# Elastic wave equations in modified patchy-saturated media and its numerical simulation

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## SUMMARY

Based on the modified patchy-saturated porous model, according to the method Biot used for the foundation of elastic wave equations in porous media, we established the stress-strain relations and obtained the dissipation function and kinetic energy in patchy-saturated porous media. By applying the Lagrange's equations, we derived the elastic wave equations in modified patchy-saturated media. Through changing the equations to first-order stress-velocity equations, we deduced the 3D high-order finite difference schemes and numerically solved the equations in the complex domain. Numerical results show that there are two kinds of P-waves and one S-wave in patchy-saturated media. The energy of slow P-wave is very weak in "solid" phase of the patchy-saturated porous media, and can hardly be seen, even though it is stronger in the fluid phase. The fast P-wave and S-wave are clear both in the "solid" phase and fluid phase. The slow P-wave has a high dispersion. The velocities of the three waves are consistent with the theoretical results. All these show that our numerical method is correct and effective.

## INTRODUCTION/METHOD

Seismic attenuation and velocity dispersion are important properties which may be used both in exploration geophysics and development geophysics. Therefore, many scientists have studied the seismic attenuation and velocity dispersion based on different models. The main basic models are the Biot model (Biot, 1956a, 1956b), the BISQ model (Dvorkin and Nur, 1993; Dvorkin et al., 1994) and the patchy-saturated model (White, 1975). On the basis of the White model, we (Zhang and He, 2015) established a patchy-saturated model, called modified patchy-saturated model, and derived its P-wave equations. Based on the modified patchy-saturated porous model established in our previous paper, we achieved the stress-strain relations according to the method Biot used for the foundation of elastic wave equations in porous media. After derivation, we obtained the dissipation function and kinetic energy in patchy-saturated porous media. By substituting stress-strain relations and dissipation function and kinetic energy into the Lagrange's equations, we derived the elastic wave equations in modified patchy-saturated media and numerically solved the equations by using high-order finite difference method and simulated the wave propagations in the modified patchy-saturated media.

The equations we established are:

$$2\sum_j \frac{\partial}{\partial x_j}(\mu e_{ij}) + \frac{\partial}{\partial x_i}(\lambda\theta + 2\gamma D\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho u_i + \rho_{f_2} w_i)$$

$$\frac{\partial}{\partial x_i}(2\gamma D\theta + 2D\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{f_2} u_i + m w_i) + \frac{\eta_2}{\kappa}\frac{\partial w_i}{\partial t}.$$

For isotropic media

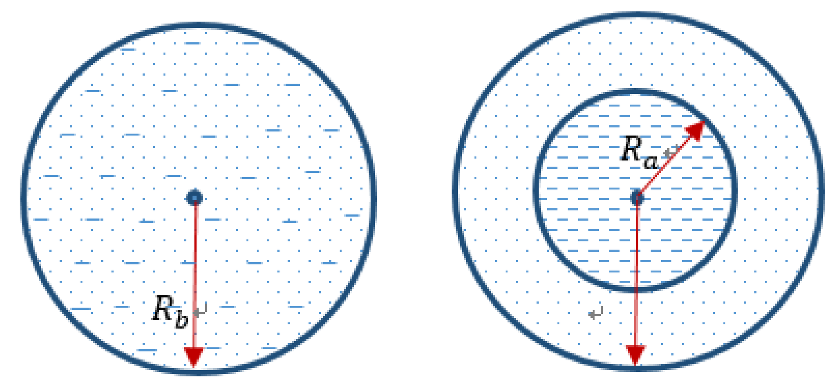
$$\mu\nabla^2\vec{u} + (H-\mu)\nabla\theta + 2\gamma D\nabla\varepsilon = \frac{\partial^2}{\partial t^2}(\rho\vec{u} + \rho_{f_2}\vec{w})$$

$$\nabla(2\gamma D\theta + 2D\varepsilon) = \frac{\partial^2}{\partial t^2}(\rho_{f_2}\vec{u} + m\vec{w}) + \frac{\eta_2}{\kappa}\frac{\partial \vec{w}}{\partial t}.$$

## CONCLUSIONS

We established the elastic wave equations in the modified patchy-saturated porous media. For the purpose of examining our equations and studying the wave propagations in partially saturated media, we perform numerical simulation of the elastic wave equations we established. In order to improve the computational precision and efficiency, we change the equations to first-order stress-velocity equations and deduced the 3D high-order finite difference schemes in the complex domain because of the complex bulk modulus in patchy-saturated media. The PML boundary conditions are used to absorb the pseudo reflections from artificial boundaries. Numerical results show that: (1) There are two kinds of P-waves and one S-wave in patchy-saturated media. (2) The energy of slow P-wave is too weak to be seen in the "solid" phase of the patchy-saturated porous media while it clearly exists in the fluid phase. (3) The high dispersion of the slow P-wave is apparent. (4) The fast P-wave and S-wave are obvious both in the "solid" phase and fluid phase. (5) The velocities of the three waves are consistent with the theoretical results. (6) The PML boundary conditions are effective in modified patchy-saturated porous media. All the above confirmed the correctness and effectiveness of our numerical method.

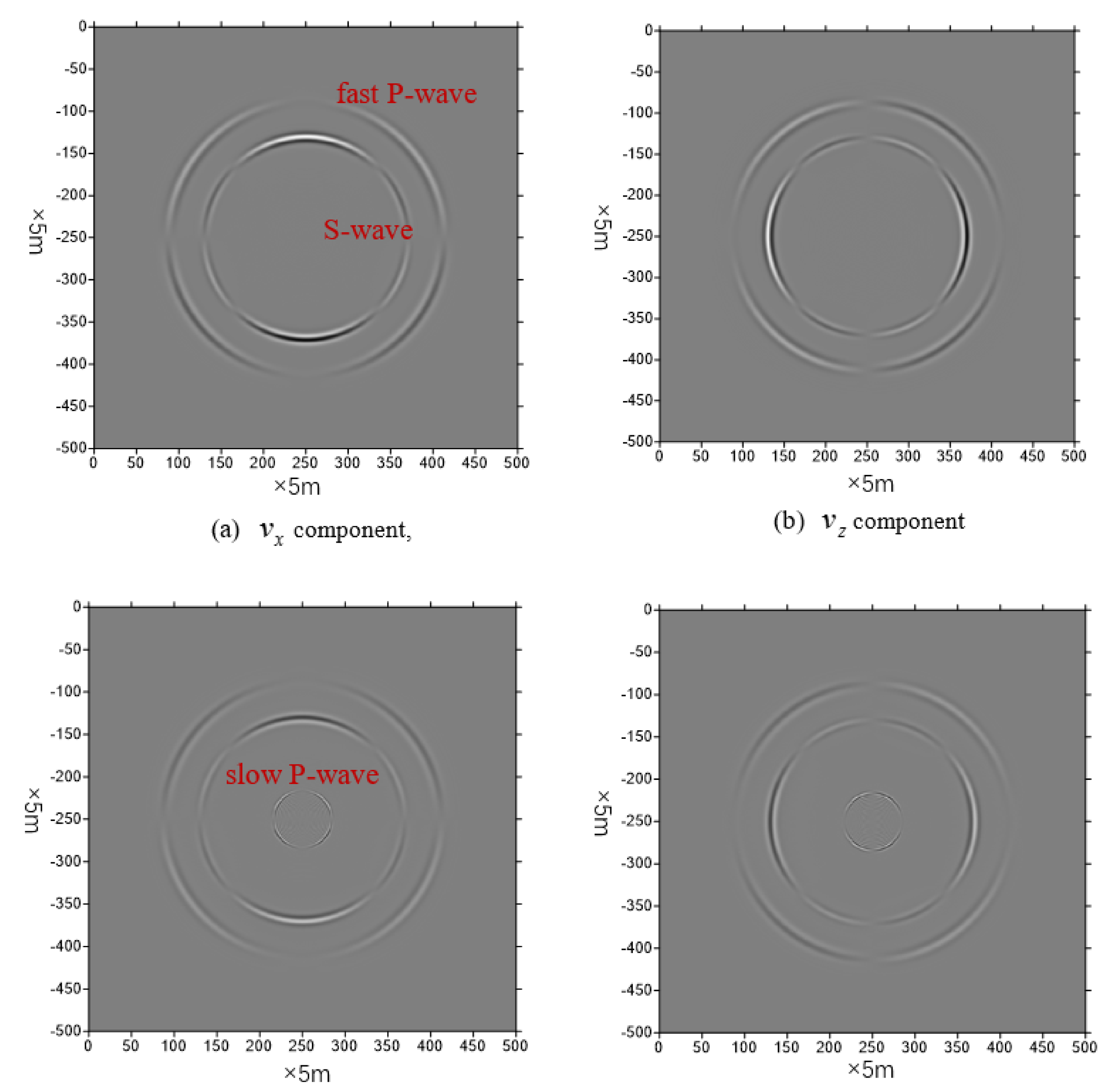
### Patchy saturated model




(a) (b)

In this model, we assumed that: The seismic wavelength is much larger than the gas pocket size and there is no interaction between two gas pockets(White's assumption), and, there is no movement between fluid 1 and the skeleton, but there is relative movement between fluid 2 and the skeleton.

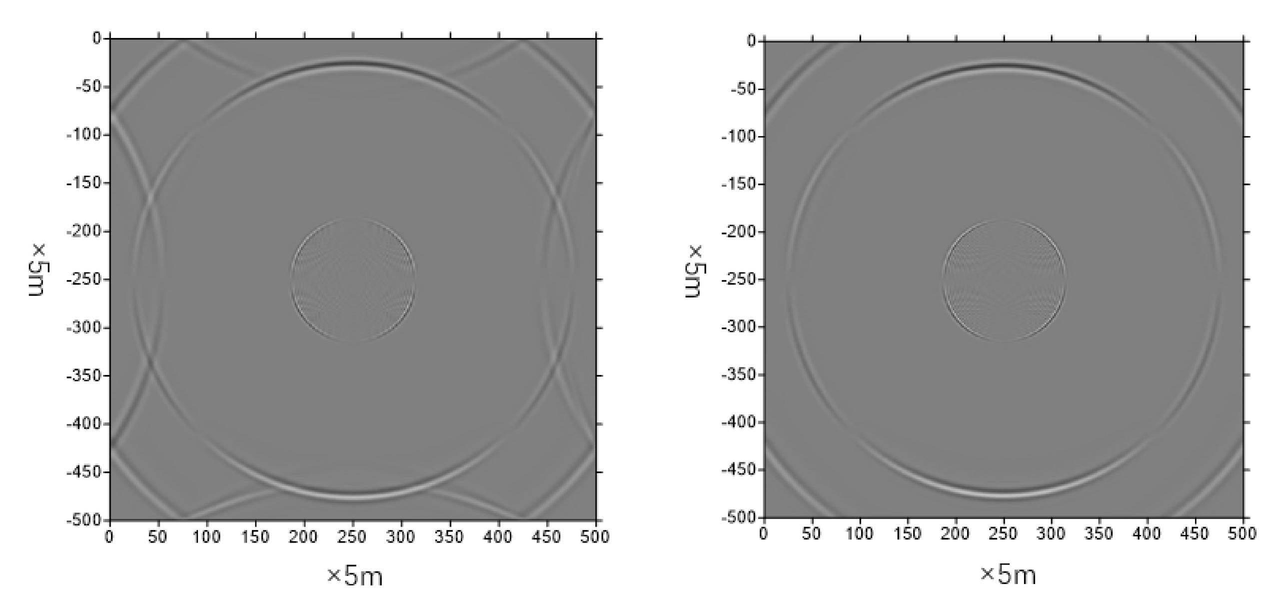
### The wave field snapshots of modified patchy-saturated porous media (t=250ms)




(a) $v_x$ component, (b) $v_z$ component

(c) $V_x$ component (d) $V_z$ component

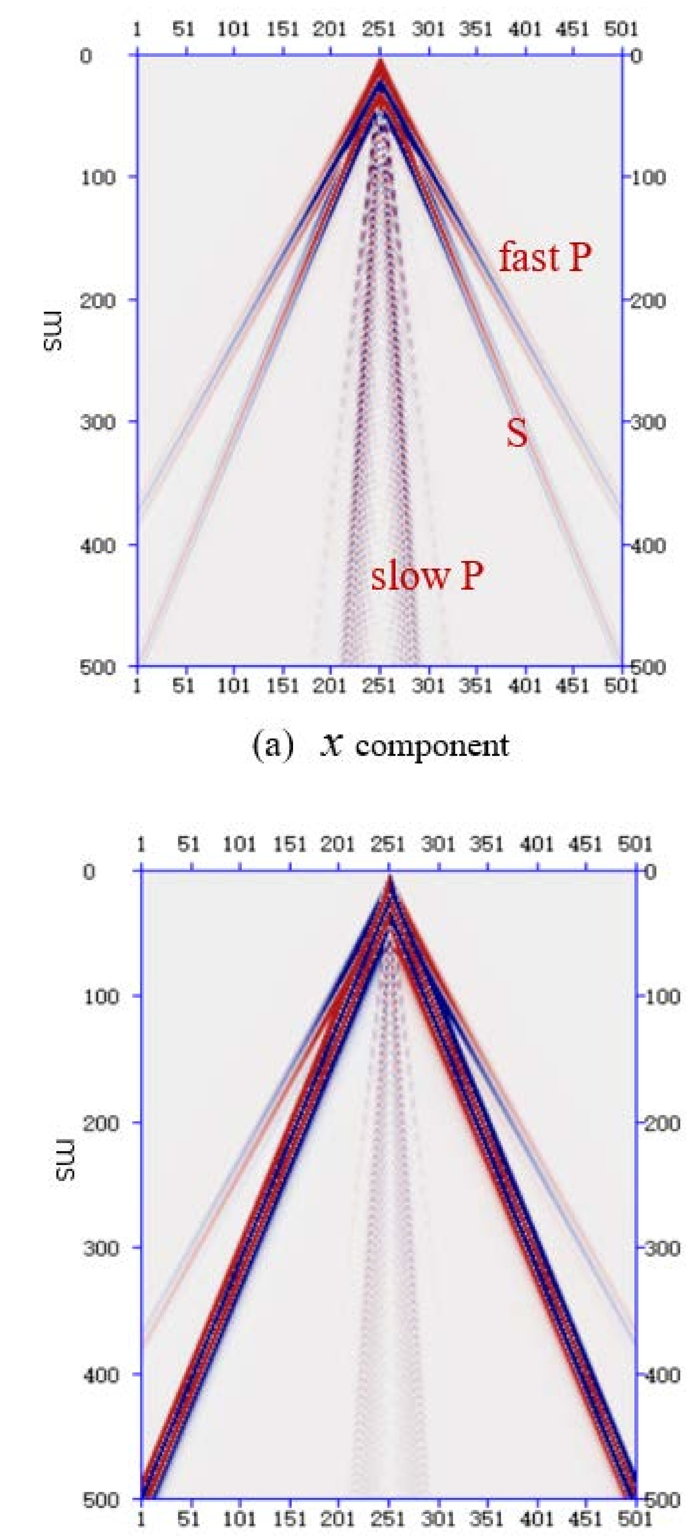
### Illustration of PML absorbing boundary conditions




(a) snapshot of $V_x$ component without using any absorbing boundary conditions

(b) snapshot of $V_x$ component after using PML absorbing boundary conditions

### Synthetic seismograms of modified patchy-saturated model




(a) $x$ component

(b) $z$ component

## ACKNOWLEDGEMENTS

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