Radiation patterns associated with the scattering from viscoelastic inclusions

Shahpoor Moradi* and Kris Innanen

*moradis@ucalgary.ca

Introduction

We obtained the radiation patterns associated with the scattering of seismic waves from five viscoelastic inclusions; density, P- and S-wave velocities and quality factors for P- and S-waves. We show that the polarization and slowness of viscoelastic waves are complex. Basically the radiation patterns from elastic and anelastic inclusions are given by the scattering potentials which are the amplitude of the spherical scattered waves from scatter points. We show that the scattering potentials are complex functions of averages in phase and attenuation angles.

Exact form of ray parameter and slowness vectors

The most important feature of the waves in a viscoelastic medium is that the wavenumber vector is a complex number whose imaginary part refers to the amplitude damping. As a result, slowness and polarization vectors are complex. The complex wavevector is given by

$$\mathbf{K} = \mathbf{P} - i\mathbf{A}, \tag{1}$$

where, propagation vector **P** is perpendicular to the wavefront and attenuation vector **A** is perpendicular to the plane of constant amplitudes and specified the direction of the maximum attenuation medium. The angle between these two vectors is called attenuation angle, δ, which is always less that 90°. In the case that the attenuation and propagation vectors are parallel the wave is called homogenous. Otherwise it is inhomogeneous. Wave speed for a homogeneous P and S-waves may be written as

$$V_{\rm H} = V_E\sqrt{\frac{2\chi_{\rm H}^2}{1+\chi_{\rm H}}}, \tag{2}$$

where

$$\chi_H = \sqrt{1+Q^{-2}}, \tag{3}$$

with quality factor Q. Complex wave-number is defined as

$$K = \sqrt{\mathbf{K}\cdot\mathbf{K}} = \frac{\omega}{V}, \tag{4}$$

where complex velocity V is defined by

$$V = \frac{V_H}{1 - i\frac{Q^{-1}}{1+\chi_H}}. \tag{5}$$

In the case of low-loss viscoelastic media as $Q^{-1} \ll 1$, we have

$$V_H \approx V_E, \qquad V \approx V_E\left(1 + \frac{i}{2}Q^{-1}\right). \tag{6}$$

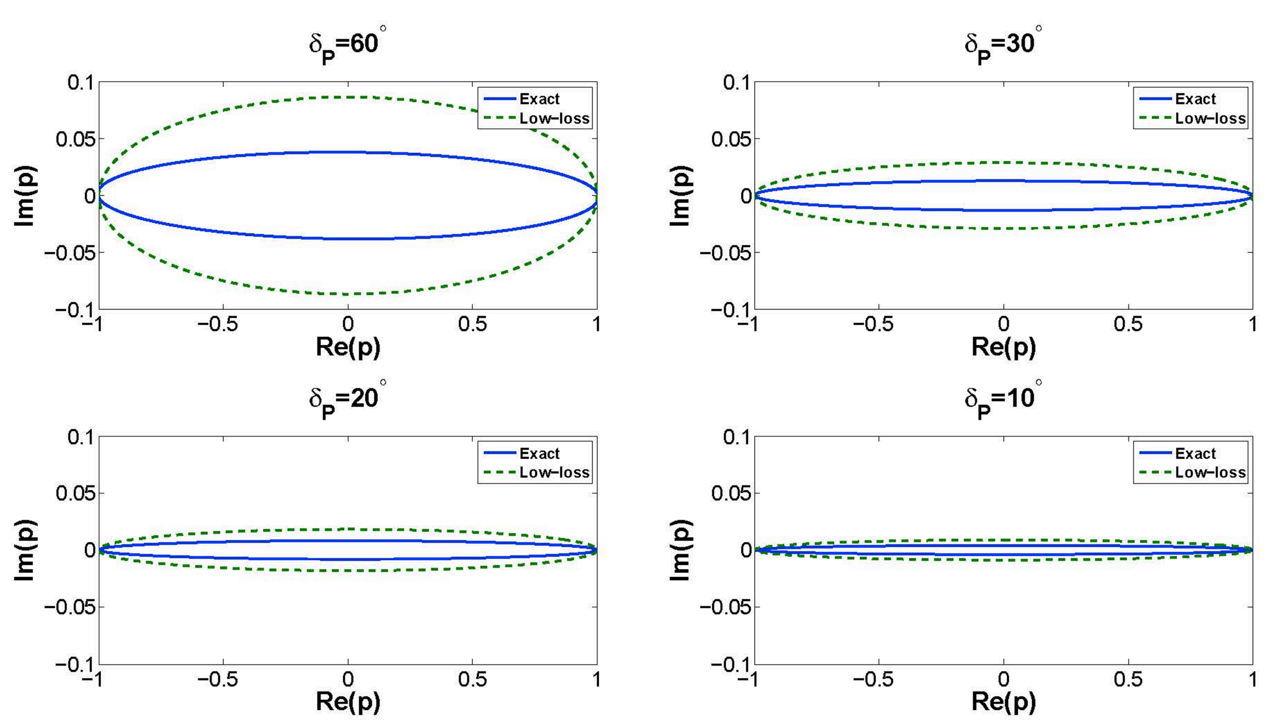

Figure 1: Digram illustrating the ray parameter versus phase angle in complex plane for different values of attenuation angle δ. Solid line refers to the general attenuation and dash line refers to the low-loss attenuation.

From study of complex vectors we know that the complex polarization displays the elliptical motion for a dynamic problem.

Linearized scattering matrix

The scattering potential in displacement space can be written as

$$^{R}_{I}\mathcal{V}_{ve} = \xi_I^T V_{ve}\xi_R. \tag{7}$$

Here ξ is the polarization vector, V_{ve} is the scattering operator and subscripts I and R respectively refer to the incident and reflected waves. The next task is to evaluate the scattering matrix in a system which naturally describes Borcherdt's viscoelastic modes $\mathrm{P}, \mathrm{SI}, \mathrm{SII}$, namely

$$\mathcal{V}_{ve} = \begin{pmatrix} {}^{P}_{P}\mathcal{V}_{ve} & 0 & {}^{P}_{SI}\mathcal{V}_{ve} \\ 0 & {}^{SII}_{SII}\mathcal{V}_{ve} & 0 \\ {}^{SI}_{P}\mathcal{V}_{ve} & 0 & {}^{SI}_{SI}\mathcal{V}_{ve} \end{pmatrix}. \tag{8}$$

Here, the diagonal elements represent the scattering which preserves the wave types and off-diagonal elements refer to scattering which converts the type of waves.

P-to-P scattering: In this case any inclusion in elastic and anelastic properties can scatter the P-wave to P-wave. In low-loss media the real part is equal to the elastic scattering potential, however for a general viscoelastic medium there is an extra term that is negligible in the low-loss limit. The components related to the inclusions in quality factors are pure imaginary terms. In Fig 2 we plot the real and imaginary parts of the P-to-P scattering potential for low-loss versus general viscoelastic medium. We can see the differences between the scattering potentials for medium with general anelastic properties and a medium with low-loss anelastic properties.

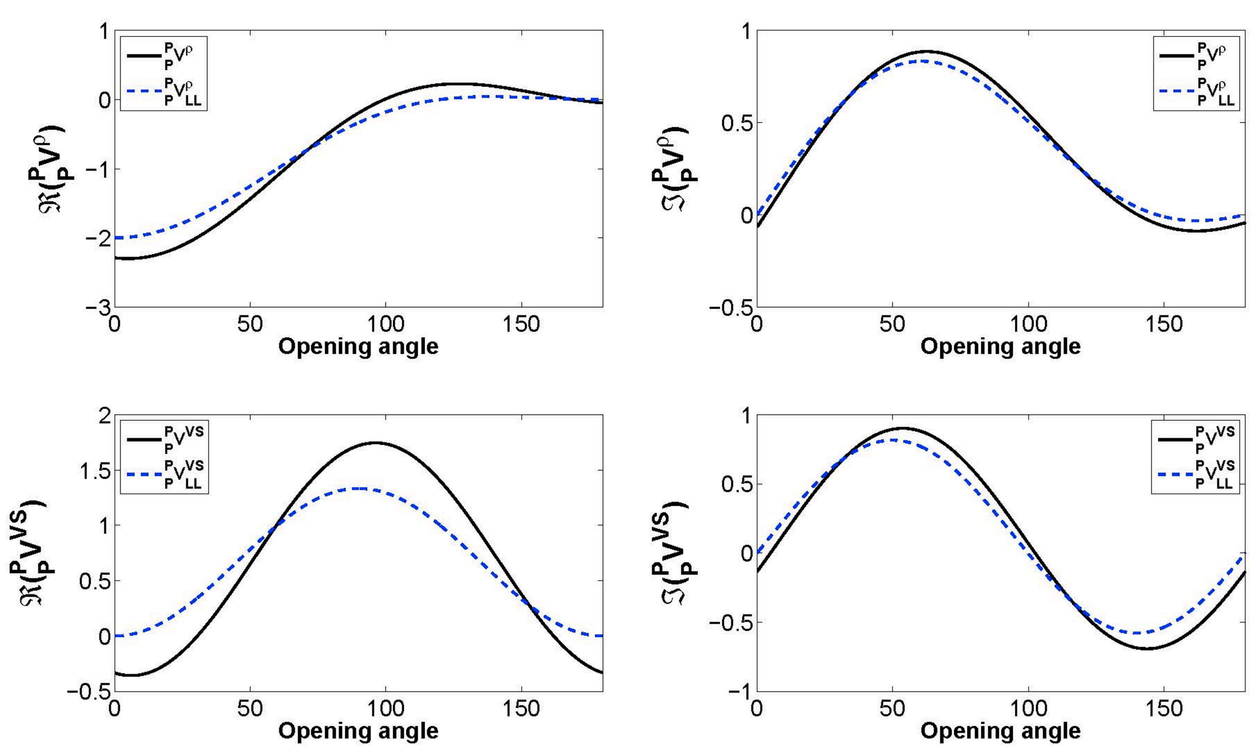

Figure 2: Digram illustrating the real and imaginary of density and S-wave velocity part of P-to-P scattering potential for $Q_P = 5$ and $Q_S = 4$ and attenuation angles $\delta_P = \delta_S = 70°$. The solid line refers to the arbitrary attenuation and dash line refers to the low-loss case.

P-to-SI scattering:In this case the scattering potential component related to the change in P-wave velocity is zero. It means that inclusion in P-wave velocity can not convert the P-wave to SI-wave. The similar situation is satisfied for the scattering of elastic case, where the P-wave velocity inclusion can not convert the elastic P-wave to the SV-wave. Consequently, the P-wave quality factor component vanishes also means that P-wave can not be converted to SI-wave due to interaction with the P-wave quality factor inclusion. In Fig 3 we plot the real and imaginary parts of the P-to-SI scattering potential for low-loss versus general viscoelastic medium.

SI-to-SI scattering:Similar to the scattering potential for P-to-SI, the non zero components are, density, S-wave velocity and S-wave quality factor. No contributions from change in P-wave velocity and P-wave quality factor in scattering of SI-wave to SI-wave. In Fig 4 we plot the real and imaginary parts of the SI-to-SI scattering potential for low-loss versus general viscoelastic medium.

Linearized scattering matrix continued

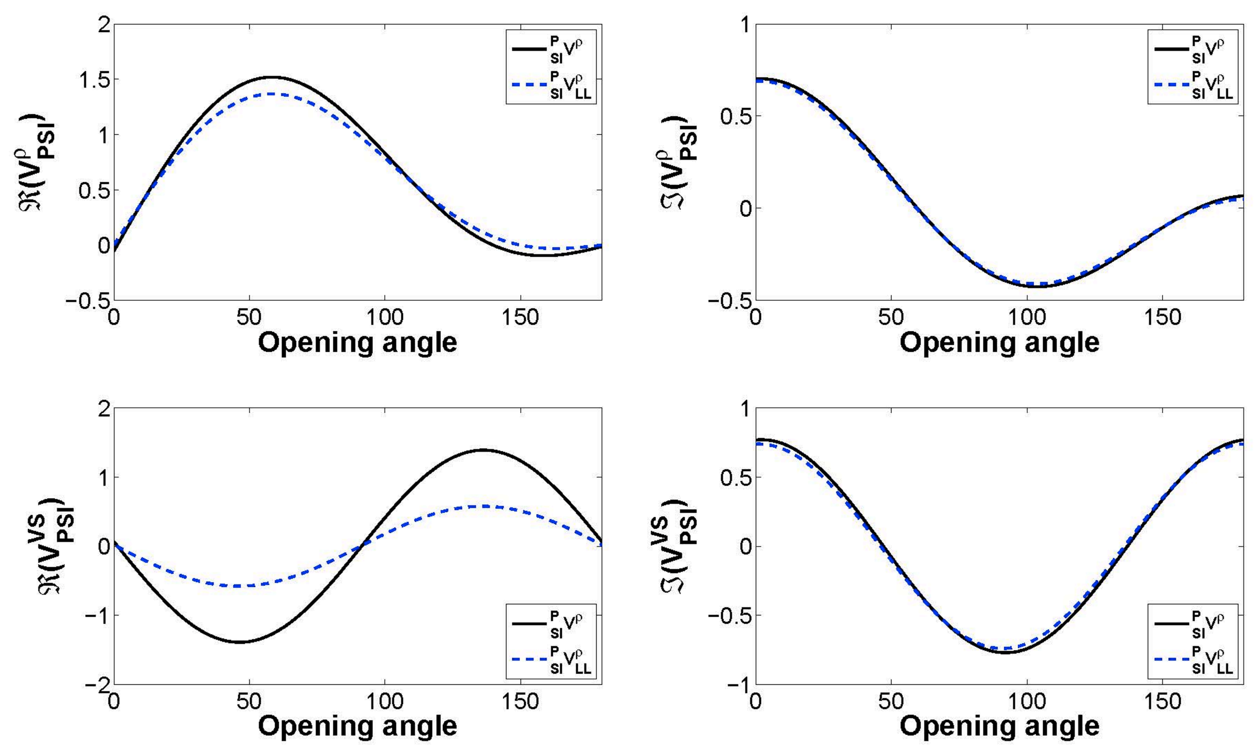

Figure 3: Digram illustrating the real and imaginary of density and S-wave velocity part of P-to-SI scattering potential. Medium properties are the same as figure 2.

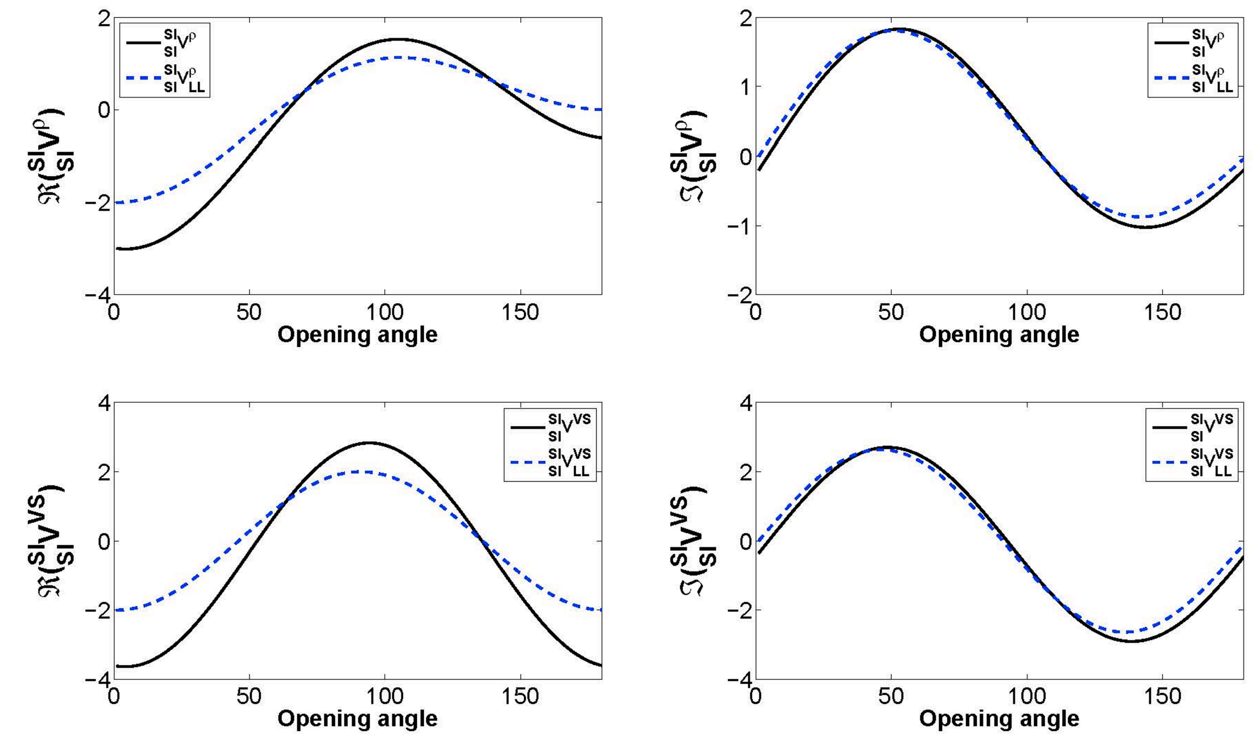

Figure 4: Digram illustrating the real and imaginary of density and S-wave velocity part of SI-to-SI scattering potential. Medium properties are the same as figure 2.

Conclusions

We introduce the scattering potentials for of viscoelastic waves using the Born approximation based on the scattering theory. These scattering potentials represent the radiation patterns generated by elastic and anelastic inclusions in a viscoelastic background. We generalized the viscoelastic scattering potentials obtained in low-loss medium to a medium with general attenuation properties. We show that the scattering potentials are complex functions of averages in phase and attenuation angles.

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Bibliography

Please see the report for references.