Higher order anacoustic AVF inversion

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Introduction

- Recovery of the attenuation parameter Q by amplitude variation with frequency analysis is possible due to the dispersive nature of the reflection coefficient between anelastic or anacoustic layers.
- The anacoustic reflection coefficient is given to second order in perturbations by

$$R \approx \frac{1}{4}a_c(1+\sin^2\theta) + a_c^2\left(\frac{1}{8}+\frac{1}{4}\sin^2\theta\right) - \frac{1}{2}a_c a_Q F(\omega)\sin^2\theta - \frac{1}{2}a_Q F(\omega)(1+\sin^2\theta) + a_Q^2 F^2(\omega)\left(\frac{1}{4}+\frac{3}{4}\sin^2\theta\right),$$

where $a_c = 1 - \frac{c_0^2}{c^2}$, $a_Q = Q^{-1}$, and

$$F(\omega) \approx \frac{i}{2} - \frac{1}{\pi}\ln\left(\frac{\omega}{\omega_0}\right) + a_Q\left(\frac{-i}{2\pi}\ln\left(\frac{\omega}{\omega_0}\right) + \frac{1}{\pi^2}\ln^2\left(\frac{\omega}{\omega_0}\right)\right).$$

- An anacoustic inversion approach was proposed in Innanen (2011), and a linearized, least squares version was suggested by Bird et al (2011).
- In this approach, two major assumptions are made;
  1. Perturbations a_c and a_Q must be small for the linearization to be valid.
  2. Perturbation a_Q must be negligible relative to a_c to allow for the real part of the reflection coefficient to be approximated by the amplitude spectrum (as the imaginary part depends only on a_Q).
- By using a second order approximation, we can relax the first constraint, and by iteratively updating an estimate of the real reflectivity, we can relax the second.

Numerical Tests

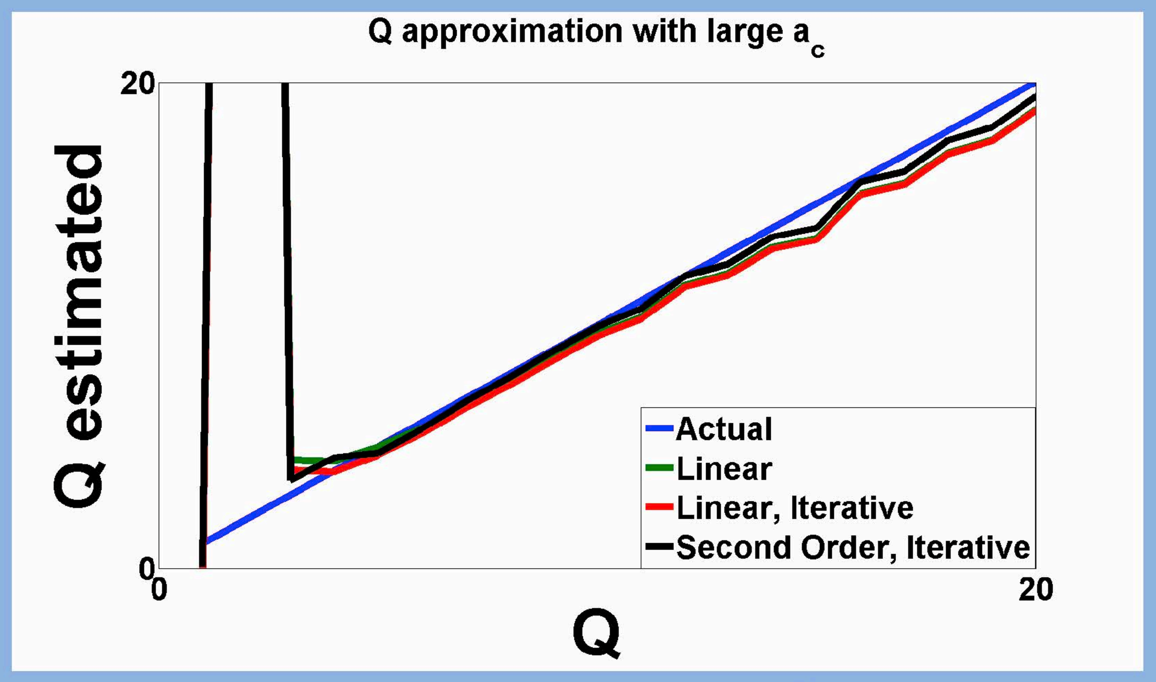

FIG. 1. The second order approximation offers limited improvement over the linearized inversion. Failure at low Q could be due to assumption 1 or 2.

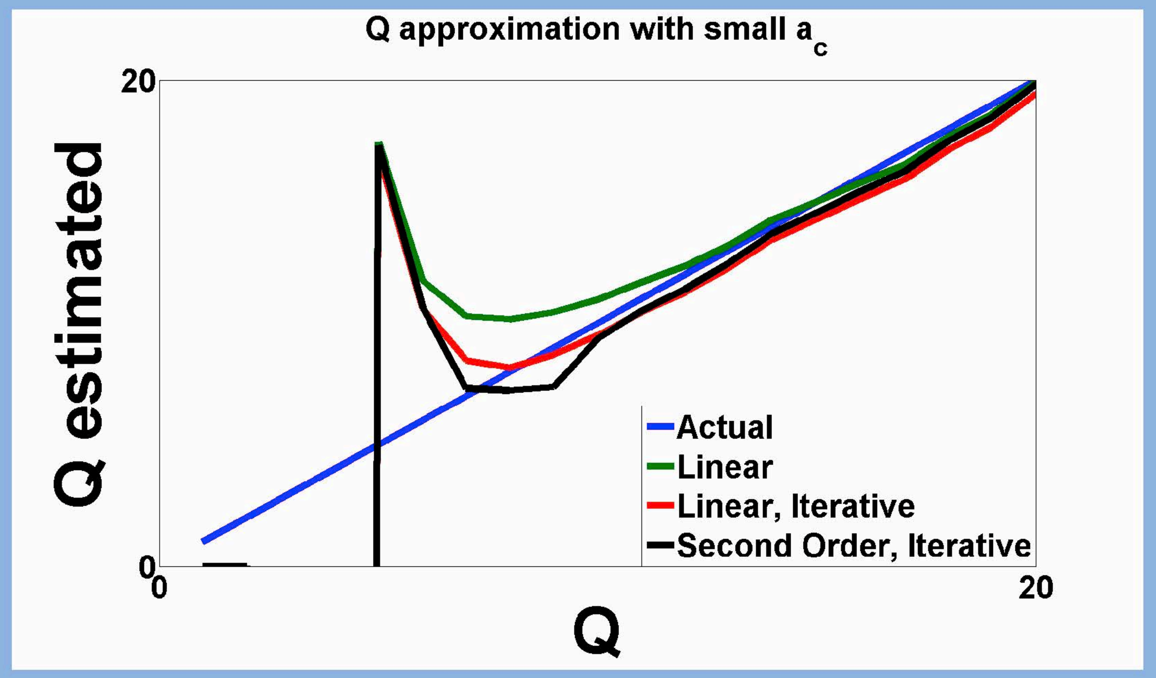

FIG. 2. A smaller a_c than Figure 1 means we better fulfill assumption 1, while violating assumption 2 to a greater extent. The failure at greater Q than in Figure 1, as well as the relative improvement from the linear to iteratively updated linear case suggest that violation of assumption 2 is causing the low Q failure.

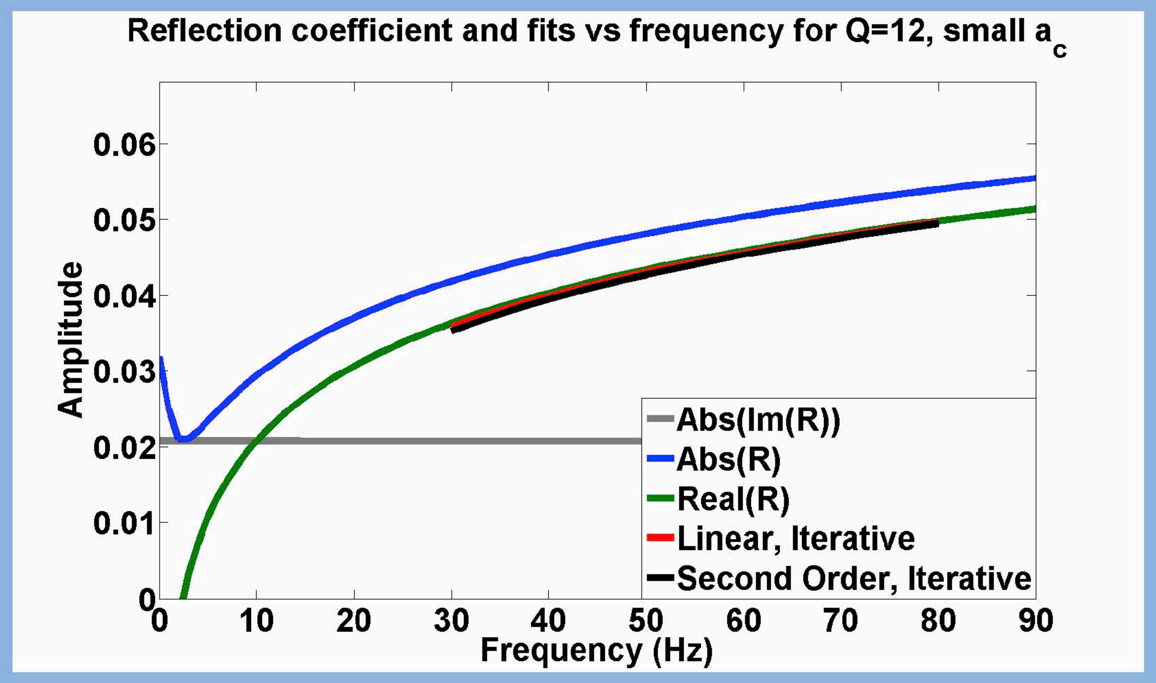

FIG. 3. At Q above the failure region, the iteratively updated estimate of the real part of the reflection coefficient is very close to the true value. Here the imaginary part is less than the real part, and assumption 2 is satisfied.

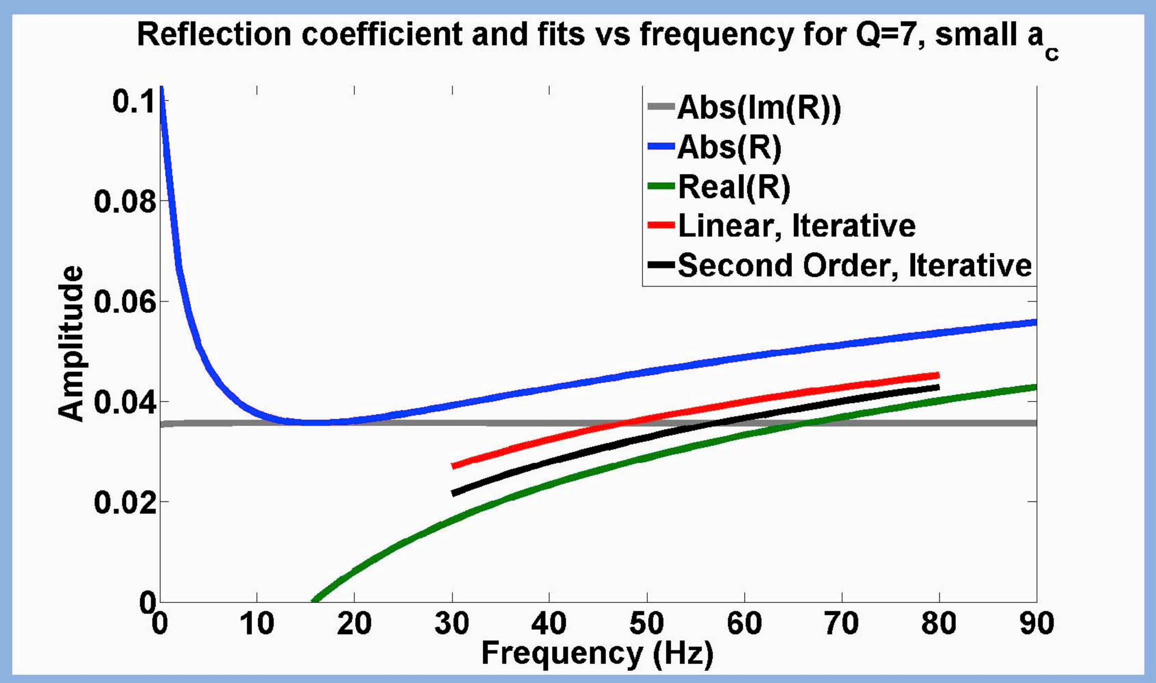

FIG. 4. At Q within the failure region, the iteratively updated estimate of the real part of the reflection coefficient does not closely match the true value. Here the imaginary part of the reflection coefficient is greater than the real part for much of the frequency band of interest, badly violating assumption 2.

Conclusions

- Iteratively updating the estimate of the real part of the reflectivity offers improvements over approaches that simply use the amplitude spectrum.
- Failures at low Q are caused by a large imaginary component in the reflection coefficient, leading to a significant violation of our assumptions.
- While the second order approximation offers slight improvements over the linearization, it does not address the cause of low Q failure, and so offers little improvement at low Q.
- If recovery of very low Q is desired, alternate formulations of the inversion which do not rely on recovering the real part of the reflectivity seem the most promising approach.

See report for a full list of references