Applications of FWI to the microseismic source problem

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Summary

We propose a modified FWI scheme that iteratively solves for both source distribution and velocity model. We show numeric examples in a 2D time-domain acoustic framework developed in Python. Examples of the source-location gradient are shown in a variety of scenarios in order to begin to understand its behaviour, as well as the potential cross-talk between the two parameters. We see that low frequencies are best for the source-term gradient to produce a correct update. This update has the form of a dipole, with a negative lobe at the incorrect source location and a positive lobe in the direction of the correct source location.

Background and Theory

Microseismic events are small-magnitude earthquakes that can be induced by a variety of processes, such as hydraulic fracturing. One of the goals of microseismic processing is to obtain accurate locations for the events.

→ This requires an accurate velocity model.

We propose a multiparameter FWI scheme that combines the conventional FWI framework to invert for velocity with the microseismic source location problem (Figure 1). We call this ***microseismic FWI***, or MFWI. In a Newton scheme, we have an update of the form

$$\begin{bmatrix}\delta\mathbf{s}_c \\ \delta\mathbf{s}_s\end{bmatrix} = -\begin{bmatrix}\mathbf{H}_1 & \mathbf{H}_2 \\ \mathbf{H}_3 & \mathbf{H}_4\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{g}_c \\ \mathbf{g}_s\end{bmatrix}$$

The subscript of *c* refers to the velocity model update, while the subscript of *s* refers to the source position update. The source-term gradient we propose is:

$$\mathbf{g}_s = -\sum_{r_g, r_s}\int dt\,\delta P(\mathbf{r}_g, \mathbf{r}_s, t|s_c, s_s)\,g(\mathbf{r}_g, \mathbf{r}, t - t^*|s_c, s_s)$$

where $\delta P(r_g, r_s, t|s_c, s_s)$ is the residuals, $g(r_g, r, t - t^*|s_c, s_s)$ is the green's function, and t^* is the origin time of the source.

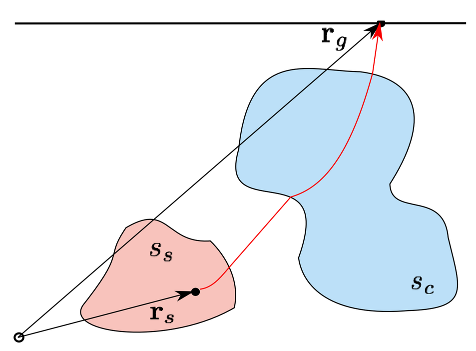


Figure 1: Schematic illustration of hypothetical source and velocity distribution

Additionally, since microseismic event magnitude is inversely proportional to the dominant frequency of an event, we have an inversion scheme where the sources have varying frequencies.

Results: source-term gradient

We use 2D acoustic time-domain FWI codes developed by Almuteri and Innanen (2016) as the starting point for this new source-term gradient.

Figure 2a shows the homogeneous background velocity model, and Figure 2b shows the gradient. The red dot is the starting position, and the green dot is the true position of the source. The gradient attempts to cancel out the incorrect location, and produces a second lobe of opposite polarity in the direction of the true location.

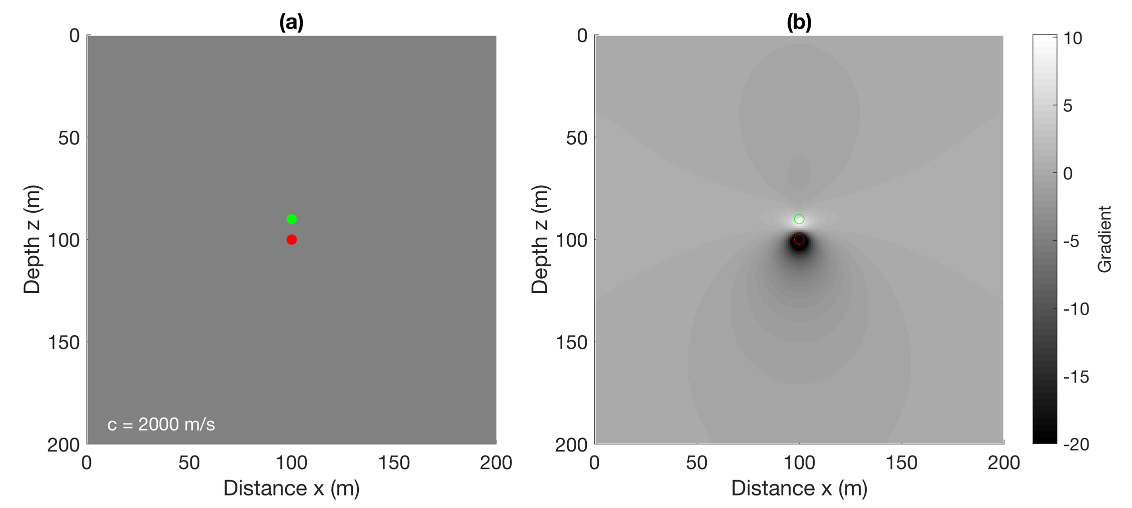


Figure 2

Figure 3 shows the gradient if the starting position is far from the true position. Again, the gradient cancels the inaccurate position, and the center of the upper lobe is at the position of the true location.

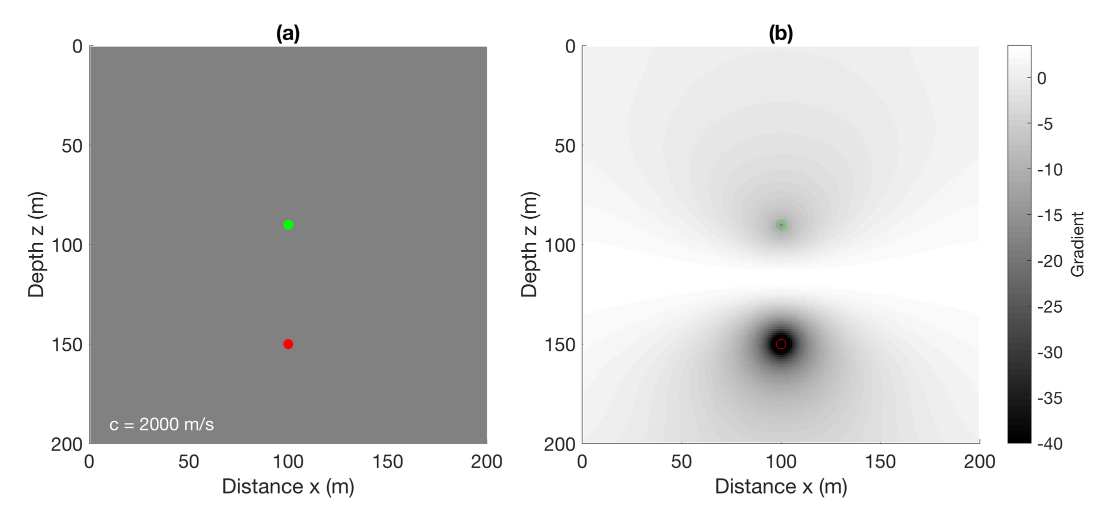


Figure 3

Figure 4 shows the gradient as a function of frequency. Only the low frequency gradient behaves like a dipole by applying a negative lobe to the incorrect location and a positive lobe to the correct location.

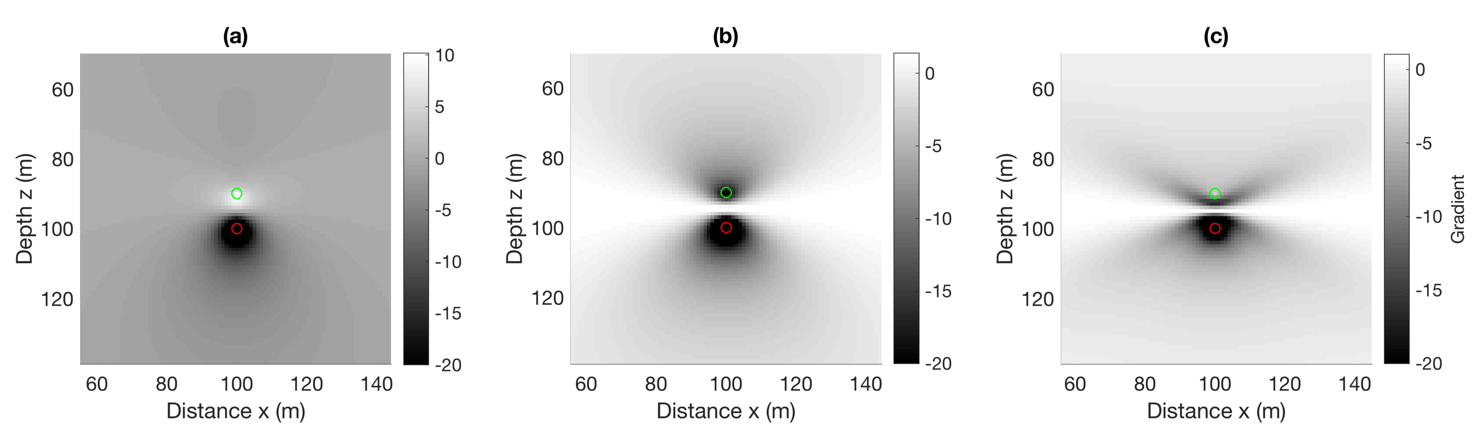


Figure 4: Gradient as a function of frequency: (a) 5 Hz, (b) 20 Hz, (c) 40 Hz.

Cross-talk

Cross talk occurs when one parameter is updated in response to data variations caused in part by a different parameter. Figure 5 shows an example of cross-talk for the velocity model gradient.

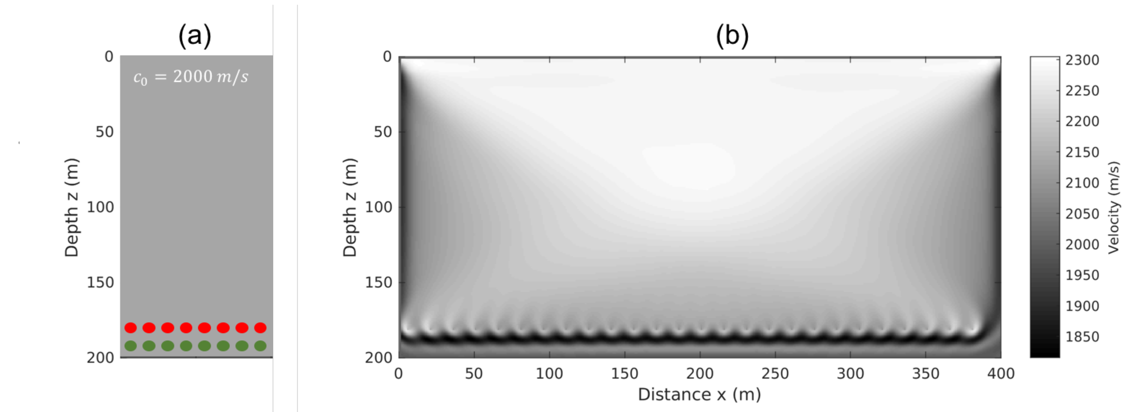
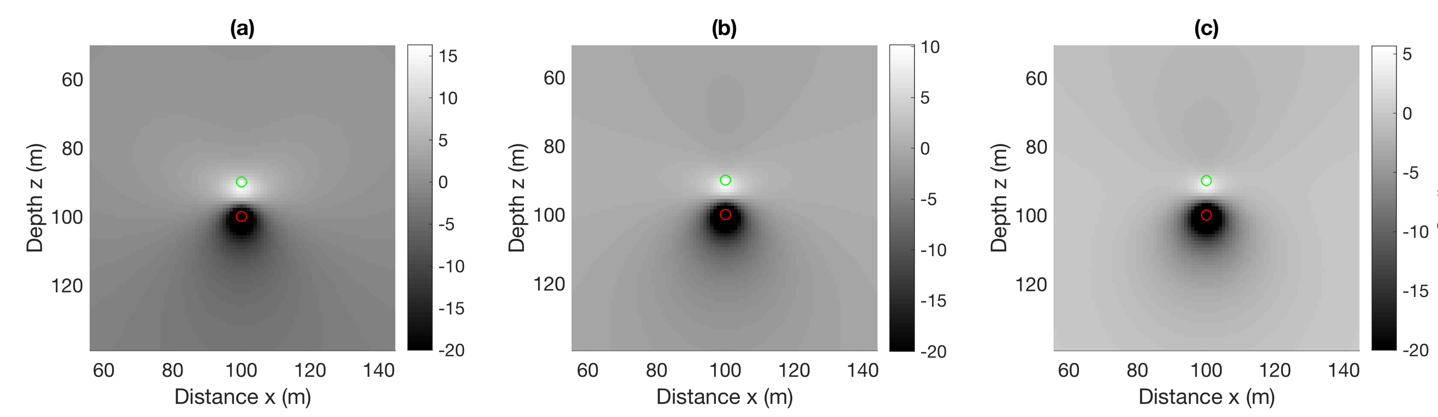


Figure 5: Moving the sources gives a bulk update to the model.

For the source-term gradient, previously the starting velocity model was set equal to the true model. In Figure 6, the background velocity for the starting model has been changed. This leads to overestimating (Figure 6a) or underestimating (Figure 6c) the source-term region.

Figure 6: Gradient as a function of change to true velocity model: (a) $v_t < v_0$, (b) $v_t = v_0$, (c) $v_t > v_0$.



Discussion

Multiparameter MFWI would solve for the source-term gradient and velocity gradient simultaneously, with the Hessian to aid with the cross-talk.

In conclusion:

- The source-term gradient moves in the correct direction of the true source location.
- Low frequency source-term updates have the character of a **dipole**.
- Due to the potential cross-talk, the Hessian is essential.

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