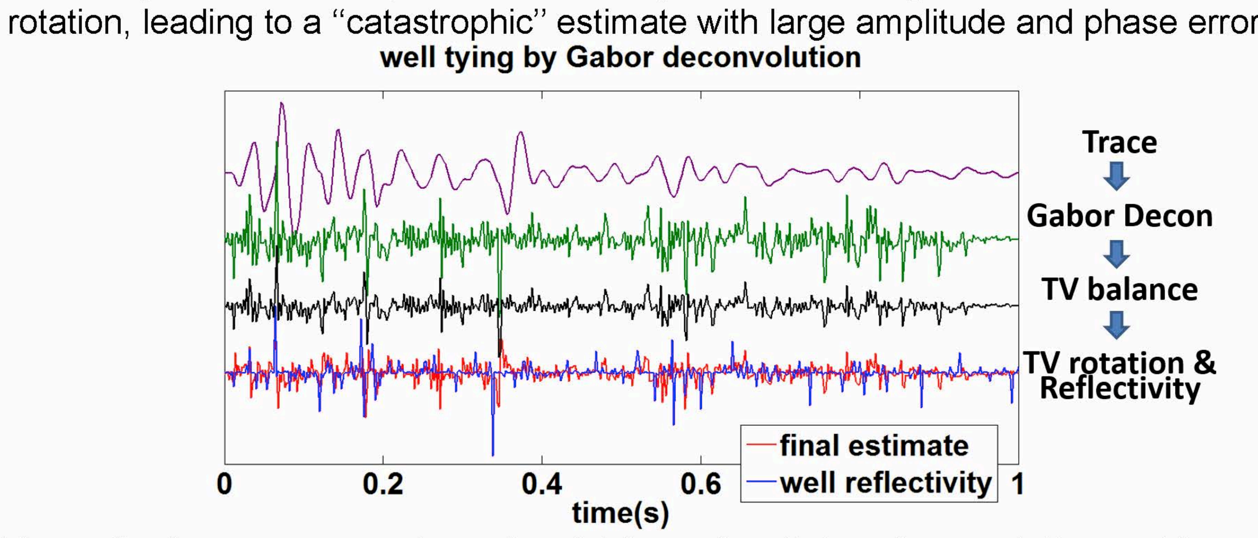

Figure 3: The same procedure of well tying using Gabor deconvolution as Figure 2, resulting in the estimate tying the well reflectivity in amplitude and spectral content, but having phase or timing errors.

Phase correction

The phase estimation errors result from the fact that the bandlimited Hilbert transform must be used (Equation 5) instead of the analytic one (Equation 4). The calculated phase is essentially with respect to the seismic Nyquist frequency f_{NYQ} (Margrave et al., 2011)

$$\varphi^H_{w_Q}(t,f)=\varphi_{w_0}(f)-2\pi ft\left(1-\frac{1}{\pi Q}ln\frac{f}{f_{NYQ}}\right) \qquad (6)$$

where $\varphi^H_{w_Q}(t,f)$ denotes the phase of the propagating wavelet at traveltime t estimated by the digital Hilbert transform. Its phase delay is less than the actual phase at traveltime t with respect to the well logging frequency f_w

$$\varphi_{w_Q}(t,f)=\varphi_{w_0}(f)-2\pi ft\left(1-\frac{1}{\pi Q}ln\frac{f}{f_w}\right). \qquad (7)$$

The difference between $\varphi_{w_Q}(t,f)$ and $\varphi^H_{w_Q}(t,f)$ is the residual phase remaining in the Gabor deconvolved seismic trace compared to the well reflectivity

$$\Delta\varphi(t,f)=\varphi_{w_Q}(t,f)-\varphi^H_{w_Q}(t,f)=\frac{2ft}{Q}ln\frac{f_{NYQ}}{f_w} \qquad (8)$$

where $\Delta\varphi(t,f)$ denotes the residual phase and it varies with traveltime t. It can be noticed from Equation 8 that the residual phase at a constant time t is a linear function of frequency f, implying that $\Delta\varphi(t,f)$ essentially acts as a time shift operator in the time domain, namely

$$\Delta\varphi(t,f)=-2\pi f\Delta drift(t) \qquad (9)$$

where $\Delta drift(t)$ is a time-variant time shift function and is called the residual drift time. According to Equations 8 and 9

$$\Delta drift(t)=\frac{t}{\pi Q}ln\frac{f_w}{f_{NYQ}}. \qquad (10)$$

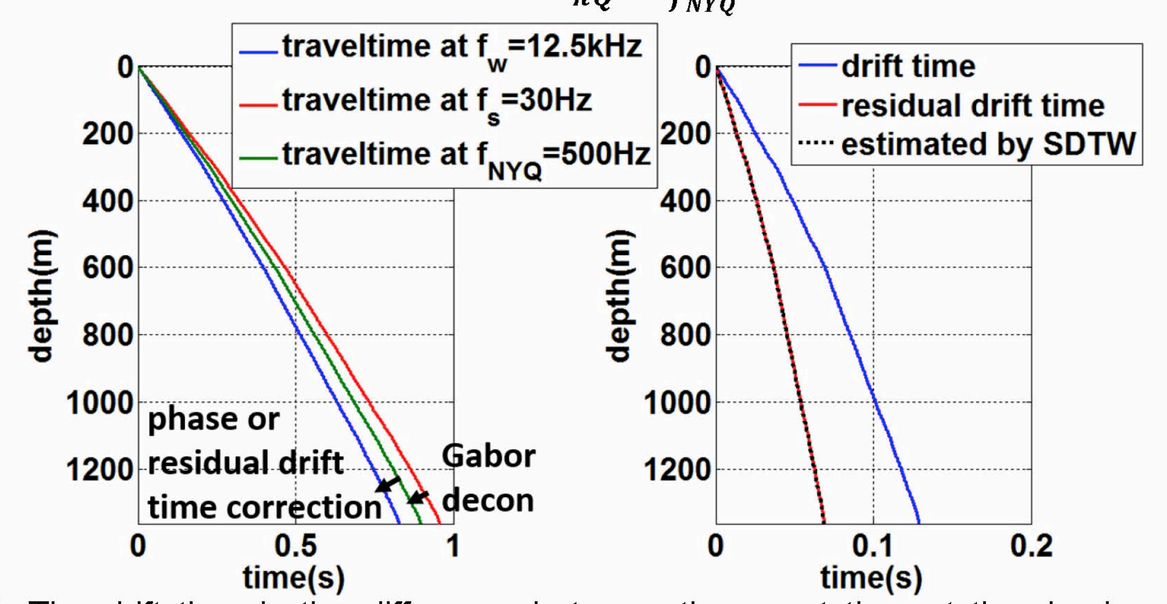

Figure 4: The drift time is the difference between the event time at the dominant seismic frequency f_s and at the sonic logging frequency f_w. Gabor deconvolution corrects the drift time from f_s to the seismic Nyquist frequency f_{NYQ} only. The residual drift time, the difference between the event time at f_{NYQ} and f_w, can be corrected by the phase correction operator or the residual drift time estimated by smooth dynamic time warping (Cui and Margrave, 2015).

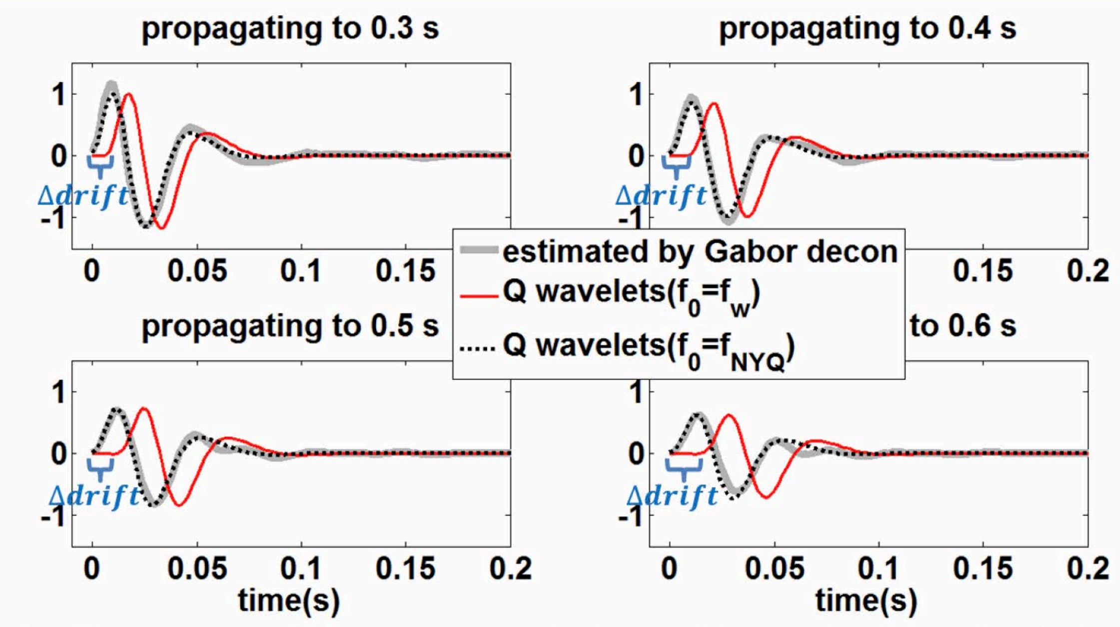

Figure 5: The propagating wavelets at four different times estimated by Gabor deconvolution are consistent with the Q wavelets with respect to f_{NYQ}, but appear earlier than those with respect to f_w. Their timing difference is residual drift time denoted by blue braces.

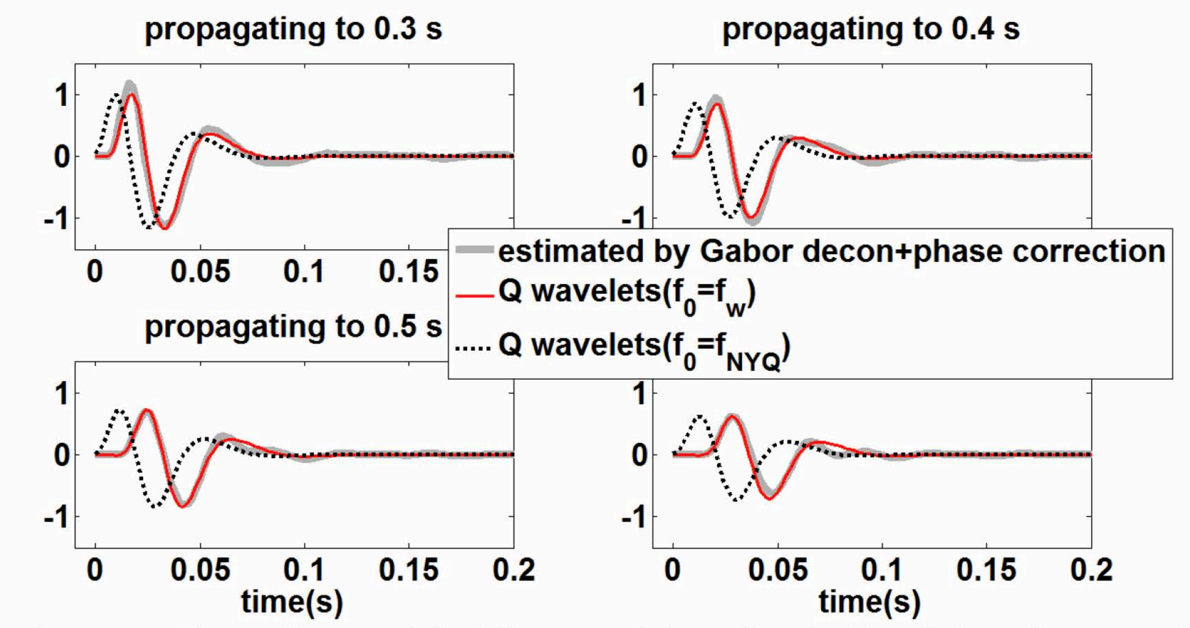

Figure 6: Same as Figure 5 except that the wavelets estimated by Gabor deconvolution are phase corrected.

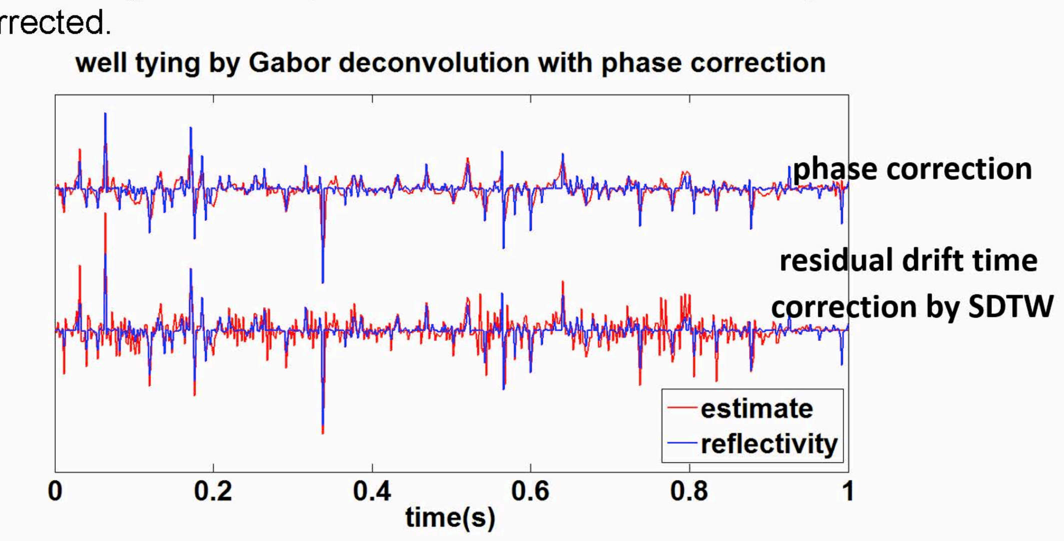

Figure 7: Gabor deconvolved seismic trace with either phase correction or residual drift time correction is precisely tied to the well reflectivity.

Conclusions

- Running Gabor deconvolution on the nonstationary trace can get reflectivity estimate tying the well reflectivity in amplitude and spectral content, but has phase errors.
- Gabor deconvolution calculates the phase spectra of the propagating wavelets by the digital Hilbert transform, which integrates within the seismic frequency band and corrects the drift time to the Nyquist frequency only.
- By correcting the estimated wavelet phase to the well logging frequency, the Gabor deconvolved trace can be well tied to the known reflectivity with very little amplitude and phase errors.
- Gabor deconvolution with either phase correction or residual drift time correction can precisely tie the nonstationary seismic trace to the well reflectivity knowing the Q values and the well logging frequency. Smooth dynamic time warping can estimate the residual drift time without knowledge of Q or the well logging frequency.

References

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