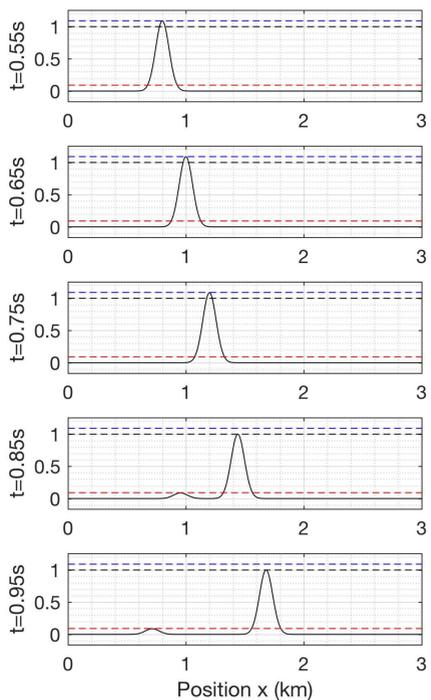
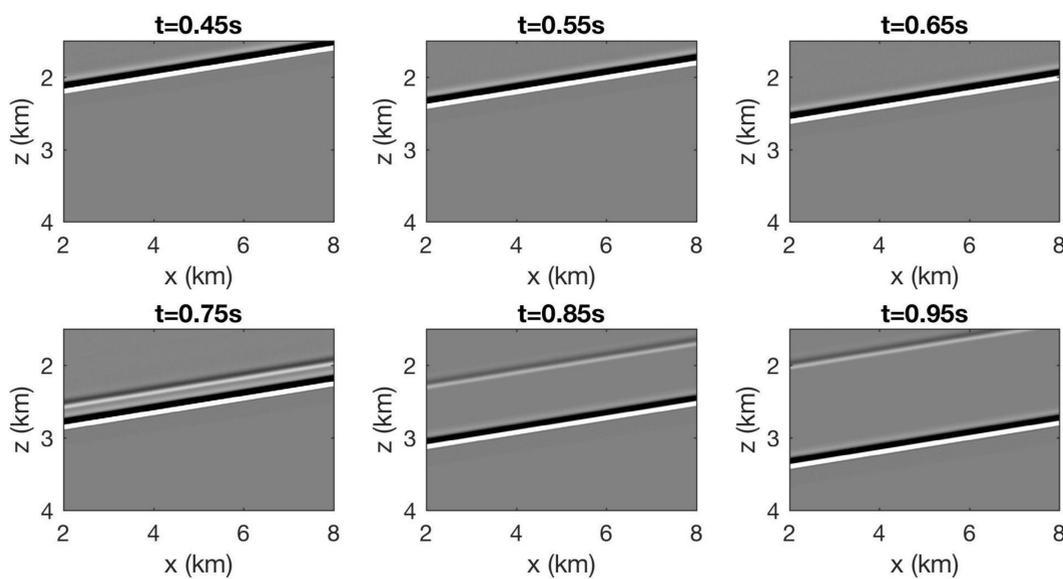


## What is a “time boundary”?

Suppose a one-way wave is propagating in a homogeneous medium. Suppose we at some instant  $t_0$  during its propagation, the entire medium has its elastic properties changed, i.e., at  $t > t_0$ , the medium remains homogeneous, but its properties are everywhere different from the ones prior to  $t_0$ . What would happen?

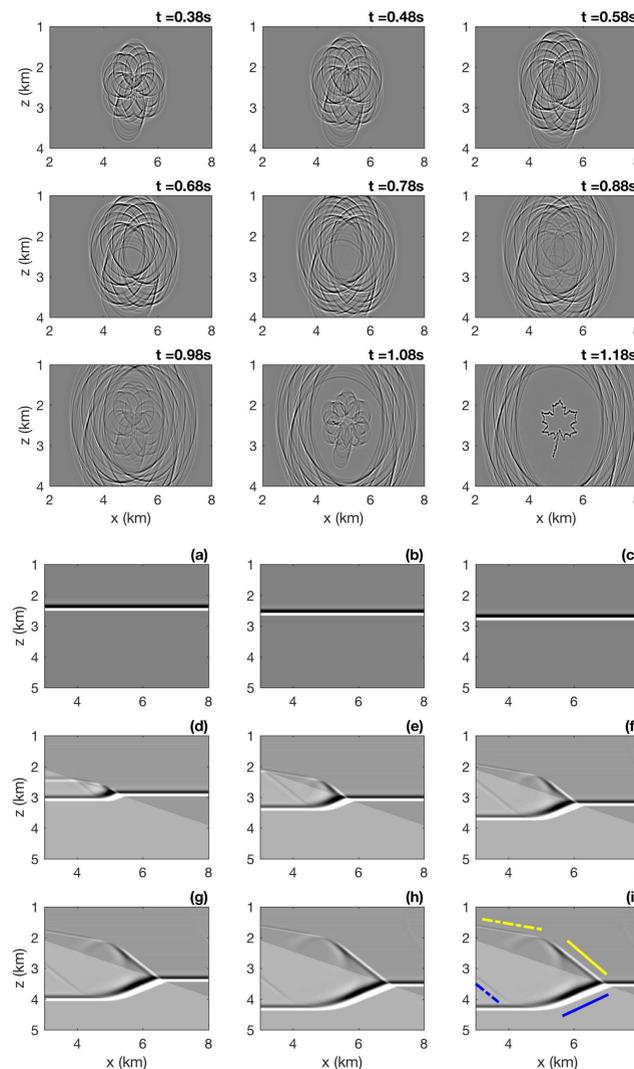


**Fig. 1.** Answer: a reflection! Because of the symmetry of the wave equation, a discontinuity in time is very similar to a discontinuity in space. In the Figure, a 1D pulse of height  $T$  propagates to the right. At  $t=0.75s$ , the medium properties change from  $v_0=2.0km/s$  to  $v_1=2.4km/s$ . The energy partitions into a right-going component of height  $I$ , and a left-going component of height  $R$ . The values  $I$ ,  $T=1.09I$ , and  $R=0.09I$  obey the rules  $R = (v_1 - v_0)/(v_1 + v_0)$  and  $T = I + R$ . But, notice that it is the “incident” wave that has height  $T$ . The wave equation is not perfectly symmetric!

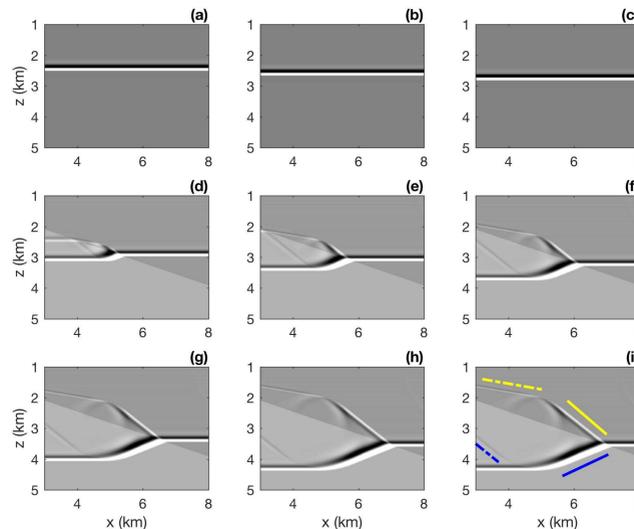


**Fig. 2.** Does it work in multiple dimensions? Yup. For plane waves upon hitting a “time boundary” a reflection normal to the incident wave is created. All current theory/results concerning time boundaries are inherently “normal incidence”.

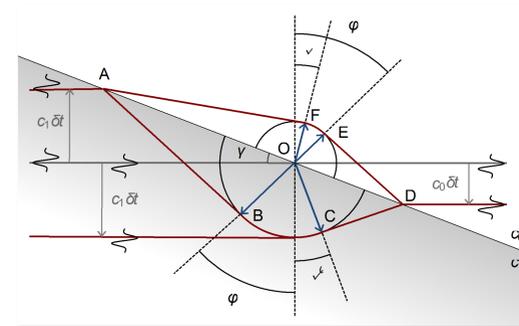
## Wave “control” and AVO without an interface



**Fig. 3.** Bacot et al. (2016) focus on time boundaries from the point of view of imaging and wave control. Suppose a maple leaf fell on the surface of a pool at time 0, exciting surface waves. By suddenly changing the wave velocity at  $0.7s$ , each point on the complex waveform generated by the leaf acts as a Huygens’ source, and at  $1.18s$  the source is clearly imaged.



**Fig. 4.** Reflections and no spatial boundaries... so, can we do AVO without an interface? Surprisingly, it is not the “A” that is hard, it is the “O” – all time boundary reflections are normal. Our main contribution is to show that by mixing time-boundaries with normal space-boundaries we can generate oblique reflections (upgoing wavefront labelled with yellow dashed line).



**Fig. 5.** The kite-shaped quadruplet of wavefronts arises when a time-boundary is introduced with space-boundary features. The “reflected” wavefront can be shown to have an angle  $\theta$ , where  $\theta = \mu - \gamma$ , and where  $\gamma$  is the interface dip angle and  $\mu$  satisfies

$$\sin \mu = (v_0/v_1) \sin \gamma$$

## Applications?

Difficult to say: monitoring? Can we illuminate a reservoir target as a rapid process of pressuring up is occurring? If injection fluids have the right properties, could we apply a magnetic field and cause an illuminating seismic wave to scatter from sites of strong injection?