Finite difference modeling of the diffusive slow P-wave in poroelastic media

Shahin Moradi*, Don C. Lawton and Edward S. Krebes
*smoradi@ucalgary.ca

Abstract

Biot's theory of poroelasticity predicts the presence of a slow P-wave in a fluid saturated medium due to the relative movement of the pore fluid with respect to the rock matrix. The slow P-wave, is highly diffusive in seismic frequencies and thus will not be observed in seismic data. However, in the case of zero fluid viscosity, this wave is a non-diffusive mode that travels through the medium. In this report both diffusive and non-diffusive modes are modeled using a previously developed finite-difference algorithm. It seems that in a uniform homogenous medium the amount of amplitude loss due to wave conversion in the diffusive case is very close to the one in the non-diffusive case. This shows that although the diffusive P-wave is not a traveling mode, it exists in the medium but dissipates quickly. The slow P-wave is particularly important where gas exists in the form of separate patches in the pore fluid. In those cases the wave conversions to the slow P-wave may dissipate a considerable amount of energy. Modeling wave propagation in such media is useful in monitoring studies for CO_2 sequestration.

Theory

Maurice Biot was the first to establish the theory of poroelasticity (Biot,1962). The low frequency partial differential equations for isotropic poroelastic media can be written as first order equations in time:

$$\frac{\partial V_i}{\partial t} = A\,\frac{\partial \tau_{ij}}{\partial x_j} - B\left(\frac{\partial P}{\partial x_i} + b\,W_i\right), \quad (1)$$

$$\frac{\partial W_i}{\partial t} = B\,\frac{\partial \tau_{ij}}{\partial x_j} + C\left(\frac{\partial P}{\partial x_i} + b\,W_i\right), \quad (2)$$

$$\frac{\partial P}{\partial t} = -\alpha M\,\frac{\partial e_{kk}}{\partial t} - M\,\frac{\partial \varepsilon_{kk}}{\partial t}, \quad (3)$$

$$\frac{\partial \tau_{ij}}{\partial t} = 2\mu\,\frac{\partial e_{ij}}{\partial t} + \left(\lambda_c\,\frac{\partial e_{kk}}{\partial t} + \alpha M\,\frac{\partial \varepsilon_{kk}}{\partial t}\right)\delta_{ij}, \quad (4)$$

where all symbols are defined in Table 1. In an earlier work we developed a finite difference program in MATLAB to simulate the wave propagation in poroelastic media. The algorithm was based on a velocity-stress staggered grid method in the time domain. However, in the previous work the algorithm was developed for the no-loss case, in which the viscosity of the fluid was assumed to be zero. The fluid viscosity has a direct effect on the mobility constant b which is the ratio of viscosity to permeability. In no-loss poroelastic medium the mobility factor b is zero and therefore the last terms in Equations 1 to 2 vanish. These terms describe the effect of fluid in the poroelastic medium and the behaviour of the slow mode. In seismic frequencies, the fluid effects become stronger than the internal effects, therefore the slow P-wave is diffusive and dies quickly in the medium. On the contrary, in the case where b=0, the internal effects dominate over the fluid effect and the slow P-wave travels through the medium without diffusion. The unknowns in the partial differential equations are the particle velocities of the solid and the fluid, the stresses and also the fluid pressure(left hand side of Equations 1 to 4). The calculated particle velocities of the solid and the fluid for the model in Table 2 are shown in Figure 1.

Table 1. List of symbols and their definitions

symbol	discerption	formula
e_{ij}	solid strain	$\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$
τ_{ij}	solid stress tensor	
$\vec{u}$	particle displacement of the solid	
$\vec{U}$	particle displacement of the fluid	
ε_{kk}	$\nabla.(\vec{u}-\vec{U})$	
μ	shear modulus of the rock	
λ_c	Lame parameter of the saturated rock	
α	Biot's coefficient	$\left(1-\frac{K_{Dry}}{K_{Solid}}\right)$
K_{Solid}	bulk modulus of the solid	
K_{Dry}	bulk modulus of the dry rock frame	
K_f	bulk modulus of the fluid	
M	coupling modulus	
P	fluid pressure	
V_i	particle velocity of the solid	$\partial u_i/\partial t$
W_i	particle velocity of the fluid relative to the solid	$\partial\left(U_i-u_i\right)/\partial t$
ρ_f	density of the fluid	
ρ_s	density of the solid	
ρ	density of the saturated rock	$\phi\rho_f+(1-\phi)\rho_s$
ϕ	porosity	
m	the fluid effective density	$T\rho_f/\phi$
T	tortuosity	
b	fluid mobility	η/κ
κ	permeability	
η	fluid viscosity	
A	Coefficients defined in this paper for simplicity	$\left(\frac{m}{m\rho-\rho_f^2}\right)$
B		$\left(\frac{-\rho_f}{m\rho-\rho_f^2}\right)$
C		$\left(\frac{-\rho}{m\rho-\rho_f^2}\right)$

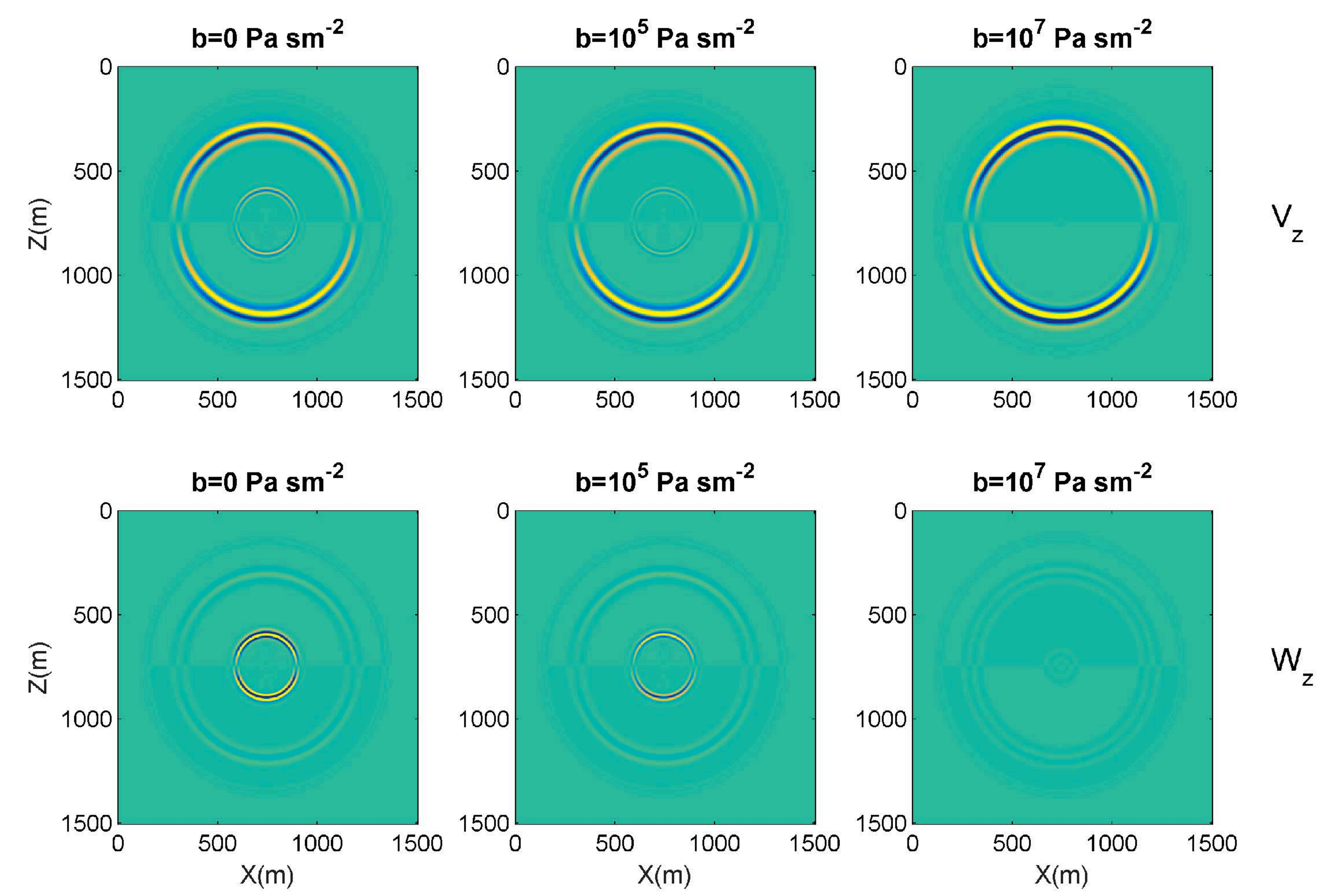

FIG 1: Vertical particle velocities of the solid (top) and the fluid (bottom) for three different values of mobility b. As expected, in the seismic frequency, for b=0 the slow P-wave is a traveling wave while for none-zero values of b, it is diffusive. For the case with b=10^5 Pa sm^{-2} which are the two images in the middle (top and bottom), the slow P-wave seems weaker than the case with b=0. Therefore, the larger b value, the more diffusive the slow mode becomes. In the case of b=10^7 Pa sm^{-2} the slow P-wave does not exist in this time because it had been attenuated due to the fluid effect. Another point that worth mentioning is that in the fluid snapshots (three images on the bottom), the slow P-wave is larger in amplitude compared to the fast P-wave, which is probably because this wave is originated from the fluid movement.

Numerical Model

The numerical model used here is based on the well log data from the Quest Carbon Capture and storage project. The purpose of the Quest project is to store CO_2 in the Basal Cambrian Sandstone, which is a saline aquifer within the Western Canadian Sedimentary Basin (WCSB). The log data and the numerical model generated based on this data were explained in detail in the previous work by the authors (Moradi et al.,2014; Moradi and Lawton, 2012).

Table 2. Physical properties of the model used in this report

Property	The model
ρ_s	$2650\ kg/m^3$
ρ_f	$937\ kg/m^3$
K_{solid}	$38.00\times10^9\ Pa$
K_f	$0.25\times10^9\ Pa$
V_P	$3800\ m/s$
V_S	$2410\ m/s$
ϕ	16%
b	zero $10^5 Pa\ sm^{-2}$ $10^7 Pa\ sm^{-2}$

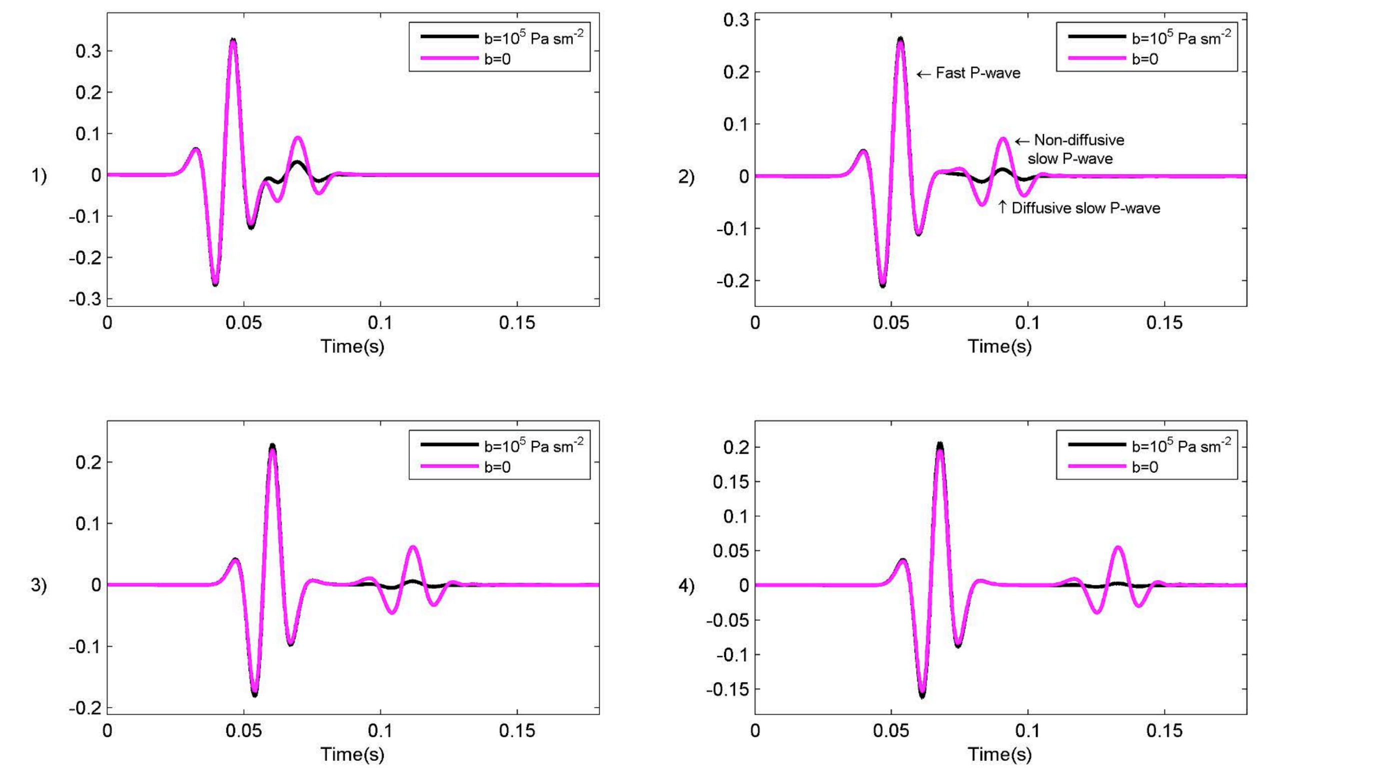

FIG. 2: Traces from the vertical particle velocities of the solid in poroelastic media for two values of b=0 and b=10^5 Pa sm^{-2}, where b (mobility constant) is the ratio of the fluid viscosity to the rock's permeability. The slow P-wave in the case of nonzero mobility dissipates quickly while in the case of zero mobility does not change through time. Furthermore, the similarity of the fast P-wave amplitude in both cases implies that the dissipated amount of energy in non-diffusive mode is close to the one in the diffusive mode. Therefore, when the mobility is not zero, although the slow P-wave is not observed, it exists as a diffusion mode and dissipates the energy of the wave.

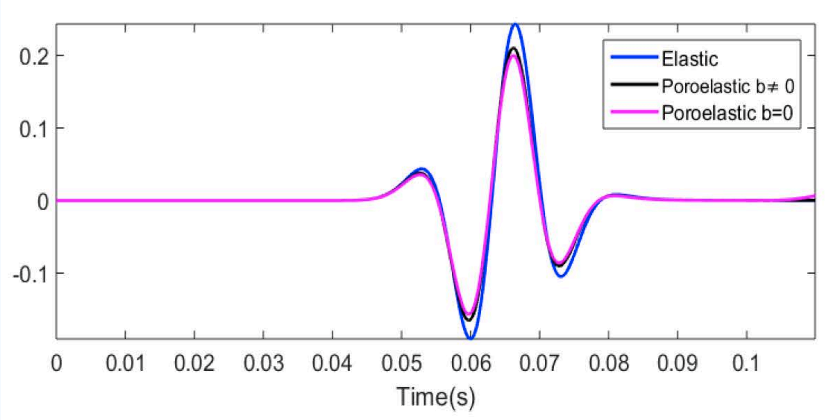

FIG 3: Traces from an elastic algorithm are compared with the ones from poroelastic case with zero and non-zero mobility values. Due to conversion to the slow P-wave, the fast P-wave in the poroelastic case is smaller in magnitude compared to the elastic one.

Conclusions

The fluid mobility constant was added to a previously developed finite-difference modeling program to simulate the wave propagation in poroelastic media. In seismic frequencies the slow P-wave is a diffusive wave and is not observed as a traveling mode unless the mobility parameter is zero. In this work we showed that although the diffusive slow wave is not a traveling mode wave, it causes energy loss that is due to conversion of the fast P-wave to slow P-wave. It is known that in finely layered poroelastic media, or in a rock saturated with fluid and patches of gas, the mode conversions could cause a significant loss in the seismic data. This algorithm could be used for modeling sequestration of CO_2, and any other cases in which the fluid saturation changes through time.

Acknowledgements

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