AVO inversion in P-, S- moduli with a combination of series reversion and Gauss-Newton iteration

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Abstract

Fluid-property discrimination is always an important goal we desired in seismic exploration, and P-, S- wave moduli can be applied to calculate fluid term. We first derived a non-linear approximation of Zoeppritz equation in terms of P-, S- moduli and density, and then implemented a non-linear AVO inversion using new approximation with a combination of series reversion and Gauss-Newton iteration. A comparison between linear and new approach was discussed.

Poroelasticity AVO approximation

The poroelasticity theory of Biot (1941) and Gassmann (1951) are most frequently way to express the P- and S- wave velocity in terms of elastic moduli in porous saturated rocks. In the formation, the fluid term f represents the interaction of the fluid filling the porosity with solid dry, which can be calculated by

$$f = \beta^2 M_v = M_{sat} - \gamma_{dry}^2 \mu_{sat}$$

where, M_v is the bulk modulus, M_{sat} and μ_{sat} are P- and S- wave moduli, and γ_{dry} is the v_p/v_s ratio. The fluid influence can be obtained with P- and S- moduli.

Starting with Zoeppritz equation, we derived a non-linear AVO approximation in terms of P-, S- wave moduli by applying Cramer's rule, both in perturbation form,

$$R_{pp}^{(1)} = \left(\frac{\sec^2\theta_0}{4}\right)a_M - \left(\frac{2}{\gamma_{sat}^2}\sin^2\theta_0\right)a_\mu + \left(\frac{1}{2} - \frac{\sec^2\theta_0}{4}\right)a_\rho$$

$$R_{pp}^{(2)} = \Gamma_{2M}^{pp}a_M^2 + \Gamma_{2\mu}^{pp}a_\mu^2 + \Gamma_{2\rho}^{pp}a_\rho^2 + \Gamma_{M\rho}^{pp}a_M a_\rho + \Gamma_{\mu\rho}^{pp}a_\mu a_\rho + \Gamma_{M\mu}^{pp}a_M a_\mu$$

$$R_{pp}^{(3)} = \Gamma_{3M}^{pp}a_M^3 + \Gamma_{3\mu}^{pp}a_\mu^3 + \Gamma_{3\rho}^{pp}a_\rho^3 + \Gamma_{2\mu\rho}^{pp}a_\mu^2 a_\rho + \Gamma_{2\mu M}^{pp}a_\mu^2 a_M + \Gamma_{\mu 2\rho}^{pp}a_\mu a_\rho^2 + \Gamma_{\mu 2M}^{pp}a_\mu a_M^2 + \Gamma_{M2\rho}^{pp}a_M a_\rho^2 + \Gamma_{\rho 2M}^{pp}a_\rho a_M^2 + \Gamma_{M\mu\rho}^{pp}a_M a_\mu a_\rho$$

and in ratio (reflectivity) form,

$$R_{pp}^{(1)} = \left(\frac{\sec^2\theta_0}{4}\right)\frac{\Delta M}{M} - \left(\frac{2}{\gamma_{sat}^2}\sin^2\theta_0\right)\frac{\Delta\mu}{\mu} + \left(\frac{1}{2} - \frac{\sec^2\theta_0}{4}\right)\frac{\Delta\rho}{\rho}$$

$$R_{pp}^{(2)} = \Lambda_{2M}^{pp}\left(\frac{\Delta M}{M}\right)^2 + \Lambda_{2\mu}^{pp}\left(\frac{\Delta\mu}{\mu}\right)^2 + \Lambda_{2\rho}^{pp}\left(\frac{\Delta\rho}{\rho}\right)^2 + \Lambda_{M\mu}^{pp}\frac{\Delta M}{M}\frac{\Delta\mu}{\mu} + \Lambda_{M\rho}^{pp}\frac{\Delta M}{M}\frac{\Delta\rho}{\rho} + \Lambda_{\mu\rho}^{pp}\frac{\Delta\mu}{\mu}\frac{\Delta\rho}{\rho}$$

$$R_{pp}^{(3)} = \Lambda_{3M}^{pp}\left(\frac{\Delta M}{M}\right)^3 + \Lambda_{3\mu}^{pp}\left(\frac{\Delta\mu}{\mu}\right)^3 + \Lambda_{3\rho}^{pp}\left(\frac{\Delta\rho}{\rho}\right)^3 + \Lambda_{2\mu\rho}^{pp}\left(\frac{\Delta\mu}{\mu}\right)^2\frac{\Delta\rho}{\rho} + \Lambda_{2\mu M}^{pp}\left(\frac{\Delta\mu}{\mu}\right)^2\frac{\Delta M}{M} + \Lambda_{\mu 2\rho}^{pp}\frac{\Delta\mu}{\mu}\left(\frac{\Delta\rho}{\rho}\right)^2 + \Lambda_{\mu 2M}^{pp}\frac{\Delta\mu}{\mu}\left(\frac{\Delta M}{M}\right)^2 + \Lambda_{M2\rho}^{pp}\frac{\Delta M}{M}\left(\frac{\Delta\rho}{\rho}\right)^2 + \Lambda_{\rho 2M}^{pp}\frac{\Delta\rho}{\rho}\left(\frac{\Delta M}{M}\right)^2 + \Lambda_{M\mu\rho}^{pp}\frac{\Delta M}{M}\frac{\Delta\mu}{\mu}\frac{\Delta\rho}{\rho}$$

Therefore, reflection coefficient of PP-wave can be written as

$$R_{pp} = R_{pp}^{(1)} + R_{pp}^{(2)} + R_{pp}^{(3)} + \ldots$$

where, $R_{pp}^{(i)}$ is the i^{th} order in series expansion of R_{pp}. Reflection coefficient versus incidence (or average angle) in different truncated forms were shown in Figure 1. In view of the comparisons, 2^{nd} order truncated R_{pp} in ratio form with angle incidence was considered as the approximated formula for AVO inversion.

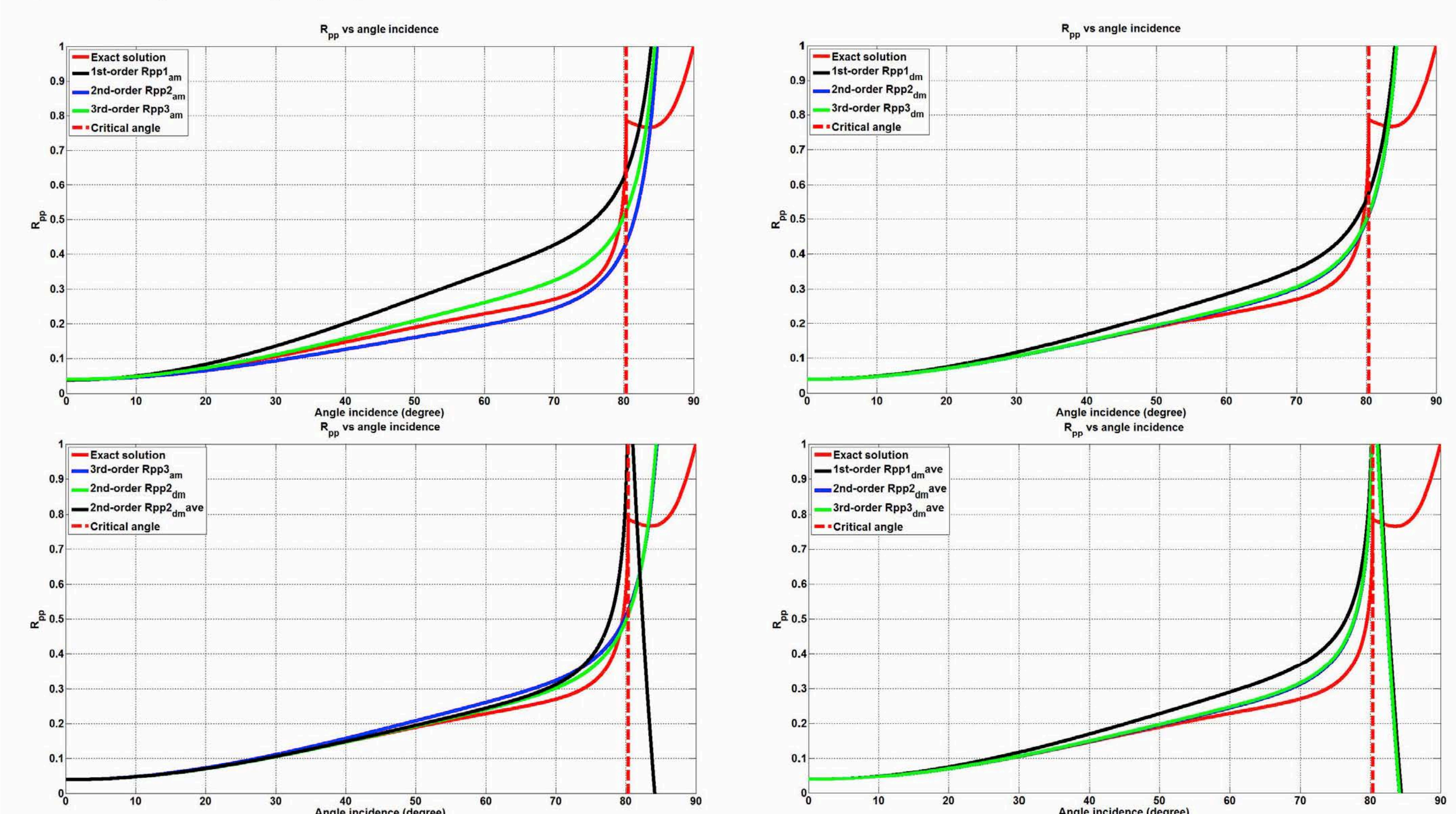

FIG. 1. Comparisons of R_{pp} in different truncated forms.

AVO with series reversion and Gauss-Newton

As mentioned, reflection coefficient can be written in a series expansion,

$$R_{pp} = R_{pp}^{(1)} + R_{pp}^{(2)} = A_1 d_M + B_1 d_\mu + C_1 d_\rho + A_2 d_M^2 + B_2 d_\mu^2 + C_2 d_\rho^2 + D_{M\mu} d_M d_\mu + D_{M\rho} d_M d_\rho + D_{\mu\rho} d_\mu d_\rho;$$

Which is a non-linear problem, and Gauss-Newton algorithm is a classic way to solve non-linear least square problem, the objective function can be expressed as

$$F = \sum_{i=1}^{N} r_i^2 = \sum_{i=1}^{N} (\varphi_i - s_i)^2$$

where, φ_i is the predicted value, and s_i is the observed data. and the i^{th} iteration can be described as

$$x_{i+1} = x_i - H_i^{-1}\overrightarrow{g_i}$$

One of the problems of Gauss-Newton iteration is local convergence, and a proper initial input is the key for that. To solve that, considering series reversion, the idea of solving series reversion is equating both sides of equation like orders in series expansion, then we have,

$$R_{pp} = R_{pp}^{(1)}; 0 = R_{pp}^{(2)}; 0 = R_{pp}^{(3)}; \ldots$$

The preliminary result from series reversion can be a valuable input for Gauss-Newton iteration. And Well 12-27 is applied to build a synthetic model to examine the approach.

Figure 2 shows well-log 12-27 collected by CREWES at Hussar in 2011, which includes P-, S- sonic logs, density and gamma logs. A synthetic angle gather from 0 to 50 degrees was generated using Zoeppritz equation. Linear inversed results and iterative inversion are shown in Figure 3, and errors analysis are shown in Figure 4.

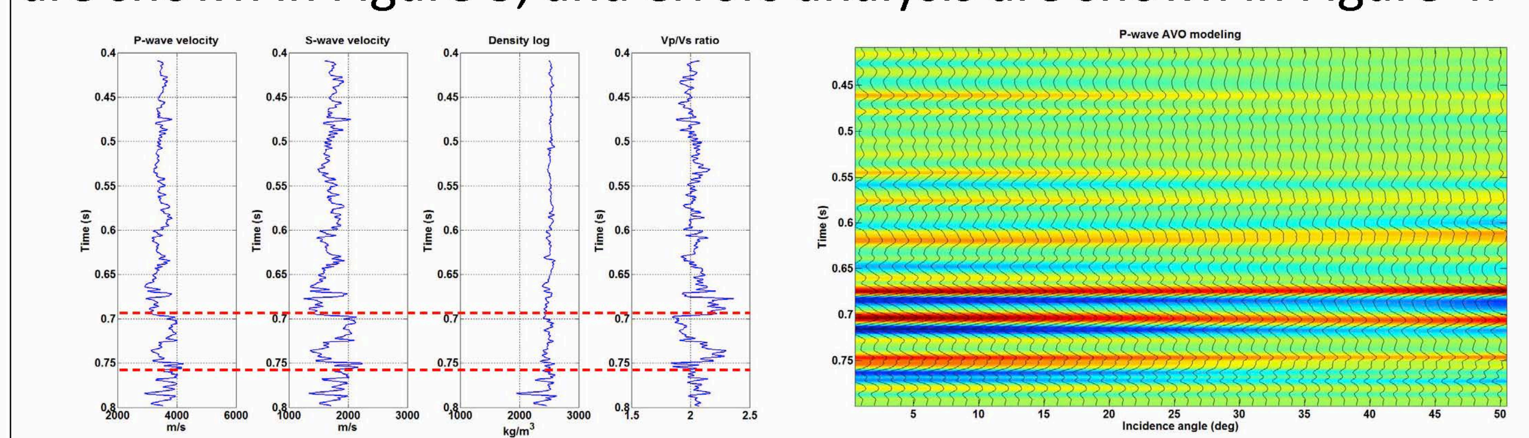

FIG. 2. P-, S-wave velocity and density of well 12-27 at Hussar, and a synthetic angle gather generated using Zoeppritz equation

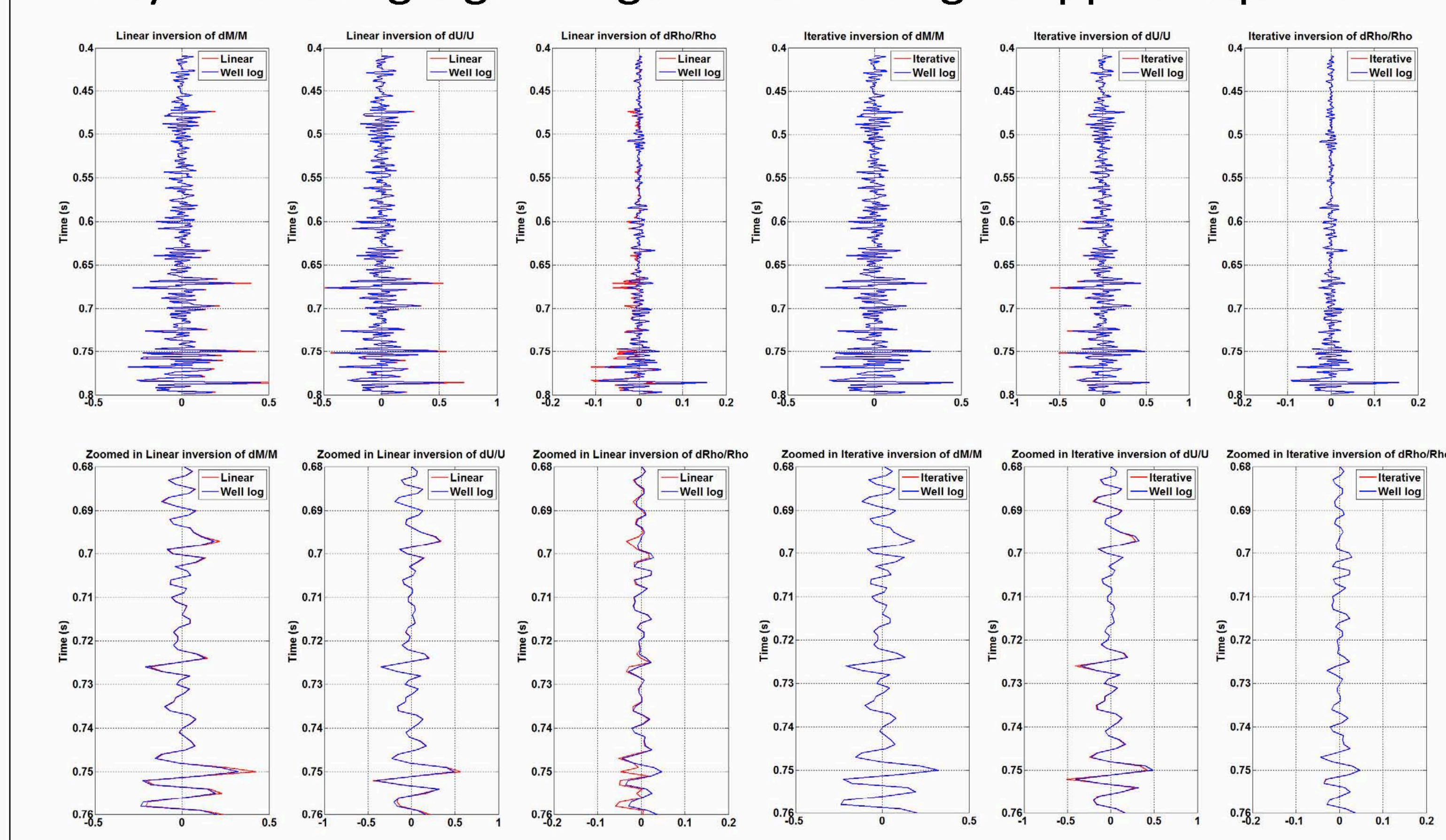

FIG. 3. Linear results and iterative inversions (enlarged view)

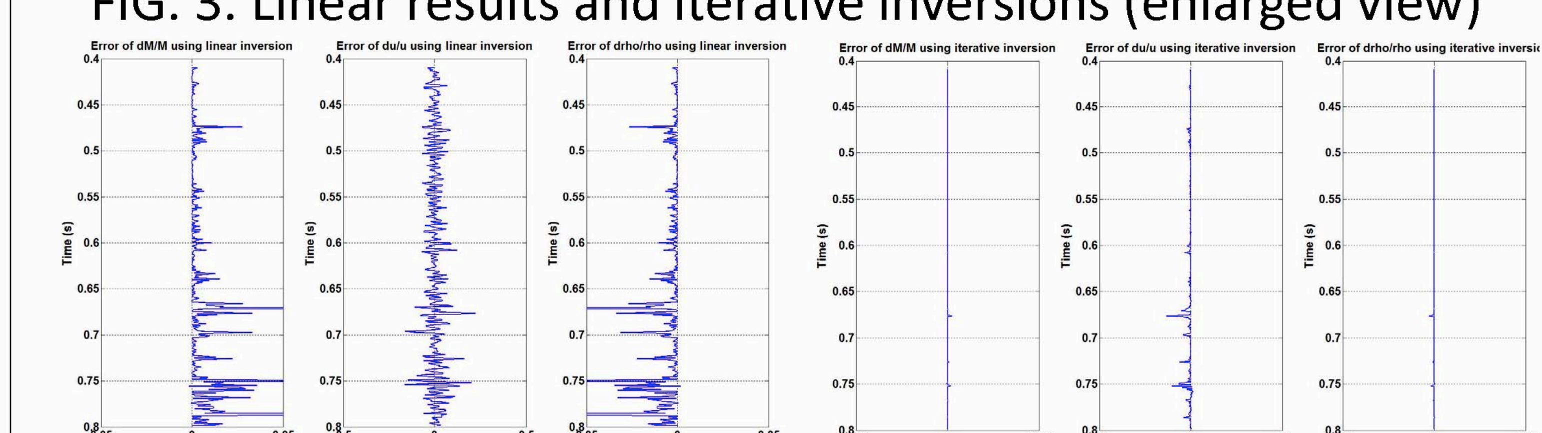

FIG. 4. Errors of linear and iterative inversions, respectively

Conclusion

Non-linear AVO approximation in terms of P-, S- moduli and density are derived, both in perturbation form and in ratio form. A non-linear AVO inversion with series reversion and Gauss-Newton are implemented.

Acknowledgements and References

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References can be found in relevant report.