Pure P- and S-wave elastic reverse time migration with adjoint state method imaging condition

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Abstract

We implemented an elastic reverse time migration based on a coupled system of pure P- and S-wave particle velocities. The system utilizes finite difference wavefields for P- and S-wave particle velocity in vertical and horizontal directions (v_{px}, v_{pz}, v_{sx} and v_{sz}), and for 2-D displacement divergence and curl (A and B). In contrast with the usual elastic imaging conditions that cross-correlates vertical displacements to obtain the P-wave image and vertical and horizontal displacements to obtain the converted wave image, we devised P- and S-wave imaging conditions using the adjoint state method. The resulting imaging conditions cross-correlate spatial derivatives of A and B wavefields with P- and S-wave displacements. The proposed migration shows a better reflector definition and more balanced amplitudes than the usual vertical and horizontal particle displacement cross-correlations.

Pure P- and S-wave finite differences

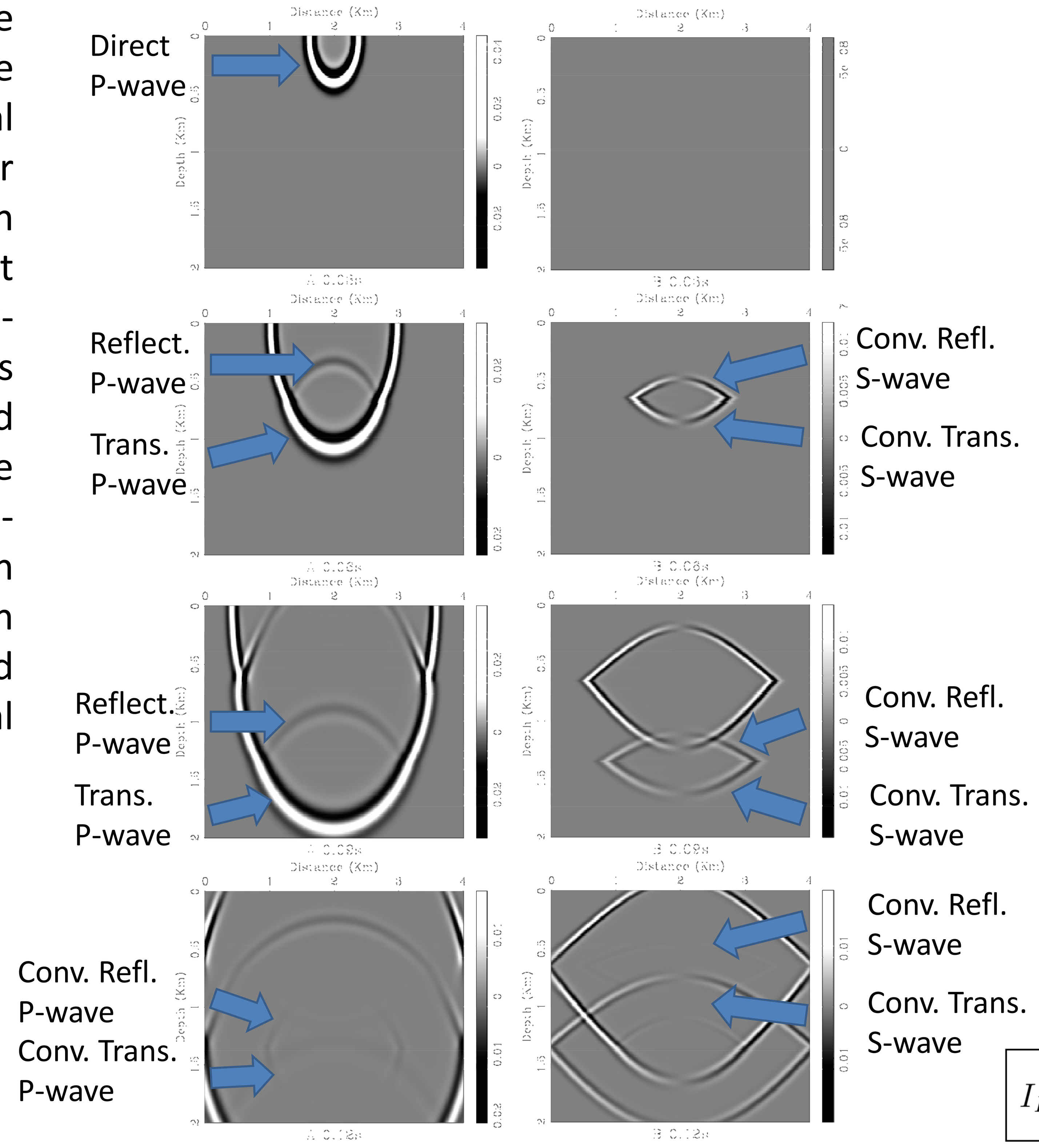
The elastic RTM migration is based on the pure P- and S-wave finite difference system of Chen (2014):

$$v_x = \frac{\partial u}{\partial t} \qquad v_z = \frac{\partial w}{\partial t}$$
$$v_x = v_{px} + v_{sx} \qquad v_z = v_{pz} + v_{sz}$$
$$\frac{\partial v_{px}}{\partial t} = \alpha^2 \frac{\partial A}{\partial x} \qquad \frac{\partial v_{pz}}{\partial t} = \alpha^2 \frac{\partial A}{\partial z}$$
$$\frac{\partial v_{sp}}{\partial t} = \beta^2 \frac{\partial B}{\partial z} \qquad \frac{\partial v_{sz}}{\partial t} = -\beta^2 \frac{\partial B}{\partial x}$$
$$A = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \qquad B = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial z}$$
$$\frac{\partial A}{\partial t} = \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} + f_A \qquad \frac{\partial B}{\partial t} = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} + f_B.$$

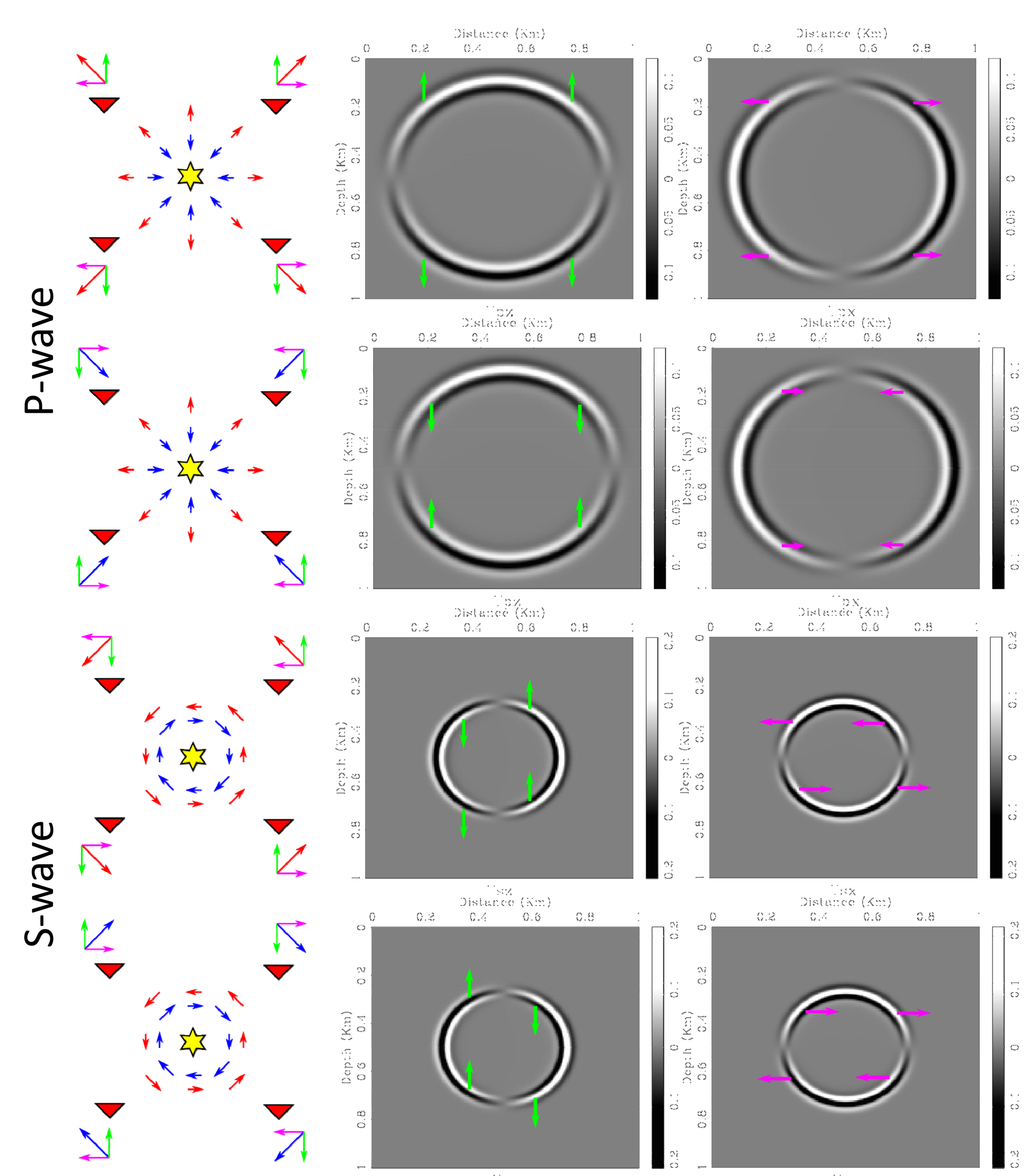
In this system, u and w are the horizontal and vertical particle displacements while v_x and v_z are their time derivatives or particle velocities. However, the system splits these velocities in their P- and S-wave components v_{px}, v_{pz}, v_{sx} and v_{sz} to explicitly evolve them. The wavefields A and B are the divergence and second curl component of the vector (u, 0, w). Although not explicitly addressed by Chen (2014), the P-wave source f_A and S-wave source f_B are injected to wavefields A and B, respectively. Finally, α and β are the P- and S-wave velocities. Density does not appear explicitly but in combination with Lamé moduli in both of these velocities.

Mode conversion

A wavefield **B wavefield**



P- and S-wave polarity



Elastic imaging conditions

$$I_{PP} = \int_0^T v_z \hat{v}_z dt$$

$$I_{PP} = \int_0^T (\sigma_{xx} + \sigma_{zz})(\hat{\sigma}_{xx} + \hat{\sigma}_{zz}) dt$$

$$I_{PP} = \int_0^T \nabla \cdot (v_x, v_z) \nabla \cdot (\hat{v}_x, \hat{v}_z) dt$$

$$I_{PP} = \int_0^T (v_{px}, v_{pz}) \cdot (\hat{v}_{px}, \hat{v}_{pz}) dt$$

$$\boxed{I_{PP} = 2\alpha \int_0^T \left(\frac{\partial A}{\partial x}, \frac{\partial A}{\partial z} \right) \cdot (\hat{v}_{px}, \hat{v}_{pz}) dt}$$

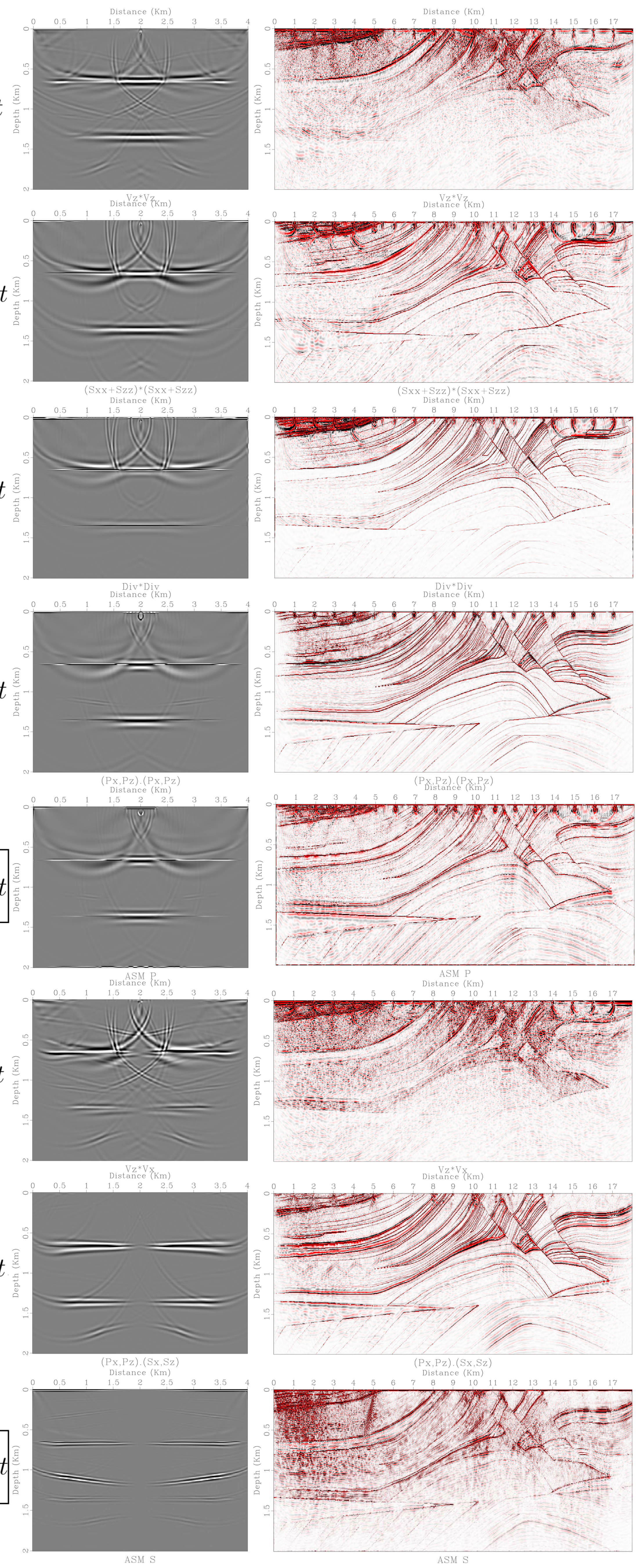
$$I_{PS} = \int_0^T v_z \hat{v}_x dt$$

$$I_{PS} = \int_0^T (v_{px}, v_{pz}) \cdot (\hat{v}_{sx}, \hat{v}_{sz}) dt$$

$$\boxed{I_{PS} = 2\beta \int_0^T \left(\frac{\partial B}{\partial z}, -\frac{\partial B}{\partial x} \right) \cdot (\hat{v}_{sx}, \hat{v}_{sz}) dt}$$

Numerical Experiments

3 layer model **Marmousi**



Note: v_z=vertical vel, v_x=horizontal vel, σ_{xx}=normal x stress, σ_{zz}=normal z stress, "^" indicates back-propagation and ASM is adjoint state method. The highlighted equations are the ASM imaging conditions.

Bibliography

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