Tweaking minimum phase calculations

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Abstract

The minimum phase characterization of impulsive seismic sources is an essential step in the deconvolution process, to remove source signature with appropriate phase. Calculation of the minimum phase equivalent of a given signal is numerically sensitive, given the presence of logarithmic singularities in certain algorithms.

We propose a simple oversampling in the frequency domain that accommodates the singularity, and show with a few examples that the performance is improved.

Introduction

Two popular methods for numerically computing the minimum phase source wavelet are the Wiener-Levinson double inverse method, and the Kolmogorov or Hilbert transform methods. Both begin with an estimation of the source's amplitude spectrum based on an autocorrelation of the seismic record, and then apply a numerical algorithm to estimate the phase spectrum. There is also a close connection between minimum phase signals and outer functions that arise in the theory of complex analysis.

Both the Hilbert transform and outer function calculations involve a direct integral of the log Fourier spectrum, which can result in numerical instabilities.

Hilbert transform: $$\int_0^1 \cot \pi(\omega - \theta) \log |\hat{f}(e^{2\pi i \theta})| \, d\theta$$

Outer function: $$\exp\left(\int_0^1 \frac{e^{2\pi i \theta} + z}{e^{2\pi i \theta} - z} \log |\hat{f}(e^{2\pi i \theta})| \, d\theta \right)$$

Examining the logarithmic singularity

The integral of log can be approximated by a finite sum, with an error term given by Stirling's formula:

$$\int_0^1 \log(x)\, dx \approx \frac{1}{N} \sum_{n=1}^{N} \log(\frac{n}{N}) \approx -1 + \frac{\log(2\pi N)}{2N}$$

The log(2pi)/2N is the error and is significantly large, even when the logarithm is stabilized by adding a small epsilon to the argument. The following table shows the error is much more sensitive to the size of N than to the size of the pre-whitening constant:

	$\epsilon = 10^{-4}$	$\epsilon = 10^{-6}$	$\epsilon = 10^{-8}$
$N = 20$	8.2	8.2	8.2
$N = 200$	0.73	0.66	0.66
$N = 2000$	0.16	0.066	0.065

Table 1: Percentage error when integrating across a logarithmic singularity. Large N oversampling is most effective in reducing the error.

Oversampling algorithm

With this suggestion, we propose a modified algorithm where the Hilbert transform in Kolmogorov, or the outer function calculation, is computed in a highly oversampled manner. With a few test runs, oversampling by a factor of 128 appears to give stable, noticeably improved results. As these computations are FFT-based, there is only a small performance penalty.

Test runs on a Ricker wavelet

As a demonstration, we compute the minimum phase equivalent of a time-limited 20Hz Ricker wavelet, with 2 mS sampling. As shown in Figure 1, the oversampled method pushes the waveform forward in time compared to the standard double inverse method. It also removes what appears to be an extraneous bump late in the waveform.

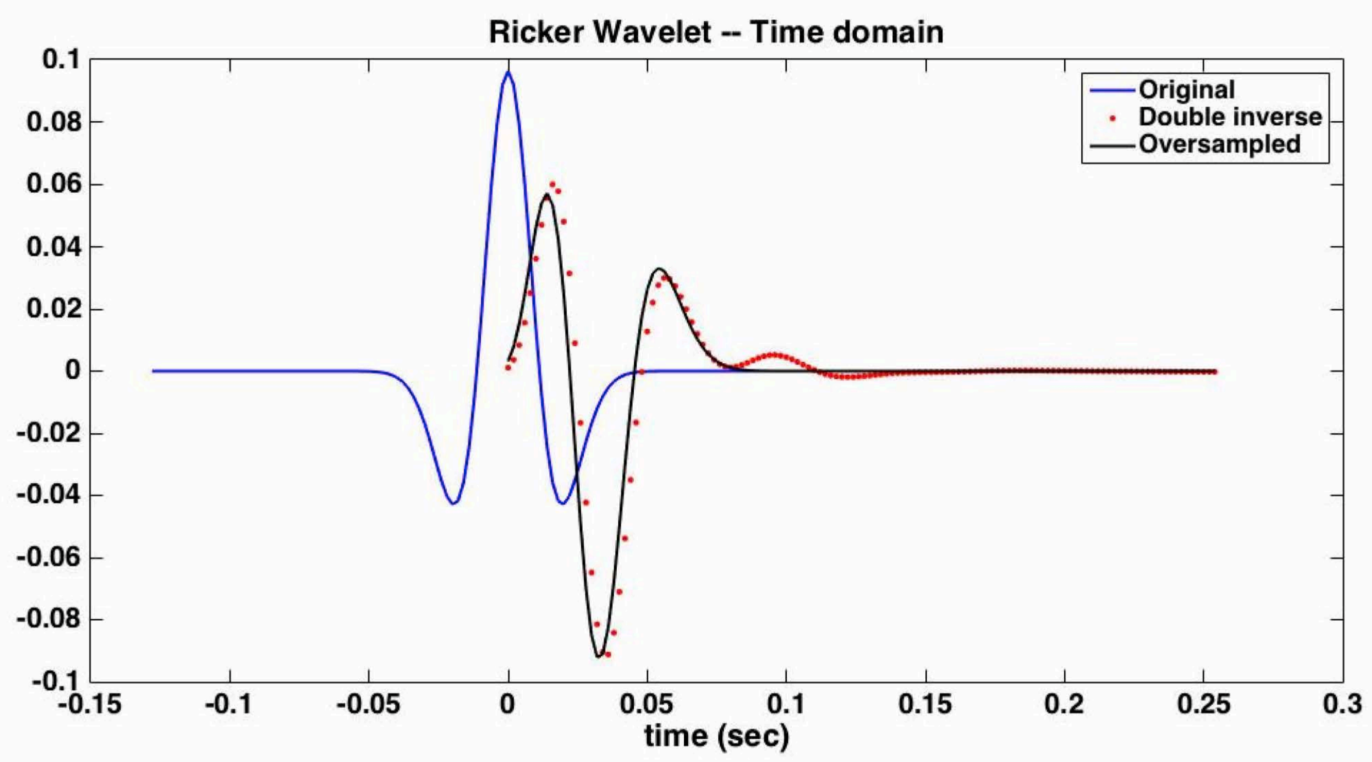

Figure 2: A Ricker wavelet and its minimum phase variant, computed by two different methods. The new oversampled method moves the waveform forward in time, and remove an extraneous bump at t = 0.1.

Test runs on Boxcar wavelet, exact solutions

A boxcar wavelet has an exact minimum phase solution, which simply moves the leading edge of the waveform to the t=0 position. Using this as a test waveform, we can see how well each algorithm works by comparing the results with the exact solution. Figure 3 shows how the new oversampled method outperforms Kolmogorov.

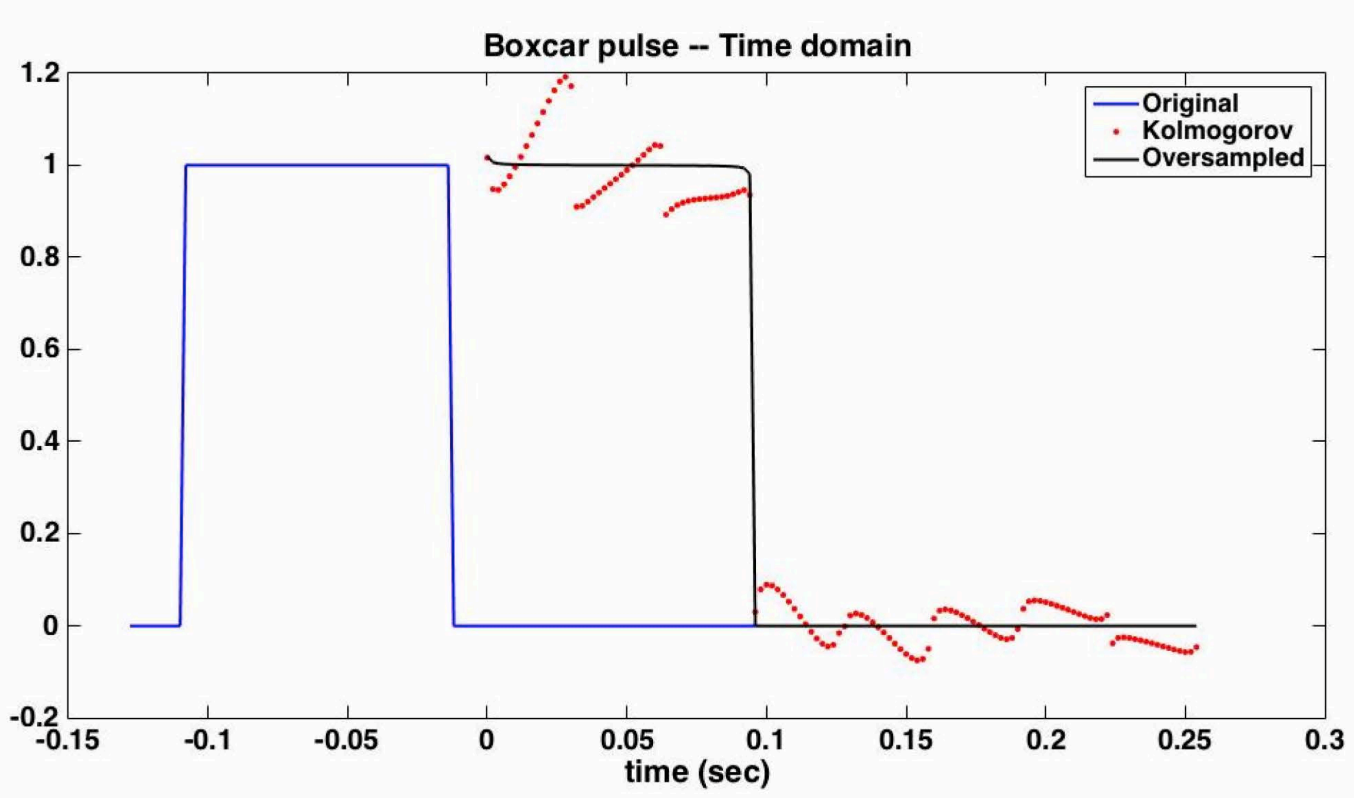

Figure 3: A Boxcar wavelet and its minimum phase variant, computed by two different methods. The new oversampled method is successful at positioning the waveform and flattening the top and base.

Test runs on an exponential ramp, exact solutions

An increasing exponential ramp also has an exact minimum phase solution, which is the mirror image as a decreasing ramp. Figure 4 compares the performance of the new oversampled method to the usual methods. Oversampling gives a near-perfect result, with no overshoot or undershoot at the leading and trailing edges.

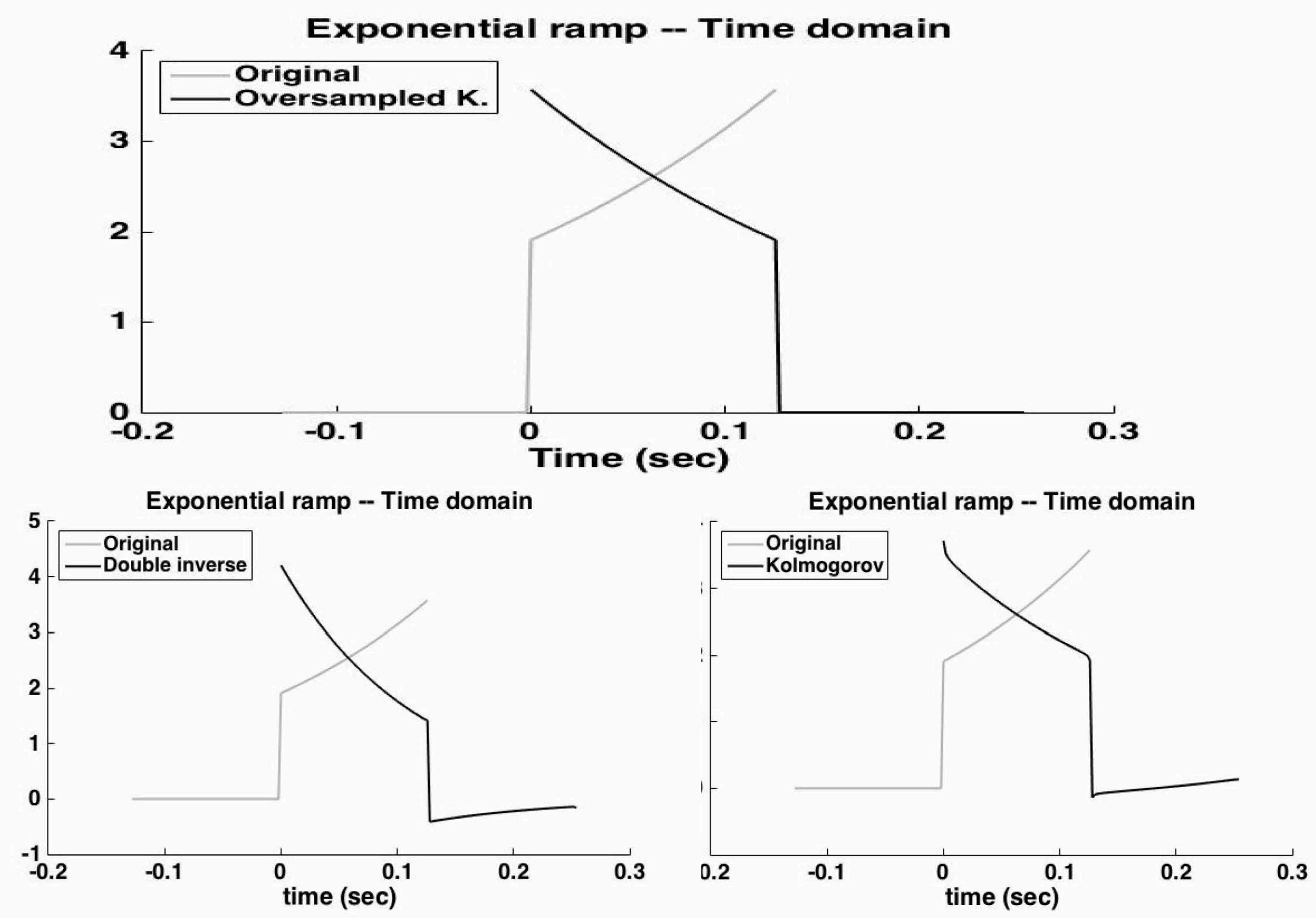

Figure 4: An exponential ramp is correctly converted to its mirror image with the new method. Double inverse and Kolmogorov overshoot at the start and end of the resulting ramp.

Future Work

We will test on real data and determine if there is any significant improvement in deconvolved seismic images. The new algorithm moves the minimum phase peak somewhat forward in time – which may give location results that are slightly different than what we are used to seeing in images. We will investigate how to reduce phase errors that occur with finite-length time and frequency windows. We will include the oversampled method as a standard routine in the CREWES toolbox.

Conclusions

The minimum phase calculation is unstable in part because of possible logarithmic singularities in the log amplitude spectrum that arise in the integral transforms. Numerical calculations are made more accurate by oversampling in the frequency domain. Sample tests with Ricker wavelets, boxcars, and exponential ramps show improvements in the modified algorithm. Work is needed to verify this gives an improvement in imaging.

Acknowledgements

The authors gratefully acknowledge the financial support of the industrial sponsors of the CREWES project at the University of Calgary. This work was funded by CREWES industrial sponsors and NSERC through a CRD grant and the authors' Discovery grants.

References

- Claerbout, J., 1976, Fundamentals of seismic data processing: McGraw-Hill, N.Y.
- Dey, A. K., and Lines, L. R., 1998, Seismic source wavelet estimation and the random reflectivity assumption: CREWES Research Report, 10, 21.1–21.28.
- Gibson, P. C., Lamoureux, M. P., and Margrave, G. F., 2011, Outer preserving linear operators: J. Functional Analysis, 261, No. 9, 2656–2668.
- Lamoureux, M. P., and Margrave, G. F., 2007, An analytic approach to minimum phase signals: CREWES Research Report, 19.
- Lines, L. R., and Ulrych, T. J., 1977, The old and the new in seismic deconvolution and wavelet estimation: Geophysical Prospecting, 25, 512–540.
- Robinson, E. A., 1967, Predictive decomposition of time series with applications to seismic exploration: Geophysics 32, 32, 418–484.
- Russell, B., 1994, Seismic inversion: SEG course notes.