Significance and behaviour of the homogeneous and inhomogeneous components of linearized viscoelastic reflection coefficients

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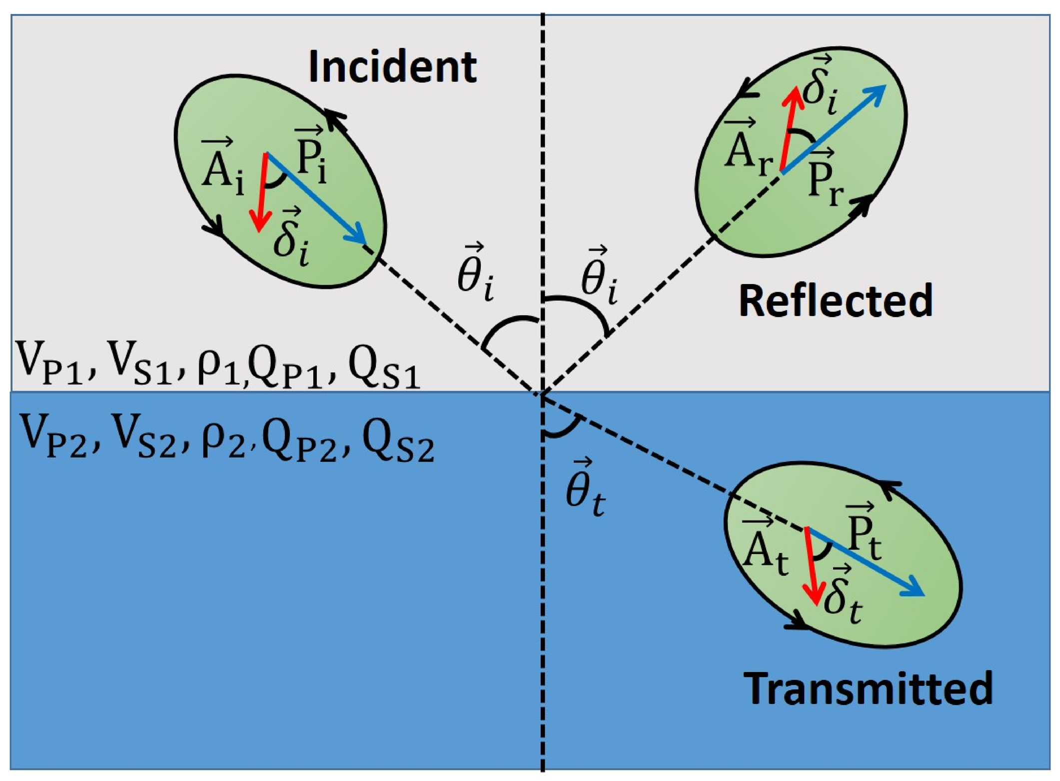
Introduction

The formulation of the amplitude-versus-offset equations for viscoelastic media is of increasing interest and importance, with quantitative interpretation of seismic data being deployed to characterize fluid presence, type, and viscosity in hydrocarbon reservoirs, CO_2 injection sites, and other exploration and monitoring settings. Properly formulated, these equations also provide insights into the character of eventual viscoelastic full waveform inversion algorithms. To date, investigations and analysis of anelastic reflection coefficients have been constructed on the assumption that the attenuation angle is unchanged across the boundary, which cannot be generally justified. We believe that a more fruitful approach approach is to apply an appropriate version of Snell's law in such way that transmitted and reflected attenuation angles are expressed in terms of the incident attenuation angle. This approach allows changes in attenuation angle to be expressed in terms of changes in velocity and quality factors, leading to new terms in the relevant AVO equations with a wider capture of anelastic reflection and transmission phenomena incorporated.

Viscoelastic Zoeppritz equations

Our interest is to be able to separately analyze and predict behaviour of the homogeneous versus inhomogeneous components of viscoelastic waves having reflected from and transmitted through a planar boundary.



	Shale	Salt	Limestone	Limestone(gas)
V_{PE}(km/s)	3.811	4.537	5.335	5.043
V_{SE}(km/s)	2.263	2.729	2.957	2.957
ρ(gm/cm^3)	2.40	2.005	2.65	2.49

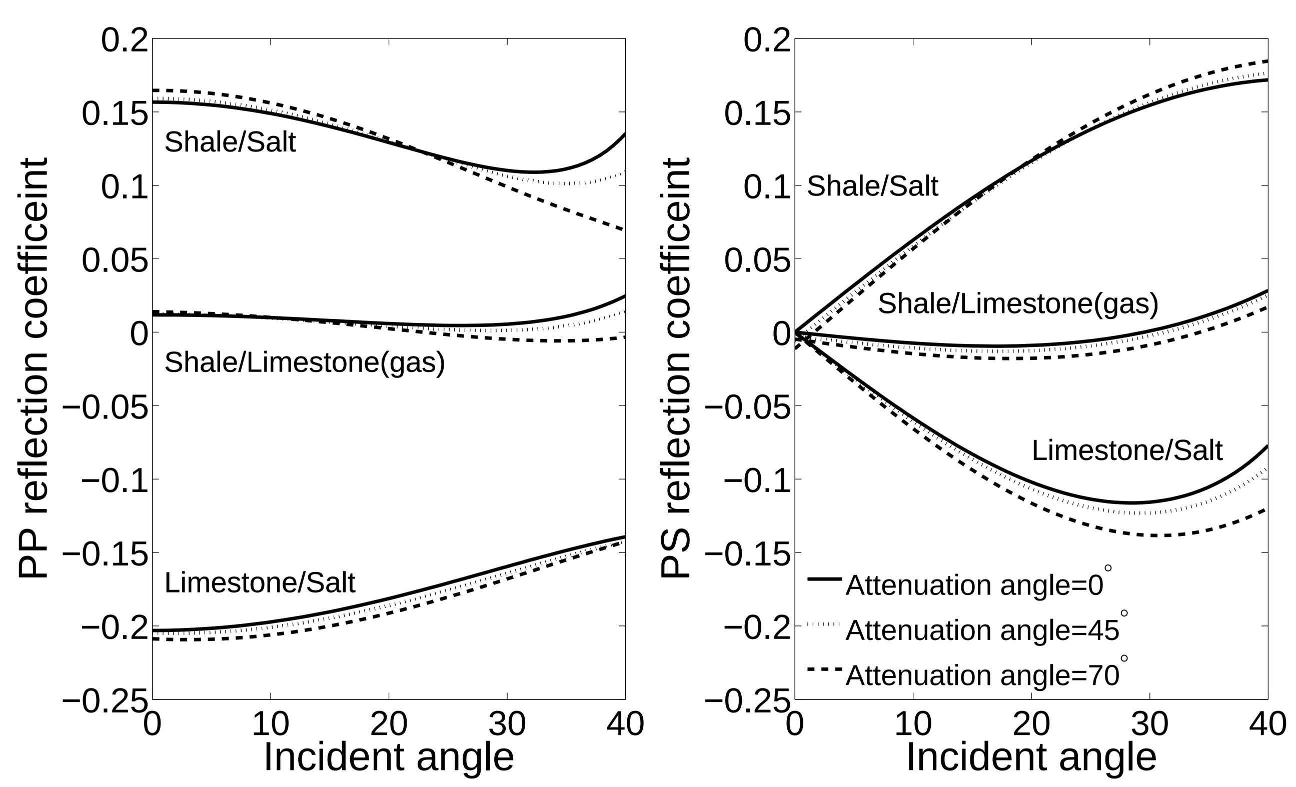


Figure 1: Comparing the real part of the exact viscoelastic PP/PS-reflectivity for $\delta_P = 0^\circ, 45^\circ, 70^\circ$ for four selected models from table .

The viscoelastic Shuey approximation

The small-offset linearized P-to-P reflection coefficient for an inhomogeneous seismic wave reflected from boundary of two isotropic viscoelastic media under the assumption of small contrast interface is given by

$$\mathrm{R_{PP}}(\theta_P,\delta_P)=\mathrm{R^E_{PP}}(\theta_P)+i\mathrm{R^H_{PP}}(\theta_P)+i\mathrm{R^{IH}_{PP}}(\theta_P,\delta_P),$$

with elastic, anelastic-homogenous and anelastic-inhomogeneous terms given by

$$\begin{aligned}\mathrm{R^E_{PP}}(\theta_P)&=A^E_{PP}+B^E_{PP}\sin^2\theta_P+C^E_{PP}(\tan^2\theta_P-\sin^2\theta_P),\\ \mathrm{R^H_{PP}}(\theta_P)&=A^H_{PP}+B^H_{PP}\sin^2\theta_P+A^H_{PP}(\tan^2\theta_P-\sin^2\theta_P),\\ \mathrm{R^{IH}_{PP}}(\theta_P,\delta_P)&=A^{IH}_{PP}\tan\theta_P+B^{IH}_{PP}\tan\theta_P\sin^2\theta_P+C^{IH}_{PP}\tan\theta_P(\tan^2\theta_P-\sin^2\theta_P),\end{aligned}$$

where

$$A^E_{PP}=\frac{1}{2}\left[\frac{\Delta\rho}{\rho}+\frac{\Delta V_P}{V_P}\right]$$

$$B^E_{PP}=\frac{1}{2}\frac{\Delta V_P}{V_P}-2\left(\frac{V_S}{V_P}\right)^2\left[\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right]$$

$$C^E_{PP}=\frac{1}{2}\frac{\Delta V_P}{V_P},\ A^H_{PP}$$

$$B^H_{PP}=-2\left(\frac{V_S}{V_P}\right)^2\left[(Q_P^{-1}-Q_S^{-1})\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)+Q_S^{-1}\frac{\Delta Q_S}{Q_S}\right]-\frac{1}{4}Q_P^{-1}\frac{\Delta Q_P}{Q_P},$$

$$A^{IH}_{PP}=Q_P^{-1}\tan\delta_P\left[\frac{1}{2}\frac{\Delta V_P}{V_P}-2\left(\frac{V_S}{V_P}\right)^2\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)\right]$$

$$B^{IH}_{PP}=Q_P^{-1}\tan\delta_P\left[\frac{1}{2}\frac{\Delta V_P}{V_P}+2\left(\frac{V_S}{V_P}\right)^2\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)\right]$$

$$C^{IH}_{PP}=Q_P^{-1}\tan\delta_P\frac{1}{2}\left[\frac{\Delta V_P}{V_P}\right].$$

For converted PS wave using standard approximations for trigonometric functions for small angles, and collecting the powers of $\sin\theta_P$, we obtain

$$\mathrm{R_{PS}}(\theta_P,\delta_P)=\mathrm{R^E_{PS}}(\theta_P)+i\mathrm{R^H_{PS}}(\theta_P)+i\mathrm{R^{IH}_{PS}}(\theta_P,\delta_P),$$

where the elastic, homogenous and inhomogeneous terms are given by

$$\begin{aligned}\mathrm{R^E_{PS}}(\theta_P)&=A^E_{PS}\sin\theta_P+B^E_{PS}\sin^3\theta_P,\\ \mathrm{R^H_{PS}}(\theta_P)&=A^H_{PS}\sin\theta_P+B^H_{PS}\sin^3\theta_P,\\ \mathrm{R^{IH}_{PS}}(\theta_P,\delta_P)&=A^{IH}_{PS}+B^{IH}_{PS}\sin^2\theta_P,\end{aligned}$$

where

$$A^E_{PS}=-\left(\frac{1}{2}+\frac{V_S}{V_P}\right)\frac{\Delta\rho}{\rho}-2\frac{V_S}{V_P}\frac{\Delta V_S}{V_S},$$

$$B^E_{PS}=\frac{V_S}{V_P}\left[\left(\frac{1}{2}+\frac{3}{4}\frac{V_S}{V_P}\right)\frac{\Delta\rho}{\rho}+2\left[\frac{1}{2}+\frac{V_S}{V_P}\right]\frac{\Delta V_S}{V_S}\right],$$

$$A^H_{PS}=\frac{V_S}{V_P}\left\{Q_S^{-1}\frac{\Delta Q_S}{Q_S}-\frac{1}{2}(Q_S^{-1}-Q_P^{-1})\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)\right\},$$

$$\begin{aligned}B^H_{PS}=&-\frac{V_S}{V_P}\left[\frac{1}{2}+\frac{V_S}{V_P}\right]Q_S^{-1}\frac{\Delta Q_S}{Q_S}-\frac{1}{4}\left(\frac{V_S}{V_P}\right)^2(Q_S^{-1}-Q_P^{-1})\frac{\Delta\rho}{\rho}\\ &+\frac{1}{4}\frac{V_S}{V_P}\left(1+4\frac{V_S}{V_P}\right)(Q_S^{-1}-Q_P^{-1})\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right),\end{aligned}$$

$$A^{IH}_{PS}=-\frac{1}{2}\frac{V_S}{V_P}\left[\left(1+\frac{1}{2}\frac{V_P}{V_S}\right)\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right]Q_P^{-1}\tan\delta_P,$$

$$\begin{aligned}B^{IH}_{PS}=&\frac{1}{8}\left[1-3\left(\frac{V_S}{V_P}\right)^2\right]\frac{\Delta\rho}{\rho}Q_P^{-1}\tan\delta_P\\ &+\frac{V_S}{V_P}\left(1+\frac{3}{2}\frac{V_S}{V_P}\right)\left(\frac{\Delta\rho}{\rho}+2\frac{\Delta V_S}{V_S}\right)Q_P^{-1}\tan\delta_P.\end{aligned}$$

Numerical examples

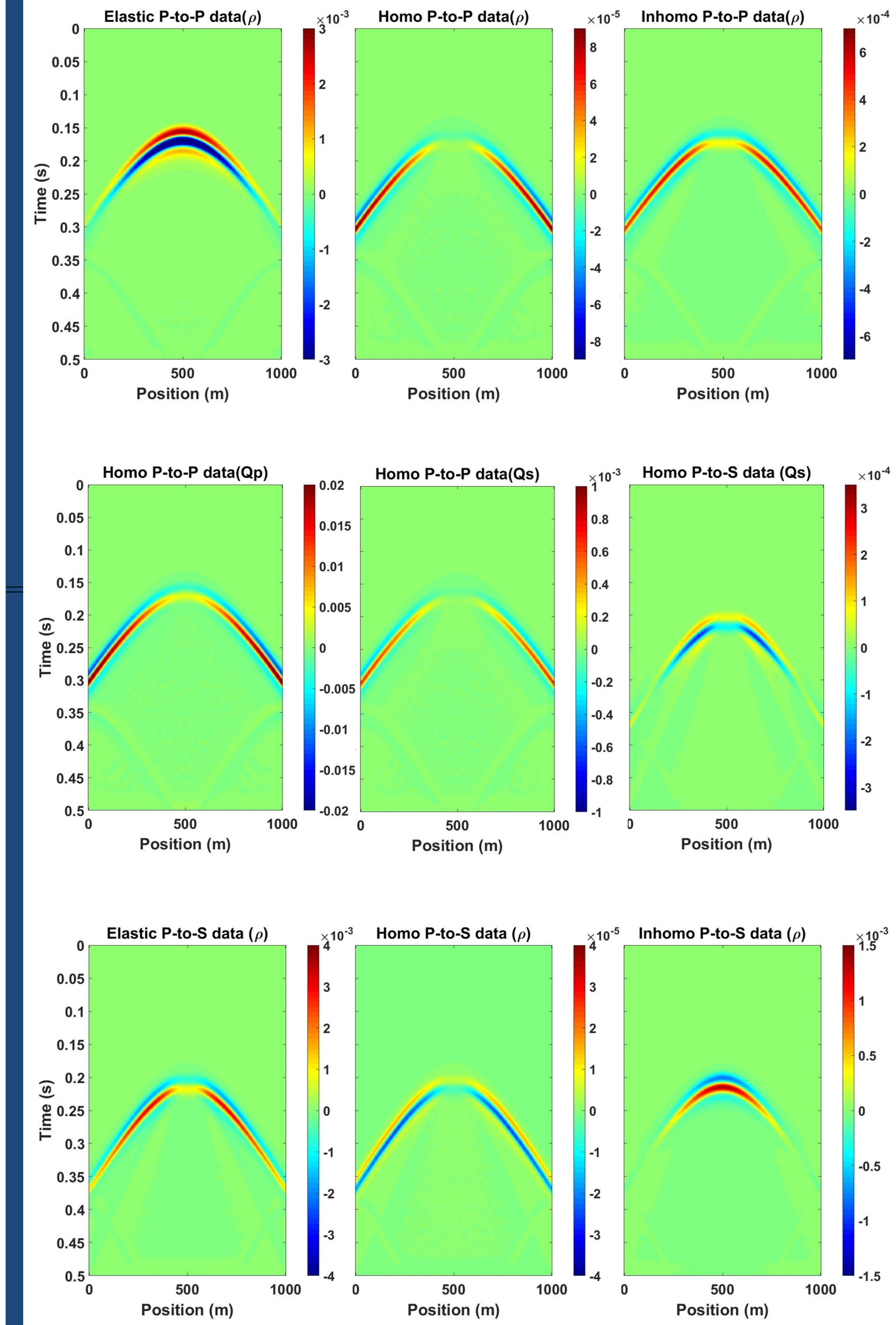


Figure 2: Modeling of the elastic, homogeneous and inhomogeneous components of the viscoelastic AVO equations.

Conclusions

The result presented in this research indicate new approximations and the decomposition of reflectivity into three terms indicate that intercepts and gradients can be used in future research to determine the quality factor and attenuation angle in an appropriate inversion strategy.

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Bibliography

Please see the report for references.