Tomography without traveltime picking

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Motivation

An accurate starting velocity model is very important to depth migration and full waveform inversion (FWI). Depth migration can update the velocity model iteratively using migration velocity analysis (MVA) methods; however, each iteration requires an update to the velocity model and a depth migration process. The goal of FWI is to converge to the global minimum of the objective function and to arrive at the correct model. However, FWI is an ill-posed problem, its solution often represents only a local minimum. Therefore, an accurate initial model can improve the efficiency and accuracy of depth migration and FWI. Traditional tomography methods inverse traveltime picks to a velocity model and requires interpretive traveltime picking of continuous reflection events. The controlled directional reception (CDR) method uses the ray parameters of waves transmitted from a shot and a receiver to invert for the velocity model. The ray parameters of a locally coherent reflection event can be picked interactively or automatically on localized slant stacks of shot and geophone gathers. We will review the CDR method and investigate its potential as a velocity model building tool.

Controlled directional reception method

The CDR method (Sword 1987) characterizes a locally coherent event with the source position x_s, receiver position x_g, traveltime T_{sr} and ray parameters p_s at the source and p_g at receiver. These parameters are referred to as the reciprocal parameters. They can be determined using localized slant stacks on common shot gather and common receiver gather. This locally coherent event is associated with a ray segment pair that is characterized by the reflector or diffractor position X, ray shooting angles θ_s, θ_g and traveltime T_s and T_g.

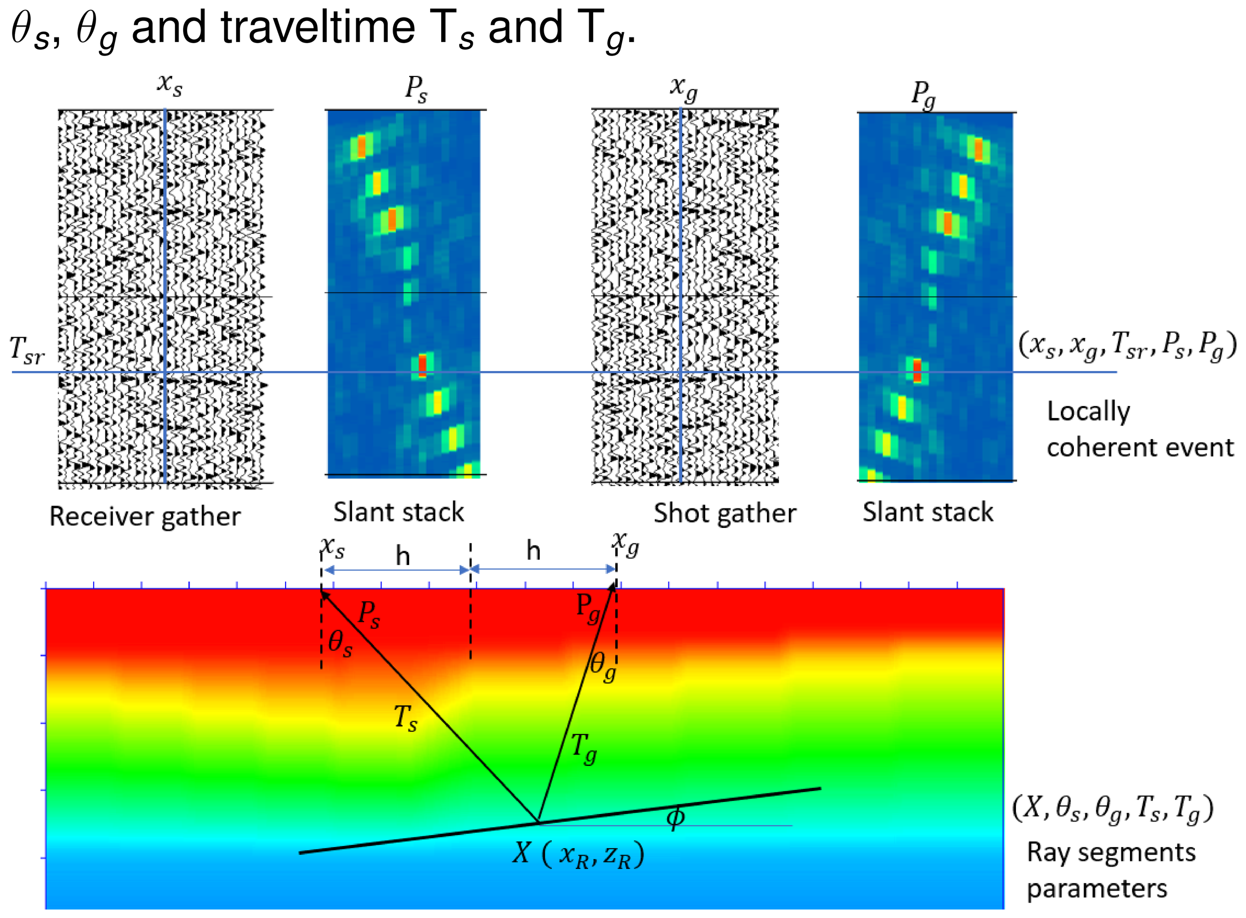

Figure 1: A locally coherent event can be picked on the localized shot and receiver slant stacks. The event is characterized by the traveltime T_{sr} and ray parameters p_s and p_g and is associated with a ray segment pair in the velocity model

Controlled directional reception method

The ray shooting angles θ_s, θ_g can be obtained according to the formulas

$$sin\theta_s = p_s \quad (1)$$

$$sin\theta_g = vp_g \quad (2)$$

The dip angle ϕ for the reflecting segment can be computed by:

$$tan\phi = \frac{v(p_s+p_g)}{\sqrt{1-v^2p_s^2}+\sqrt{1-v^2p_g^2}} \quad (3)$$

With the dip angle ϕ,the location of the reflector X can be determined by computing the horizontal position X_R and depth Z_R:

$$X_R = 0.5*(x_s+x_g)+\frac{0.5vt(tan\phi)}{\sqrt{1-(4h^2/(v^2t^2))+tan^2\phi}} \quad (4)$$

$$Z_R = \frac{0.5vt(1-(4h^2/(v^2t^2)))}{\sqrt{1-(4h^2/(v^2t^2))+tan^2\phi}} \quad (5)$$

v_{CDR} is the velocity determined using the reciprocal parameters and the half offset h. It is not the velocity used in tomographic inversion; however, it can be used to display the dip bars. Stereo velocity displays can be created by using two plots of dip bars. One of the plot shifts the dip bars laterally proportional to the value of v_{CDR} to create the 3D effect of v_{CDR} differences. This can be used to identify and remove multiple events.

$$v_{CDR}^2 = \frac{1-(h/t)(p_s-p_g)}{(p_s-p_g)(t/4h)+p_sp_g} \quad (6)$$

CDR tomographic velocity inversion

Distance error x_{err} is less sensitive to error in p_s and p_g than traveltime errors; therefore, it is used in the cost function for velocity inversion:

$$J(v) = ||X_{err}(v)||^2 \quad (7)$$

Damping factor is added to avoid rapid variation in velocity:

$$J(v) = ||X_{err}(v)||^2 + \lambda_x^2||\frac{\partial v^2}{\partial x}|| + \lambda_z^2||\frac{\partial v^2}{\partial z}|| \quad (8)$$

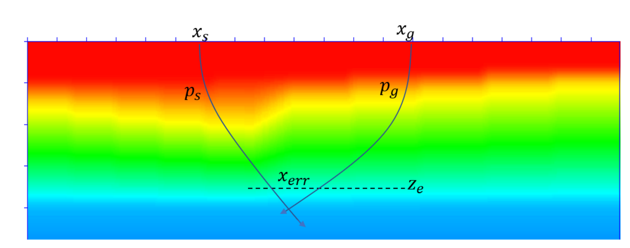

Figure 2: If error exists in the velocity model, rays traced from source and receiver will not meet at the depth where traced time equals measured time. The distance error x_{err} is used in the cost function for tomographic inversion.

The tomographic inversion problem is solved by finding the value of v that minimizes J(v). This can be done by solving the following least-squares system

$$A^{(k)} \triangle v = -x_{err}^{(k)} \quad (9)$$

where $x_{err}^{(k)} = x_{err}(v^{(k)})$ and is computed by ray tracing using the current velocity model and takeoff angles computed from p_s and p_g, $A^{(k)}$ is the Fréchet matrix derived from $x_{err}^{(k)}$, and $\triangle$ v is the value to update the velocity model with.

Stereotomography

Recent enhancements of CDR include stereotomography (Lambaré 2008, Prieux, Lambare, Operto and Virieux 2012). Stereotomography is a generalization of CDR. It extends CDR to include 3D geometry, converted wave and anisotropy. It also extends the picking to depth and time migrated domain as well as post-stack time domain.

Stereotomography

The model space of stereotomography consists of the velocity model and the parameters of the ray-segment pairs:

$$\boldsymbol{m} = [(v_m)_{m=1}^M, [X, \theta_s, \theta_g, T_s, T_g]_{n=1}^N] \quad (10)$$

The data space of stereotomography consists of the 5 reciprocal parameters for each locally coherent event picks:

$$\boldsymbol{d}_{obs} = [(x_s, x_g, T_{sr}, p_s, p_g)_{obs}]_{n-1}^N \quad (11)$$

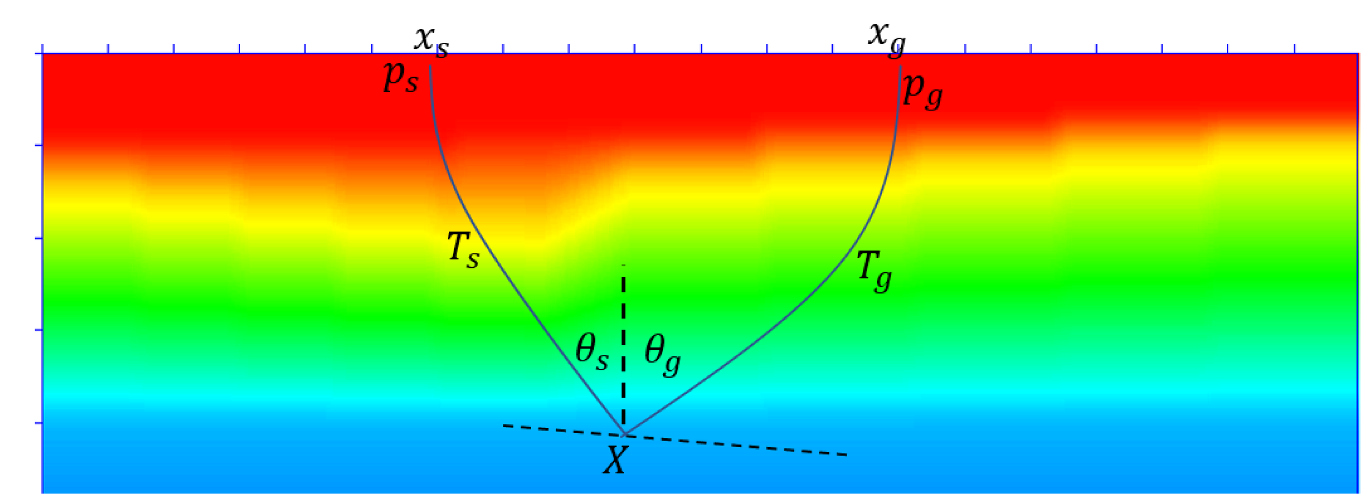

Figure 3: Forward modeling is done by shooting rays from X toward the source and receiver

The data space is forward modeled iteratively by ray tracing from the picked position X with a priori pair of ray-segments and the velocity model. All 5 reciprocal parameters and the velocity model are updated between iterations and contribute to the misfit function.

$$\boldsymbol{d}_{calc}(\boldsymbol{m}) = [(x_s, x_g, T_{sr}, p_s, p_g)_{calc}]_{n-1}^N \quad (12)$$

The L2 norm of the misfit function is:

$$S(\boldsymbol{m}) = \frac{1}{2}(d_{calc}(\boldsymbol{m})-d_{obs})^T C_D^{-1}(d_{calc}(\boldsymbol{m})-d_{obs})+\frac{1}{2}(\boldsymbol{m}-\boldsymbol{m}_{priori})^T C_M^{-1}(\boldsymbol{m}-\boldsymbol{m}_{priori}) \quad (13)$$

The inversion of $\boldsymbol{m}$ can be done by computing the gradient of the misfit function and the inverse of the Hessian matrix iteratively:

$$\boldsymbol{m}_{k+1} = \boldsymbol{m}_k - (\frac{\partial^2 \boldsymbol{S}}{\partial \boldsymbol{m}^2}(\boldsymbol{m}_k))^{-1}\frac{\partial \boldsymbol{S}}{\partial \boldsymbol{m}}(\boldsymbol{m}_k) \quad (14)$$

Summary and future work

- Stereotomography uses ray parameters of locally coherent event picked on shot and receiver slant stacks to estimate the velocity model. Since picking is done on localized slant stacks, it is less sensitive to noise than traveltime picking.
- Besides being a viable velocity model building tool for depth imaging and FWI, stereotomography has the potential in building velocity model at depths that are too deep for conventional seismic refraction inversion and too shallow and too noisy for reflection velocity analysis.
- We will investigate the resolving power of stereotomography at difference depths and noise conditions with synthetic and real data.

Acknowledgements

We thank the sponsors of CREWES and NSERC (Natural Science and Engineering Research Council of Canada) through the grant CRDPJ 461179-13 for supporting this project.