

Direct nonlinear inversion of viscoacoustic media using the inverse scattering series

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Abstract

The objective of seismic exploration is obtaining structural subsurface information from seismic data by recording seismic waves motion of the ground. The recorded data have a non-linear relationship with the property changes across a reflector. In this work, the multi-parameter multi-dimensional direct non-linear inversion is investigated based on the inverse scattering task-specific sub-series. The result is direct and non-linear and has the potential to provide more accurate and reliable earth property predictions for larger contrast and more complex. The inverse scattering method has a direct response for imaging and inversion problems for a large contrast and a multi dimensional corrugated target. We are derived the direct non-linear inversion equation for three parameter viscoacoustic cases. Numerical tests show that non-linear inversion results provide improved estimates in comparison with the standard linear inversion. When the non-linear term add to linear term the recovered value of parameters are much closer to the exact value.

Viscoacoustic scattering potential

The basic wave equations governing the wave propagation in the reference and actual medium are (Matson, 1997).

$$LG = \delta$$

$$L_0 G_0 = \delta$$

The perturbation operator, V (the difference between the reference and actual medium wave operators) is defined as

$$V = L_0 - L$$

For a homogeneous reference medium and 1D case this amounts to

$$V(z, \nabla) \cong \frac{\omega^2}{\rho_0 c_0^2} [\alpha(z) - 2\zeta(z)F(k)] + \frac{1}{\rho_0} \beta(z) \frac{\partial^2}{\partial x^2} + \frac{1}{\rho_0} \frac{\partial}{\partial z} \beta(z) \frac{\partial}{\partial z}$$

The inverse scattering series

The Lippmann-Schwinger equation is an operator identity:

$$\psi_s = G - G_0 = G_0 V G$$

By using the Born series, the scattered field can be expanded in an infinite series through self-substitution.

$$\psi_s = G_0 V G_0 + G_0 V G_0 V G_0 + \dots$$

The inversion scattering series form is:

$$D = (\psi_s)_m = (G_0 V_1 G_0)_m$$

$$0 = (G_0 V_2 G_0)_m + (G_0 V_1 G_0 V_1 G_0)_m$$

This series is a multi-D inversion procedure that directly determines physical properties using only reference medium information and reflection data.

The linear term in frequency domain:

$$D(k_g, z_g; -k_s, z_s; \omega) = -\frac{\rho_0}{4} e^{-iq(z_g+z_s)} \left[\frac{1}{\cos^2 \theta} \alpha_1(-2q_g) - 2 \frac{F(k)}{\cos^2 \theta} \zeta_1(-2q_g) + (1 - \tan^2 \theta) \beta_1(-2q_g) \right]$$

The nonlinear term in frequency domain:

$$\begin{aligned} & \frac{1}{\cos^2 \theta} \alpha_1(-2q_g) - 2 \frac{F(k)}{\cos^2 \theta} \zeta_1(-2q_g) + (1 - \tan^2 \theta) \beta_1(-2q_g) \\ &= -\frac{1}{2 \cos^4 \theta} \alpha_1^2(z) - \frac{2F(k)}{\cos^4 \theta} \zeta_1^2(z) - \frac{\tan^4 \theta}{2} \beta_1^2(z) \\ &+ \frac{2F(k)}{\cos^4 \theta} \alpha_1(z) \zeta_1(z) + \frac{\tan^2 \theta}{\cos^2 \theta} \alpha_1(z) \beta_1(z) - \frac{2F(k) \tan^2 \theta}{\cos^2 \theta} \zeta(z) \beta_1(z) + \dots \end{aligned}$$

Numeric examples: single interface

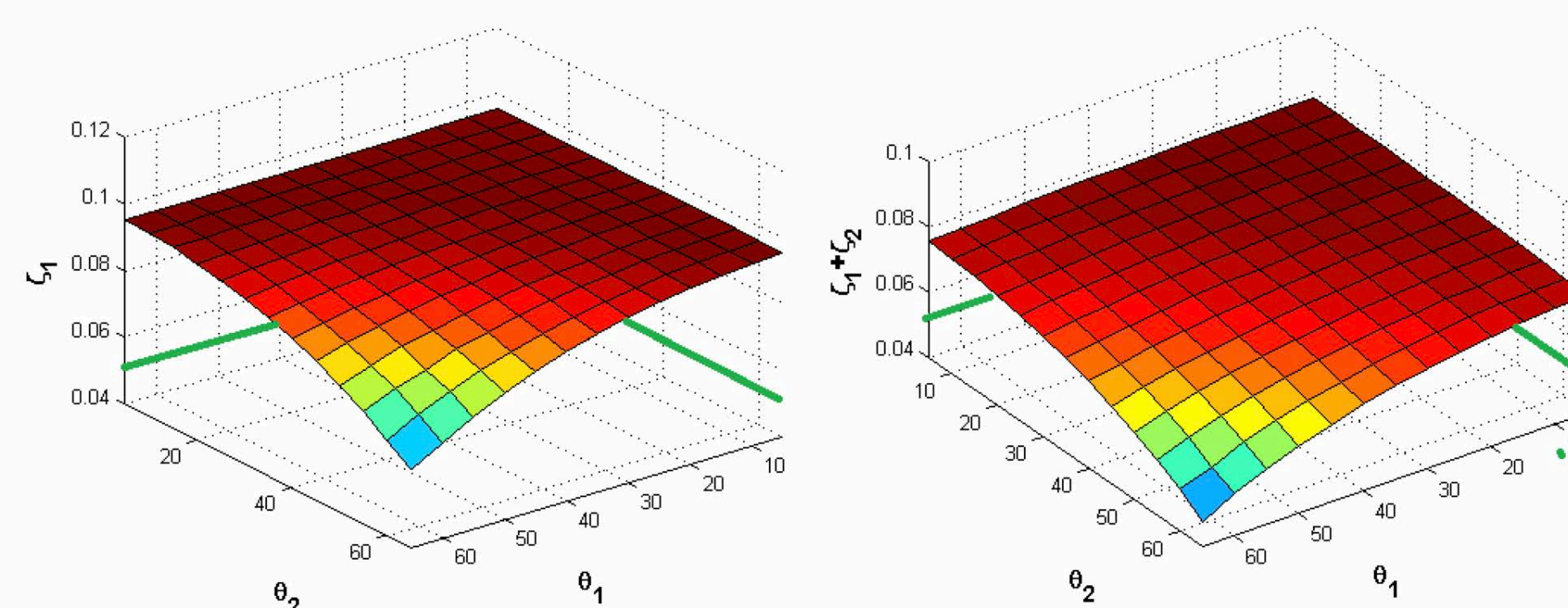


FIG. 1. Recovered parameter from the normal incidence for . The exact value of is 0.1. The linear approximation 1 (a) and the sum of linear and first non-linear 1 + 2(b). The graphs on the bottom are the corresponding contour plots of the graphs on the top.

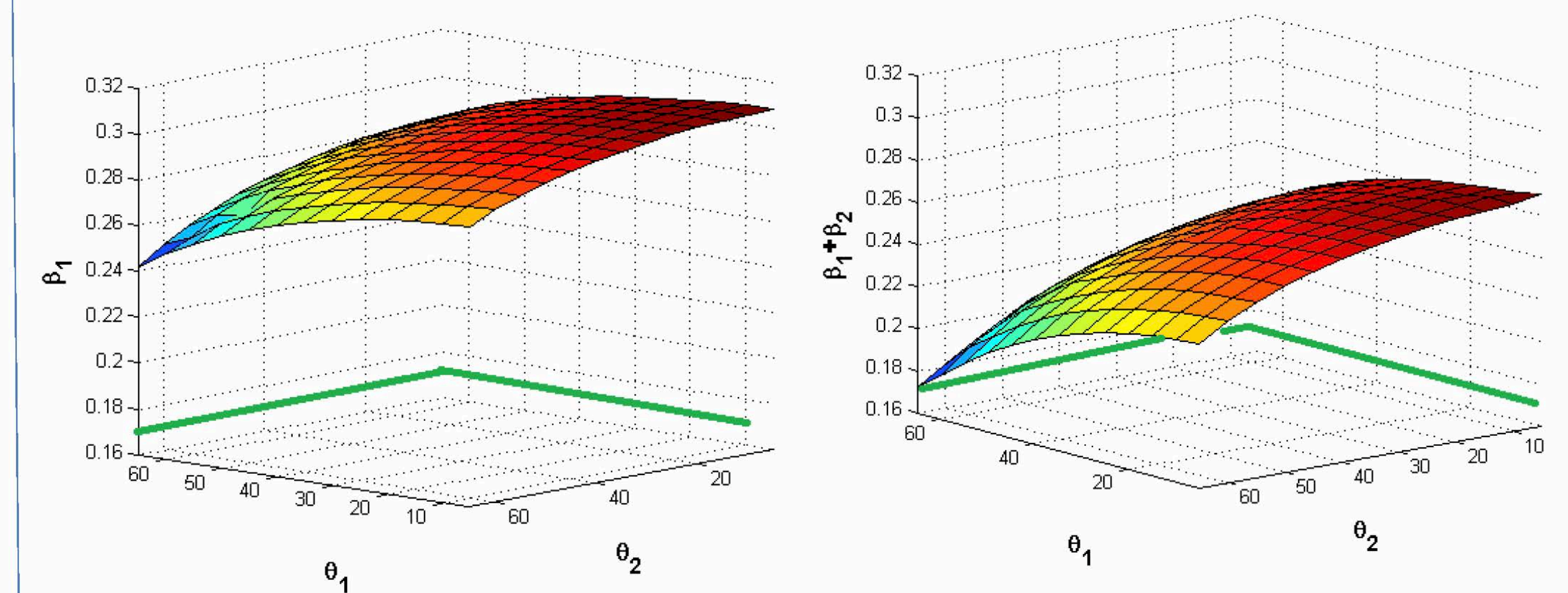


FIG. 2. Recovered parameter from the normal incidence for . The exact value of is 0.17. The linear approximation 1 (a) and the sum of linear and first non-linear 1 + 2(b). The graphs on the bottom are the corresponding contour plots of the graphs on the top.

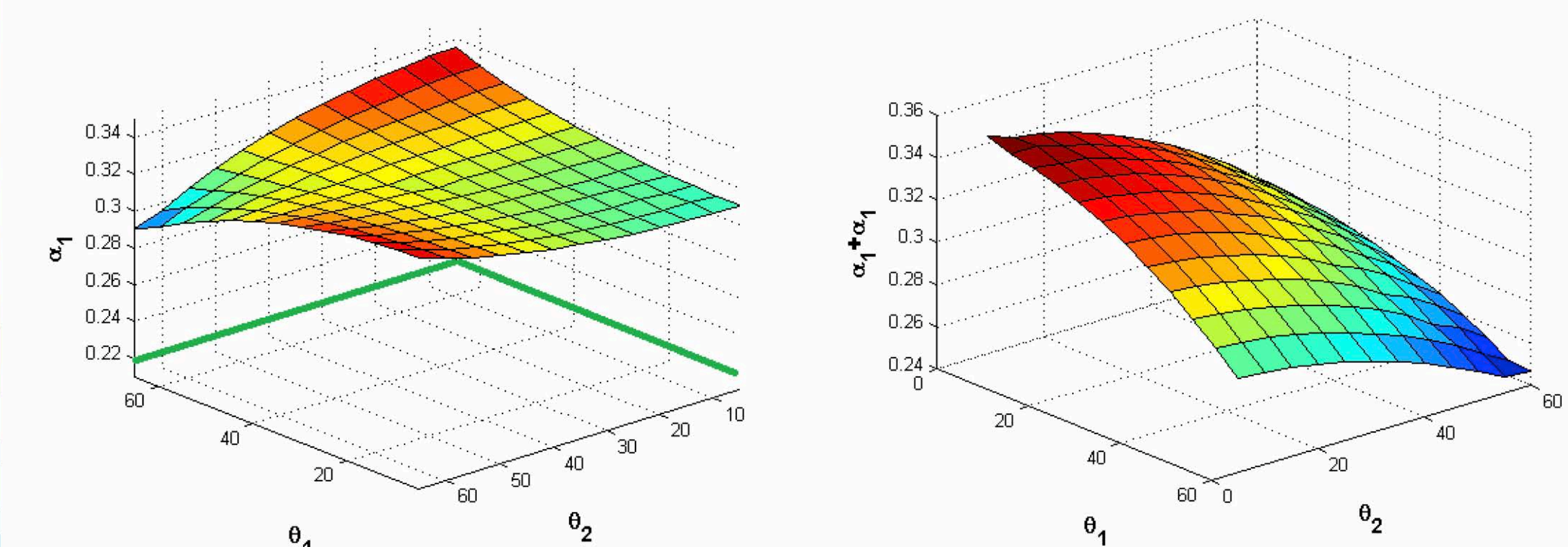


FIG. 3. Recovered parameter from the normal incidence for . The exact value of is 0.22. The linear approximation 1 (a) and the sum of linear and first non-linear 1 + 2(b).

Conclusions

In this chapter, we consider three parameters direct non linear inversion for viscoacoustic media. Both the linear and nonlinear processing and inversion are investigated. The numerical results indicate that all the second order solutions provide improvements over the linear solutions. When the second term is added to linear order, the results become much closer to the corresponding exact values. The inversion method is direct and nonlinear and has the potential to provide more accurate and reliable earth property predictions for larger contrast and more complex without knowing the specific properties of the target.

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