Velocity model building by slope tomography

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Abstract

Slope tomography methods use slopes and traveltimes of locally coherent reflected events to estimate the macro velocity model from reflection data for depth imaging and full waveform inversion (FWI). Without the requirement of picking traveltimes on continuous reflection events, slope tomography is operationally more efficient than traditional reflection tomography. We review the slope tomography methods and investigate their potential as velocity model building tools.

Controlled directional reception (CDR) method

The concept of using the apparent slopes of source and receiver ray paths of reflection arrivals to image the subsurface started in Russia (Riabinkin 1957, Sword 1987). Using straight ray and constant velocity assumption, reflector position and dip can be computed from the slopes (p_s, p_g) and arrival time T_{sr} of locally coherent reflection events.

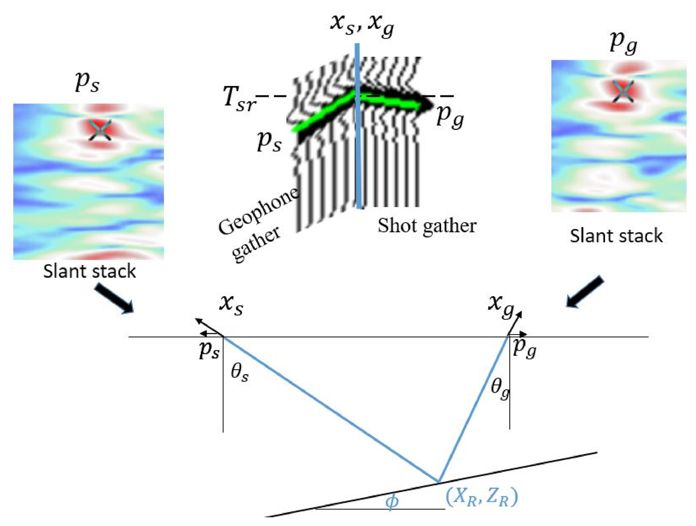


Figure 1: A locally coherent event can be picked on the localized shot and receiver slant stacks. The event is characterized by the traveltime T_{sr} and ray parameters p_s and p_g and is associated with a reflection arrival from (X_R,Z_R) with a locallized dip ϕ.

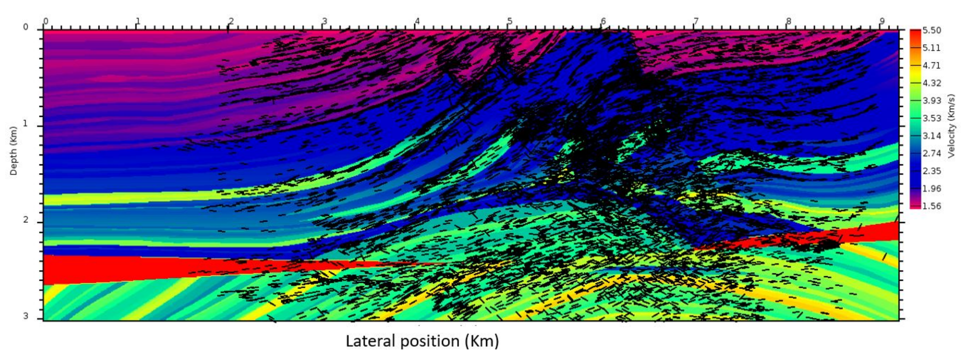
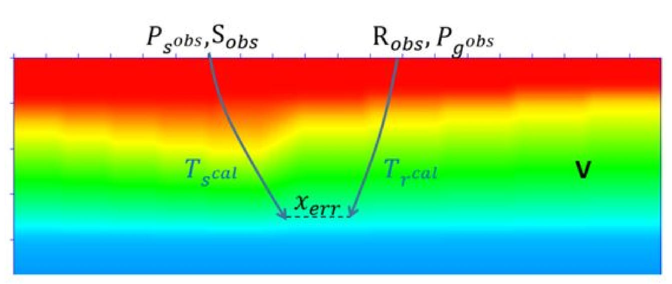


Figure 2: Dip bars computed from slopes and arrival time picks using synthetic data of the Marmousi velocity model.

CDR tomography

- CDR tomography traces rays from source and receiver using ray shooting angles computed from p_s and p_g. If error exists in the velocity model, rays traced from source and receiver will not meet at the depth where traced time equals measured time. The distance error X_{err} is used in the cost function for CDR tomographic inversion.



Model space : $[V]_{i=1,M}$

Data space : $[X_{err}]_{j=1,N}$

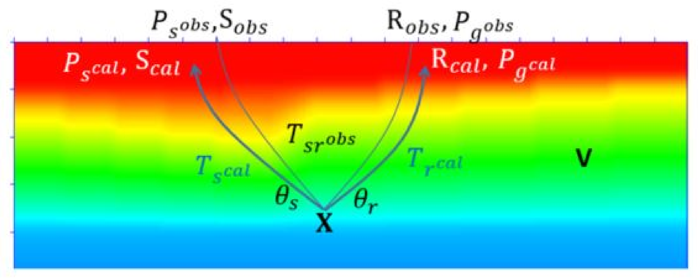
Fréchet derivative: $A_{ij} = \frac{\partial X_{err\,j}}{\partial V_i}$

Inversion: $\mathbf{A}\,\Delta\mathbf{V} = \Delta\mathbf{X}_{\mathbf{err}}$

- **Advantages:**
  - Does not require picking of continuos reflection events.
  - Uses slopes of reflection events in addition to arrival times.
- **Disadvantages:** Sensitive to picking errors in p_s and p_g.

Stereotomography

- Stereotomography is a generalized slope tomography method that takes into accounts of all measurement uncertainties.



Model space: $\mathbf{m} = [(X, \Theta_s, \Theta_r, T_s, T_r)_{i1=1,N}], [V]_{i2=1,M}]$

Data space : $\mathbf{d} = [S, R, P_s, P_g, T_{sr}]_{j=1,N}$

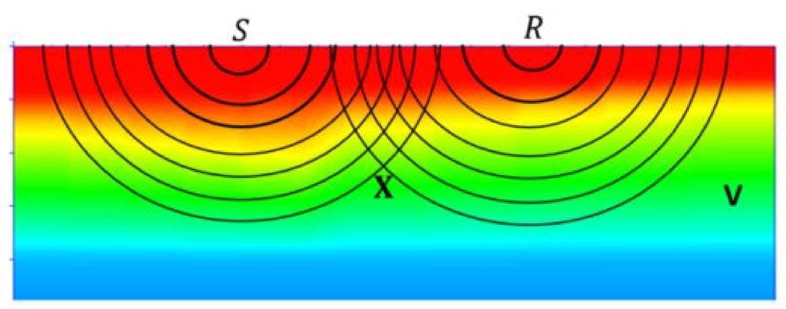
Fréchet derivative: $A_{ij} = \frac{\partial(S,R,P_s,P_r,T_{sr})}{\partial(X,\Theta_s,\Theta_r,T_s,T_r,V)}$

Inversion: $\mathbf{A}\,\Delta\mathbf{m} = \Delta\mathbf{d}$

- **Advantages:** Less sensitive to picking errors in p_s, p_g and T_{sr}.
- **Disadvantages:** Large data and model space result in a large system of linear equations that is inefficient for large 3D surveys.

Adjoint stereotomography

- Adjoint stereotomogaphy uses adjoint-state method and eikonal solver to achieve a matrix-free formulation with reduced data and model space.



Model space : $[X_{j=1,N}], [V]_{i=1,M}]$

Data space : $[T_{sr}, P_s, P_g]_{j=1,N}$

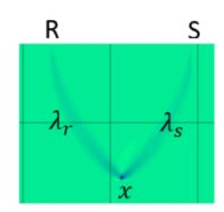
Inversion : $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \frac{\partial J}{\partial \mathbf{m}}$

- Following adjoint-state method procedure (Plessix, 2006) adjoint-state equations and gradient equations are developed:

Adjoint state equations

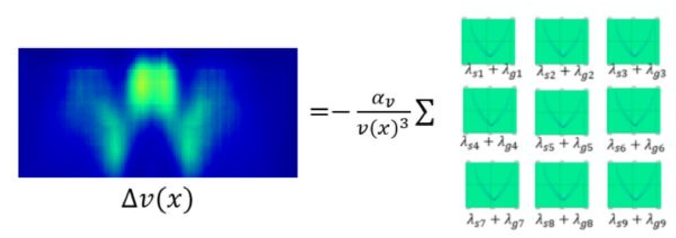
$$\frac{\partial}{\partial x}\left(\frac{\partial T_s}{\partial x}\lambda_s\right) + \frac{\partial}{\partial z}\left(\frac{\partial T_s}{\partial z}\lambda_s\right) = -\sum \Delta T_{sr} + \frac{1}{2\Delta s}\left(\sum \Delta p_{s+1} - \sum \Delta p_{s-1}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial T_r}{\partial x}\lambda_r\right) + \frac{\partial}{\partial z}\left(\frac{\partial T_r}{\partial z}\lambda_r\right) = -\sum \Delta T_{sr} + \frac{1}{2\Delta r}\left(\sum \Delta p_{g+1} - \sum \Delta p_{g-1}\right)$$

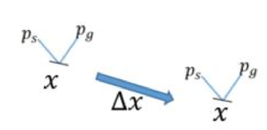


Gradient equations:

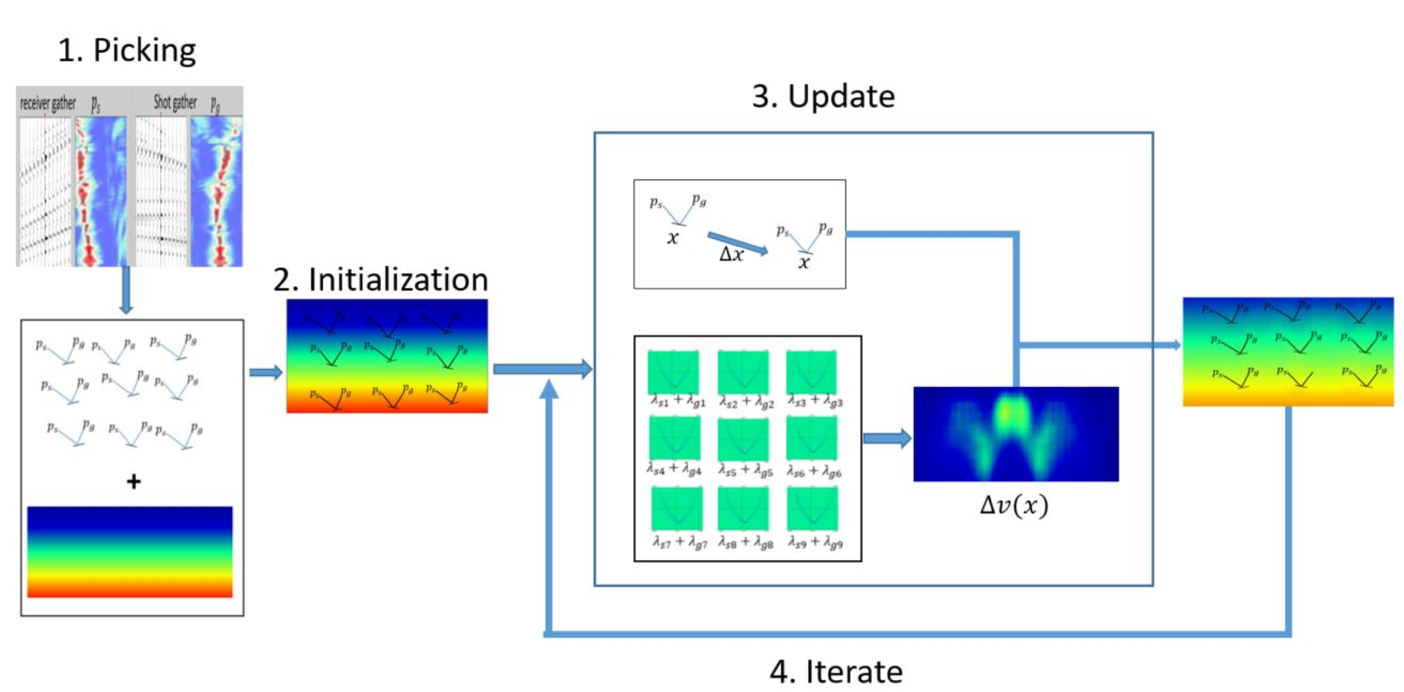
$$\frac{\partial J}{\partial v(x)} = -\frac{1}{v(x)^3}\sum(\lambda_s(x) + \lambda_r(x))$$



$$\frac{\partial J}{\partial X} = \Delta T_{sr}(\nabla T_s + \nabla T_r) + \frac{\Delta p_s}{2\Delta s}(\nabla T_{s+1} - \nabla T_{s-1}) + \frac{\Delta p_g}{2\Delta r}(\nabla T_{r+1} - \nabla T_{r-1})$$



- **Adjoint stereotomography workflow:**



- **Advantages:**
  - Takes into accounts of measurement uncertainties in p_s, p_g and T_{sr}.
  - Efficient algorithm for large surveys.
- **Disadvantages:** Potential cross-talk between $\boldsymbol{X}$ and $\boldsymbol{V}$.

Numerical example

- **Forward modeling of arrival times and slopes:**

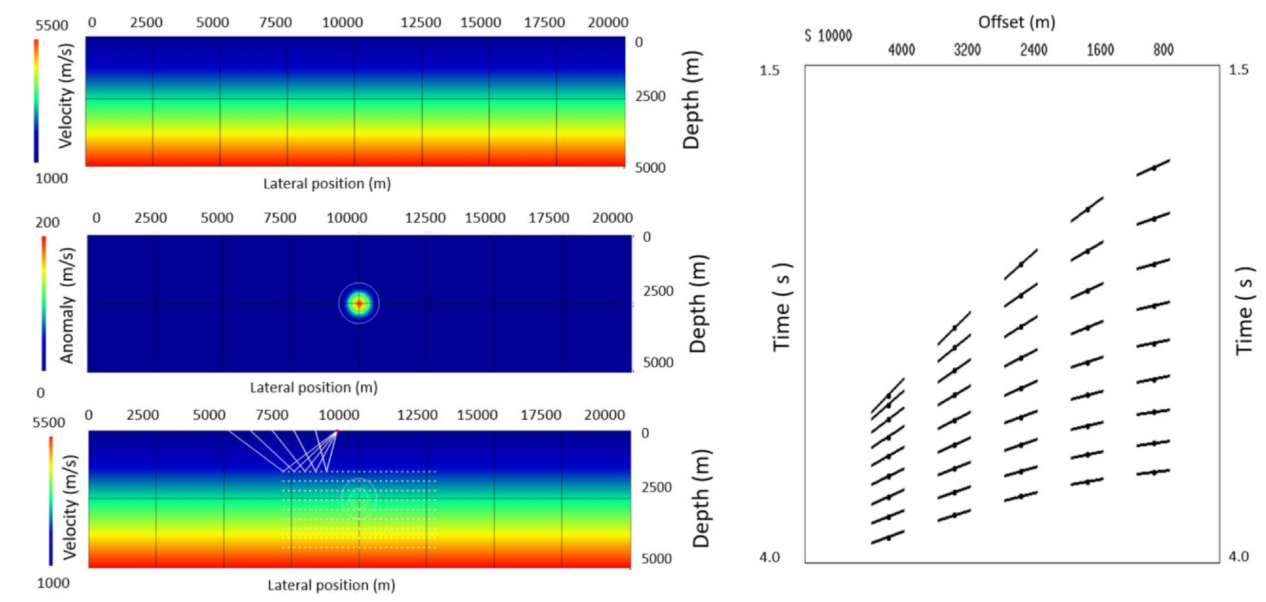


Figure 3: Arrival times and slopes are computed for a velocity model with constant vertical gradient and a circular anomlay

- **Inverting for scatter position and velocity updates:**

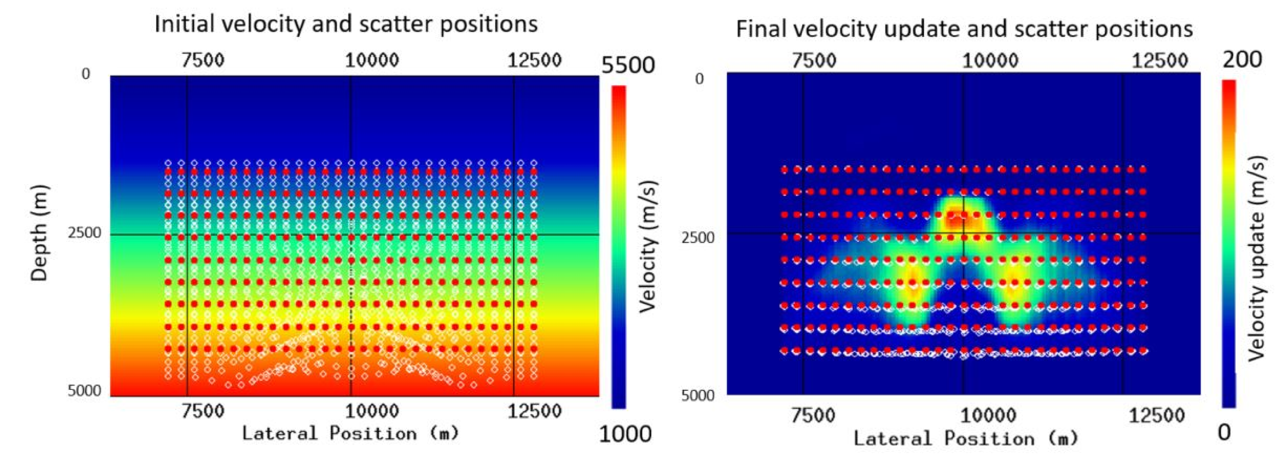


Figure 4: Inital scatter positions $\boldsymbol{X}$ (white circles) are computed with CDR method

- **Inverting for velocity update only:**

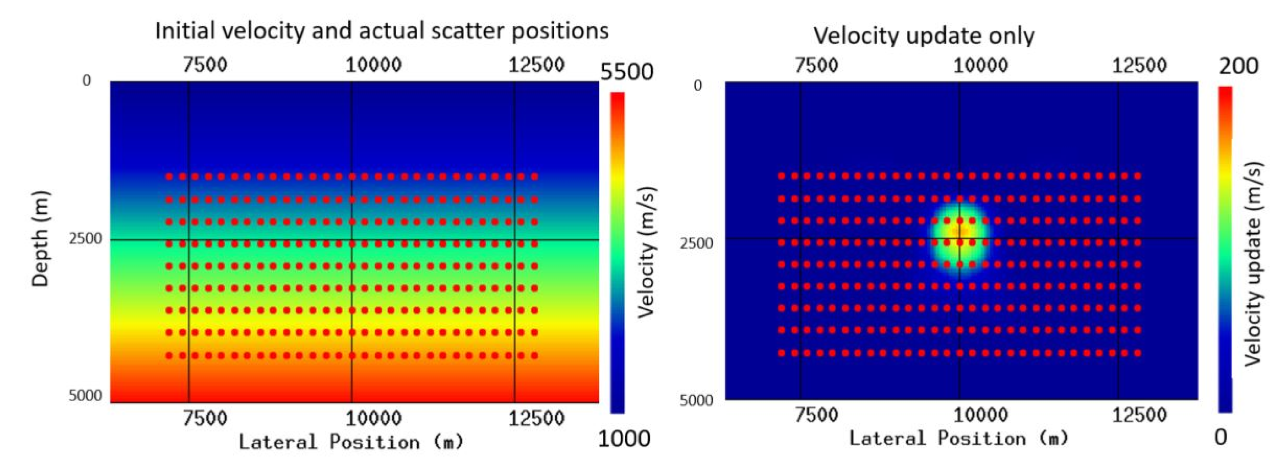


Figure 5: Actual scatter positions (red dots) are used for the iversion

Summary and future work

Adjoint stereotomography retains the benefit of the classical stereotomography method while provides a computationally and memory efficent algorithm. Future work includes:

- Investigate better simultaneous step length estimation for scatter positions and velocity updates to reduce cross talk between the two parameters classes.
- Investigate inverting for velocity only with migration image gathers to elimiate the scatter position from the model parameters.
- Test with more complex velocity model and real data.
- Test with multi-component velocity model and data.

Acknowledgements

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