Recover low frequency for FWI via band-limited impedance inversion and POCS

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Summary

Full-waveform inversion (FWI) provides high-resolution estimates of the subsurface properties by iteratively minimizing a L-2 norm misfit function, which measures the difference between the modelled data and observed data. FWI suffers from cycle-skipping difficulty arising from inaccurate initial model and lack of low frequency information in the seismic data. In this paper, we aim at recovering low frequency information from well log data through band-limited impedance inversion. Projection-onto-convex-sets (POCS) algorithm is generally used to infill the missed traces in seismic data reconstruction. In this paper, we also consider to recover the low frequency information by spectral extrapolation with POCS algorithm. The reflectivity estimate is first generated. The frequency spectrum is then extrapolated with POCS algorithm. The band-limited impedance inversion is then performed with the reflectivity section and interpolated well log model. Through this process, we build an enhanced initial model for full-waveform inversion with low frequency information. we illustrate with numerical examples that the inversion results can be improved with the enhanced initial velocity model.

Principle of FWI

The objective function of FWI is formulated as a L-2 norm:

$$\Phi(\mathbf{m}) = \frac{1}{2}\sum_{\mathbf{x}_s}\sum_{\mathbf{x}_g}\sum_{\omega}\|\Delta\mathbf{d}(\mathbf{x}_g,\mathbf{x}_s,\omega)\|^2$$

The gradient can be constructed by cross-correlating the forward modelled wavefield with the back-propagated residual wavefield with adjoint-state method:

$$\mathbf{g}(\mathbf{x}) = \sum_{\mathbf{x}_g}\sum_{\mathbf{x}_s}\sum_{\omega}\Re\left(\omega^2 f_s(\omega)\, G(\mathbf{x},\mathbf{x}_s,\omega)\, G(\mathbf{x}_g,\mathbf{x},\omega)\,\Delta\mathbf{d}^*(\mathbf{x}_g,\mathbf{x}_s,\omega)\right)$$

The search direction is obtained by solving the Newton linear system:

$$\mathbf{H}_k\Delta\mathbf{m}_k = -\mathbf{g}_k$$

where **H** is the Hessian matrix, which represents the second-order partial derivative of the misfit function with respect to the model parameter.

FWI suffers from cycle-skipping difficulty, which arises from the lack of low frequency information and inaccurate initial model.

Hence, the low frequency information is crucial for retrieving the long wavelength structure of the velocity model and overcome the cycle-skipping problem

Band-limited impedance (BLIMP) inversion

The band-limited impedance inversion is used for seismic data inversion with incorporating low frequency information from well log data. The impedance is obtained by the following equation:

$$I_{j+1} = I_1\mathbf{exp}\left(2\sum_{k=1}^{j} r_k\right)$$

where I_{j+1} indicates the impedance and r_k indicates the reflectivity.

POCS algorithm

Projection onto convex sets (POCS) algorithm is generally employed to infill missing seismic data. Here, we employ the POCS algorithm to recover low frequency information for FWI. POCS algorithm is implemented in an iterative scheme.

$$s_{k+1}(t) = F^{-1}\left(\mathcal{H}F(\Gamma_k s_k(t)) + (1-\mathcal{H})F(s_k(t))\right)$$

where F and F^{-1} indicate Fourier and inverse Fourier transform respectively. The updated trace $s_{k+1}(t)$ is obtained in terms of the trace $s_k(t)$. And $\mathcal{H} = H(f-f_0) - H(f+f_0)$.

Numerical Example

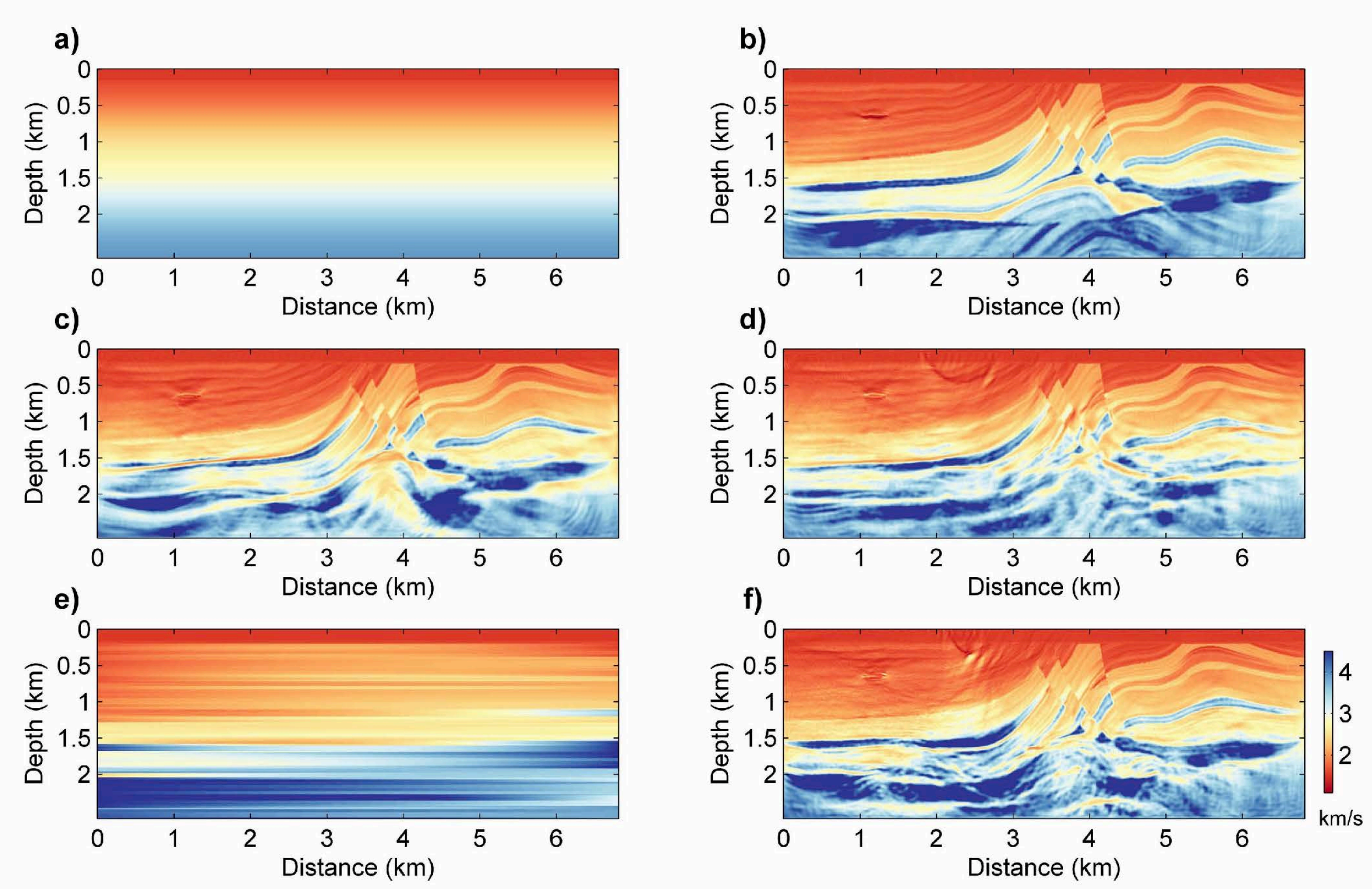

Figure 1. (a) The linear initial P-wave velocity model; (b) The inverted model (1-30 Hz); (c) 3-30 Hz; (d) 6-30 Hz; (e) The interpolated model with well log data; (f) The inverted model with well data regularization (6 Hz-30 Hz).

Recover low frequency with POCS algorithm

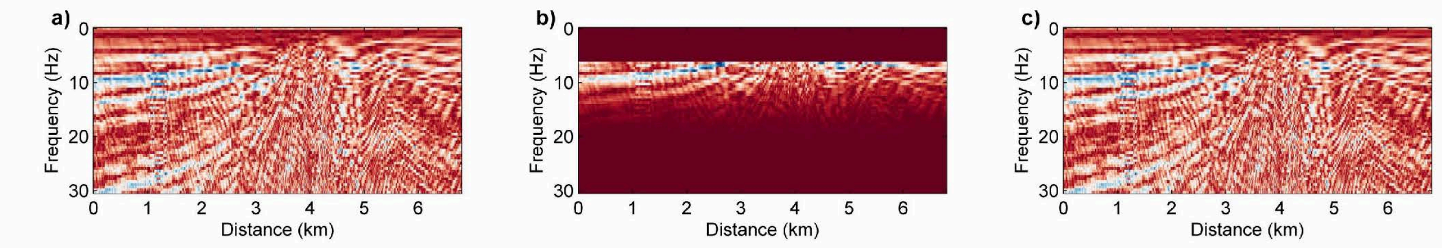

Figure 2. (a) is the amplitude spectrum of the true time reflectivity section; (b) is the amplitude spectrum of the band-limited time reflectivity section (6 Hz-30 Hz); (c) is the amplitude spectrum of the extrapolated time reflectivity section with POCS algorithm.

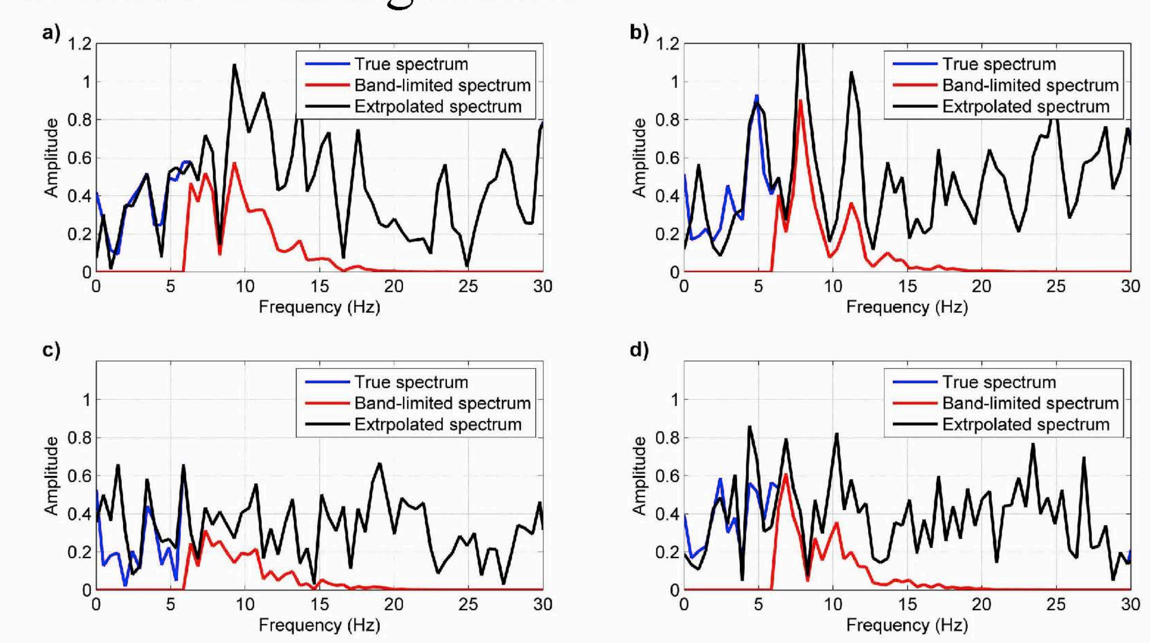

Figure 3. Comparison the spectrums of true reflectivity, band-limited reflectivity and recovered reflectivity after POCS extrapolation at 0.1 km (a), 2.0 km (b), 4.0 km (c) and 6.0 km respectively.

Recover low frequency via BLIMP

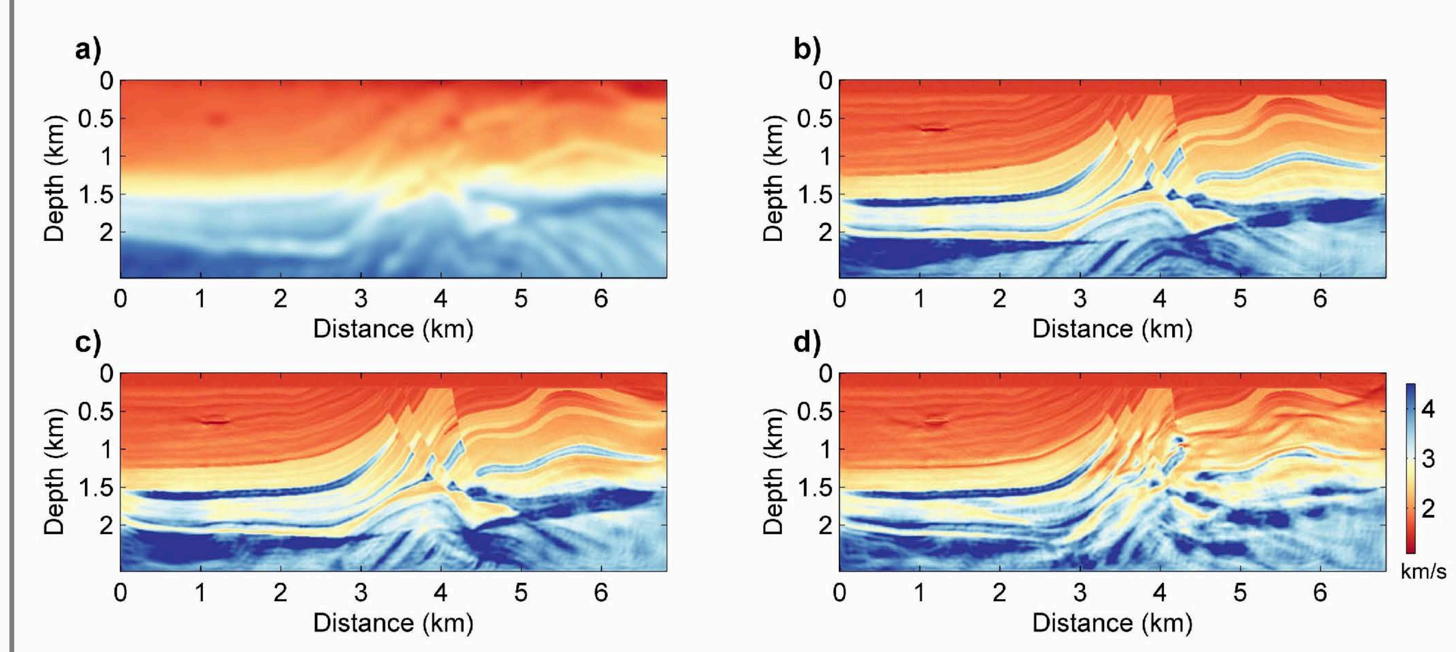

Figure 4. (a) The enhanced initial model; (b) The inverted model (1-30 Hz); (c) 3-30 Hz; (d) 6-30 Hz.

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