The relationship between scattering theory and the reflection coefficient for elastic media: Forward scattering series

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Abstract

The scattering theory can be used as a powerful theoretic approach to understand and process seismic data. Exploring inverse scattering series, which have been used to remove multiples from seismic data, depends on understanding how these series generate primaries and multiples. The inverse scattering methods depend on an understanding of how the forward scattering series generates primaries and multiples. In this work, we study the forward scattering series for elastic media in order to identify on which terms in inverse scattering series are important for imaging and inversion. Primary reflections are described by all of the terms in the series excluding the first term.

Forward scattering series

We consider the two dimensional Born series (Innanen, 2009):

$$P(x_g,z_g,x_s,z_s,\omega) = G_0(x_g,z_g,x_s,z_s,\omega)$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_0(x_g,z_g,x',z',\omega)V(x',z')G_0(x',z',x_s,z_s,\omega)dx'dz'$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_0(x_g,z_g,x',z',\omega)V(x',z')\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_0(x',z',x'',z'',\omega)$$
$$\times V(x',z')G_0(x'',z'',x_s,z_s,\omega)dx''dz''+\cdots$$
$$= P_0+P_1+P_2+\cdots$$

This equation plays a pivotal role in scattering theory. Based on this equation the wavefield in an actual medium is the sum of the wavefield in a reference medium and integral that represent the scattered wavefield due to perturbation .

P to P scattering by a single interface

The pp scattering potential in terms of velocity and density perturbations is

$$\mathcal{V}_{PP} = -\rho_0\omega^2[a_\alpha + a_\rho\left(1+\cos\sigma-\frac{2\beta_0^2}{\alpha_0^2}sin^2\sigma\right)-a_\beta\frac{2\beta_0^2}{\alpha_0^2}sin^2\sigma]$$

The first order term in the Born series is given by (when $z_g< z_1$)

$$P_1(x_g,z_g,x_s,z_s,\omega) =$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_0(x_g,z_g,x',z',\omega)V(z')G_0(x',z',x_s,z_s,\omega)dx'dz'$$

Where x_g, z_g and x_s, z_s are respectively the position of the receiver and source. The function G_0 describes propagation in the reference medium, and can be written as a 2D Green's function bilinear form.

$$G_0(x_g,z_g,x',z',\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}dk'_x\int_{-\infty}^{\infty}dk'_z\frac{e^{ik'_x(x_g-x')}e^{ik'_z(z_s-z')}}{k^2-k'^2_x-k'^2_z}$$

$$G_0(x',z',x_s,z_s,\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}dk''_x\int_{-\infty}^{\infty}dk''_z\frac{e^{ik''_x(x'-x_s)}e^{ik''_z(z'-z_s)}}{k^2-k''^2_x-k''^2_z}$$

After the Fourier transform over x_g and x_s and using $q_g^2 = k^2-k_g^2$ and $q_s^2 = k^2-k_s^2$, $\widehat{P}1$ can be obtain as

$$\hat{P}1(k_g,z_g,-k_s,z_s,\omega) = -\frac{C_P}{4cos^2\theta}e^{-iq_g(z_g+z_s)}[a_\alpha(-2q_g)$$
$$+a_\rho(-2q_g)\left(1+\cos\sigma-\frac{2\beta_0^2}{\alpha_0^2}sin^2\sigma\right)-a_\beta(-2q_g)\frac{2\beta_0^2}{\alpha_0^2}sin^2\sigma]$$

The higher order terms in the Born series have an important role when the perturbation value is lager while the higher order terms become less important for small value of perturbation and tha Born approximation is valid

$$\hat{P}_{Born} \cong \hat{P}_0 + \hat{P}_1$$

In this expression, the first term propagates outward from the source directly to receiver. The second term is a reflected wave. The reflection coefficient, which is the ration of the amplitude of the incident and reflected wave, for the Born approximation can be written as ($z_g = z_s = 0$)

$$R_1^{PP}(\theta) \approx \frac{1}{2(1+cos\sigma)}a_\alpha + \left(\frac{1}{2}-\frac{\beta_0^2}{\alpha_0^2}(1-cos\sigma)\right)a_\rho$$
$$-\frac{\beta_0^2}{\alpha_0^2}(1-cos\sigma)a_\beta$$

Where $\sigma = 2\theta$ is the opening angle.

Numeric examples: single interface

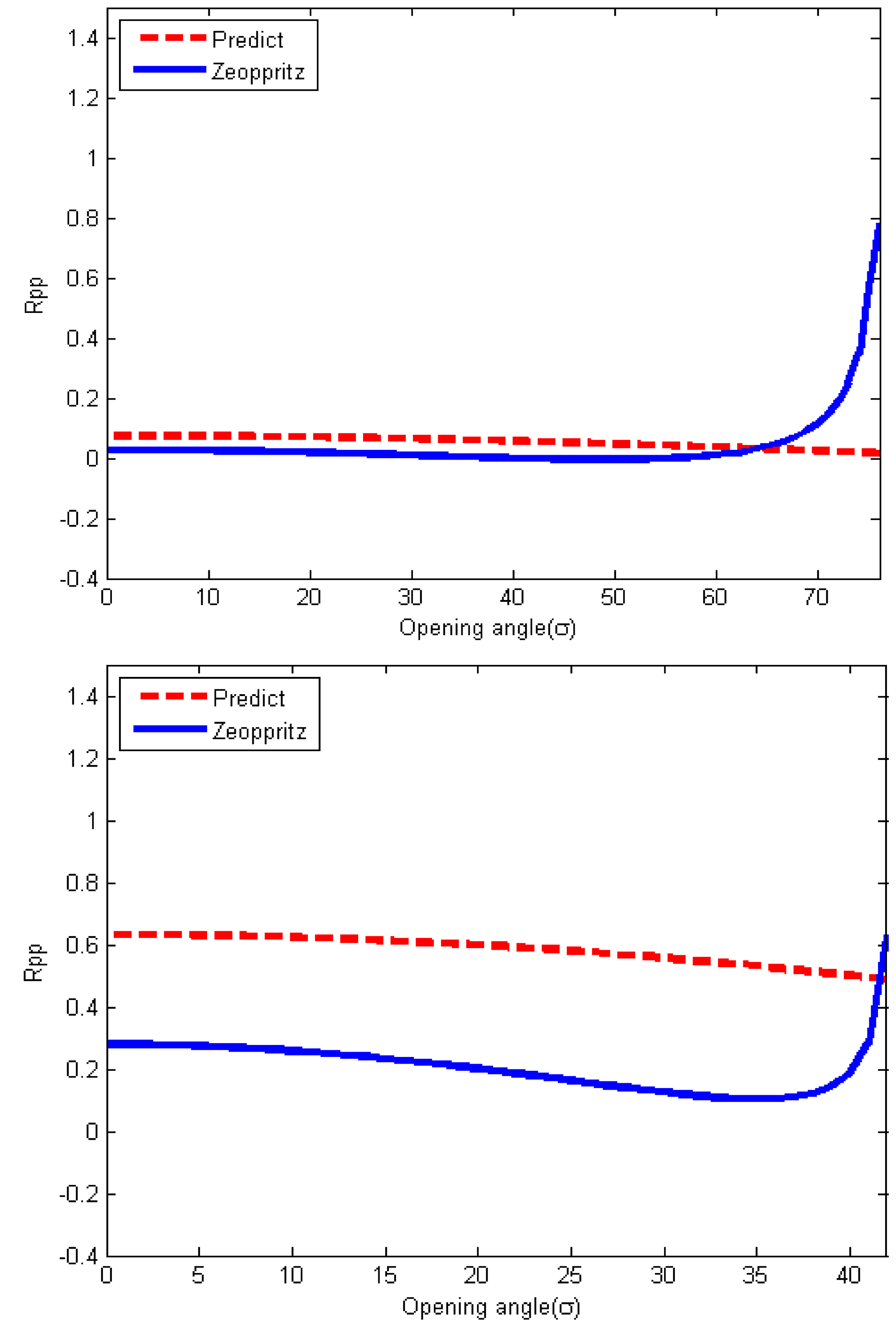

Figure 1: The comparison between the synthesized values and the actual values of Rpp for small and large layer contrast models.

Conclusions

The scattering theory is applied to investigate a mapping method between the earth model and seismic data. The Born series is established and full series terms are derived. Theses series were able to predict and interpret seismic reflection data including primary and multiple events. To identify on which terms in inverse scattering series are important for imaging and inversion the forward scattering series for elastic media is investigated. The results show that the exact curve of R_{PP} is complex beyond critical angles, while the approximation curve reminds real and decreasing for all opening angle that is smaller than the critical angles.

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