Assumptions and goals for least squares migration

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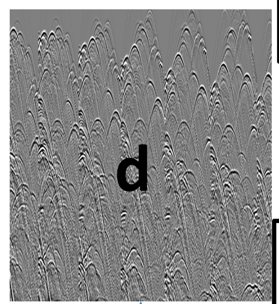
LEAST SQUARES MIGRATION FORMULATION

L: modeling operator
d: acquired data
m: model

data



$L_{Kirchhoff}$

$$d(\vec{\xi},t) = \int W(V,\vec{x},t)R(\vec{x})d\vec{x}$$

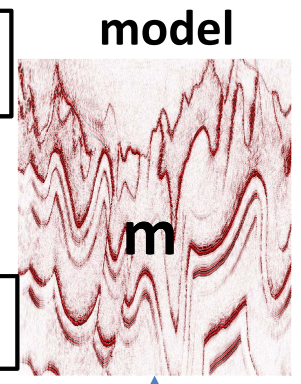
Kirchhoff modelling

L_{RTM}

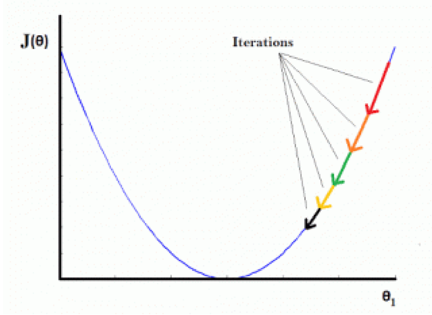
$$\phi_r(x,z,t) = \int \frac{\partial^2 \phi_s(x,z,t)}{\partial t^2} R(x,z)dt$$

Born modelling

model



$$\boxed{\mathbf{d} = \mathbf{Lm}}$$



$$J = \|\mathbf{d} - \mathbf{Lm}\|^2$$

One number!

Undesired features = prediction error + size of model

Least Squares inversion

L^H :adjoint operator (~migration).

$$J = \underbrace{\|\mathbf{d} - \mathbf{Lm}\|^2}_{\text{Data residuals in a particular norm choice}} + \lambda \underbrace{\|\mathbf{m}\|_{\mathbf{W}}}_{\text{Model size in a particular norm choice}}$$

W: weights to enforce a particular solution

$$\boxed{\mathbf{d} = \mathbf{W_d L W_m m}}$$

W_d data space weights
W_m model space weights

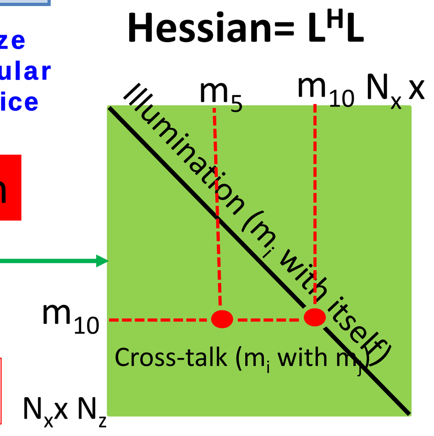
Iteratively solve...

$$\mathbf{m} = (\mathbf{L^H L})^{-1} \mathbf{L}^H \mathbf{d}$$

Migration

Solution: migration + Hessian inverse

Hessian= $L^H L$



LSMIG is defined by the choice of the modelling operator.

Optimization seeks to reduce the error energy by applying changes to the reflectivity. If the operator cannot predict the data completely then some components of the residuals cannot be decreased.

The error energy (**J**) is a global measure, not sufficient to control the optimization outcome.

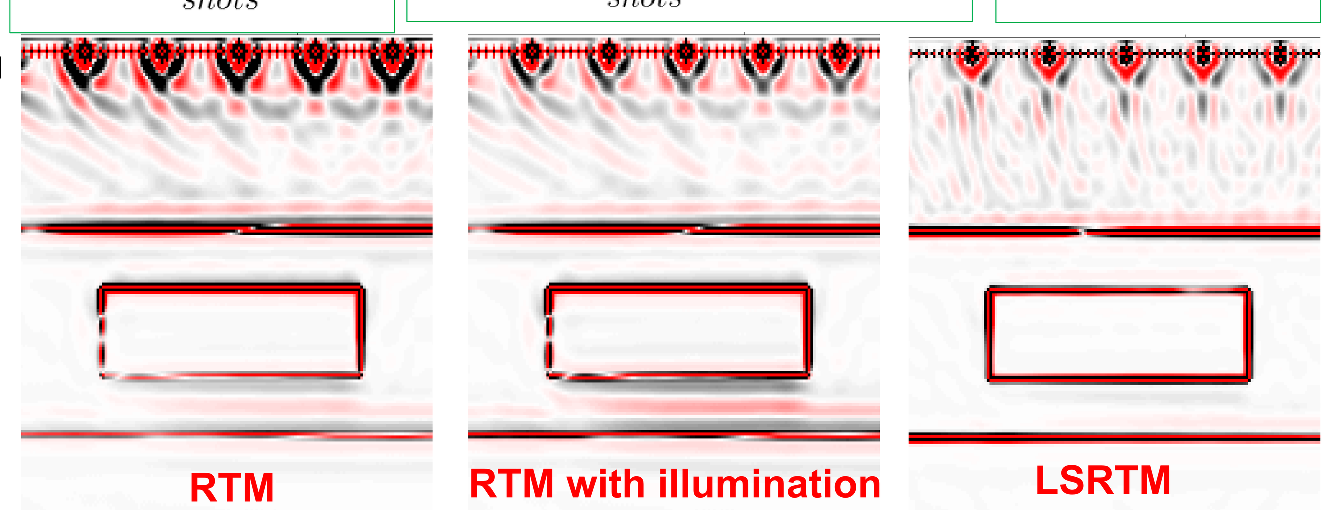
Data weights, model weights and constraints are used to enforce the outcome to be a useful result.

The solution to the normal equations implies first a migration and then the deconvolution of the Hessian.

Sampling and aperture issues are in the Hessian's off-diagonal

The Hessian diagonal can be approximated by the sum of cross-correlations for all sources. This compensates for different shot illumination but not sampling or aperture. This is commonly done in industry.
Without the summation, this is equivalent to a deconvolution imaging condition, which compensates for different energy across shots, but not shot different density.
However, the Hessian deconvolution on the right is required to achieve proper focusing.

$$\mathbf{L^H d} \Leftrightarrow \sum_{shots}^{N} \ddot{s} \otimes g^* \qquad \mathbf{PL^H d} \Leftrightarrow \sum_{shots}^{N} \frac{\ddot{s} \otimes g^*}{\sum_{shots}(s \otimes s^*)} \qquad (\mathbf{L^H L})^{-1}\mathbf{L^H d}$$

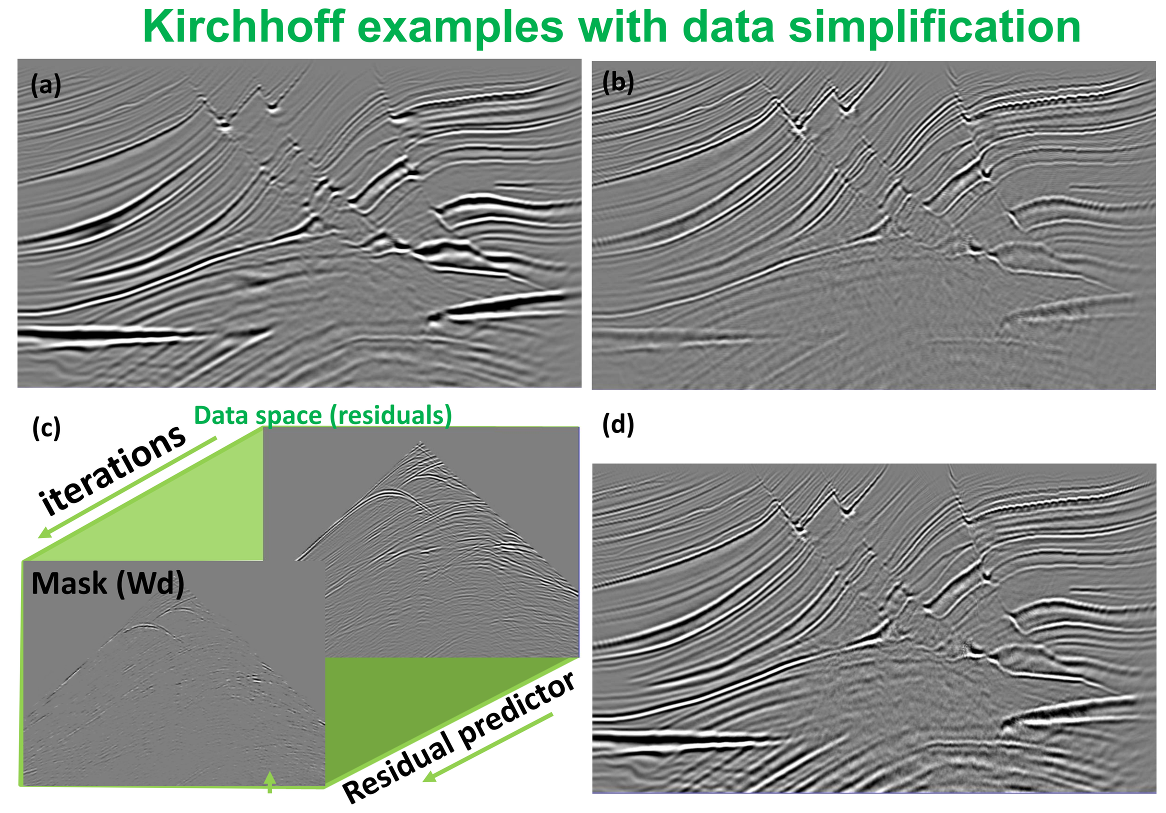


ADAPTIVE DATA SIMPLIFICATION

LSMIG Kirchhoff for a complex model shows the problems for optimization because of modelling limitations for real data. Traveltime tables cannot match the complexity of the FD data. Complex events persist in residuals calculated at different iterations producing wrong model updates.

One way to control these wrong events is tracking the residual evolution during iterations and attenuate events that cannot be properly predicted.

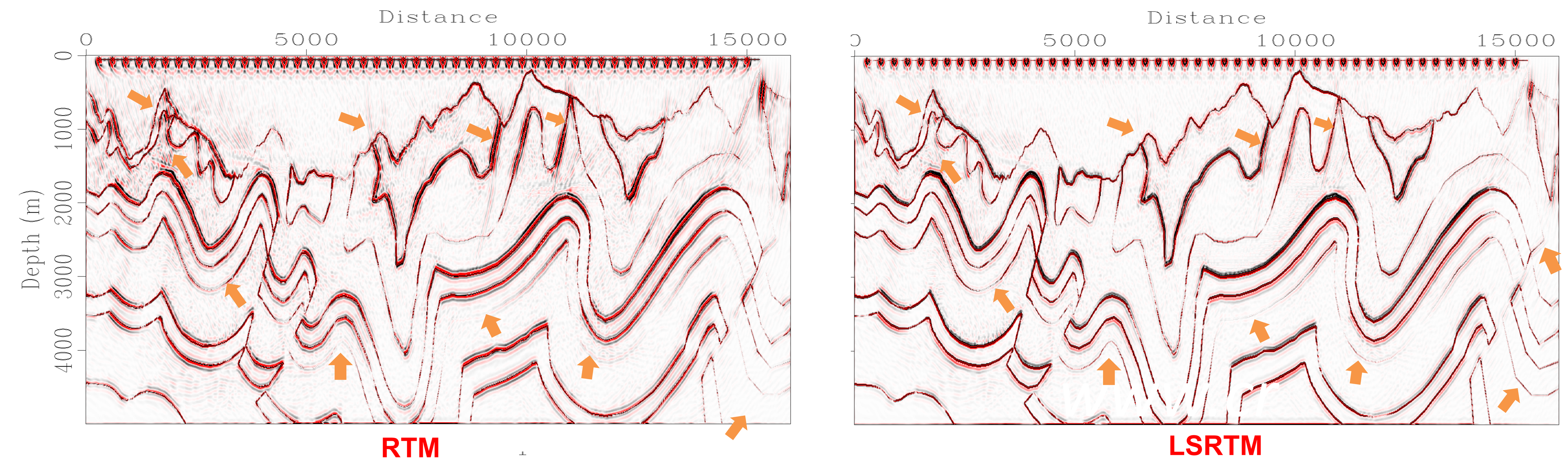
Kirchhoff examples with data simplification

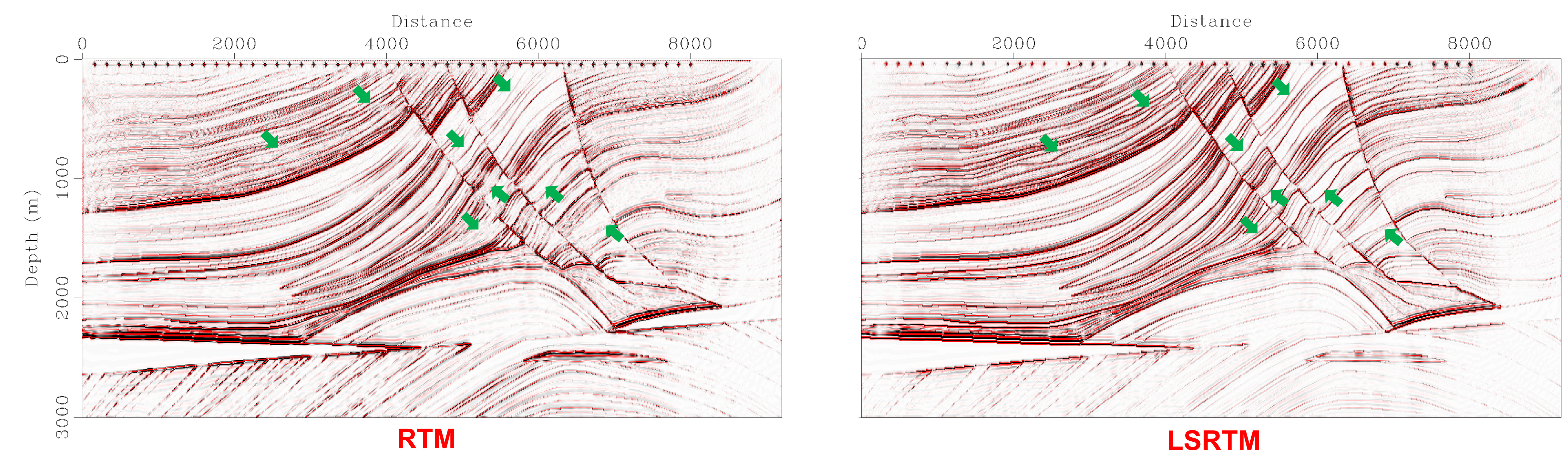


a) Kirchhoff migration; b) LSKirchhoff without data weights; c) Residuals with the data mask; d) Kirchhoff LSMIG with data mask.

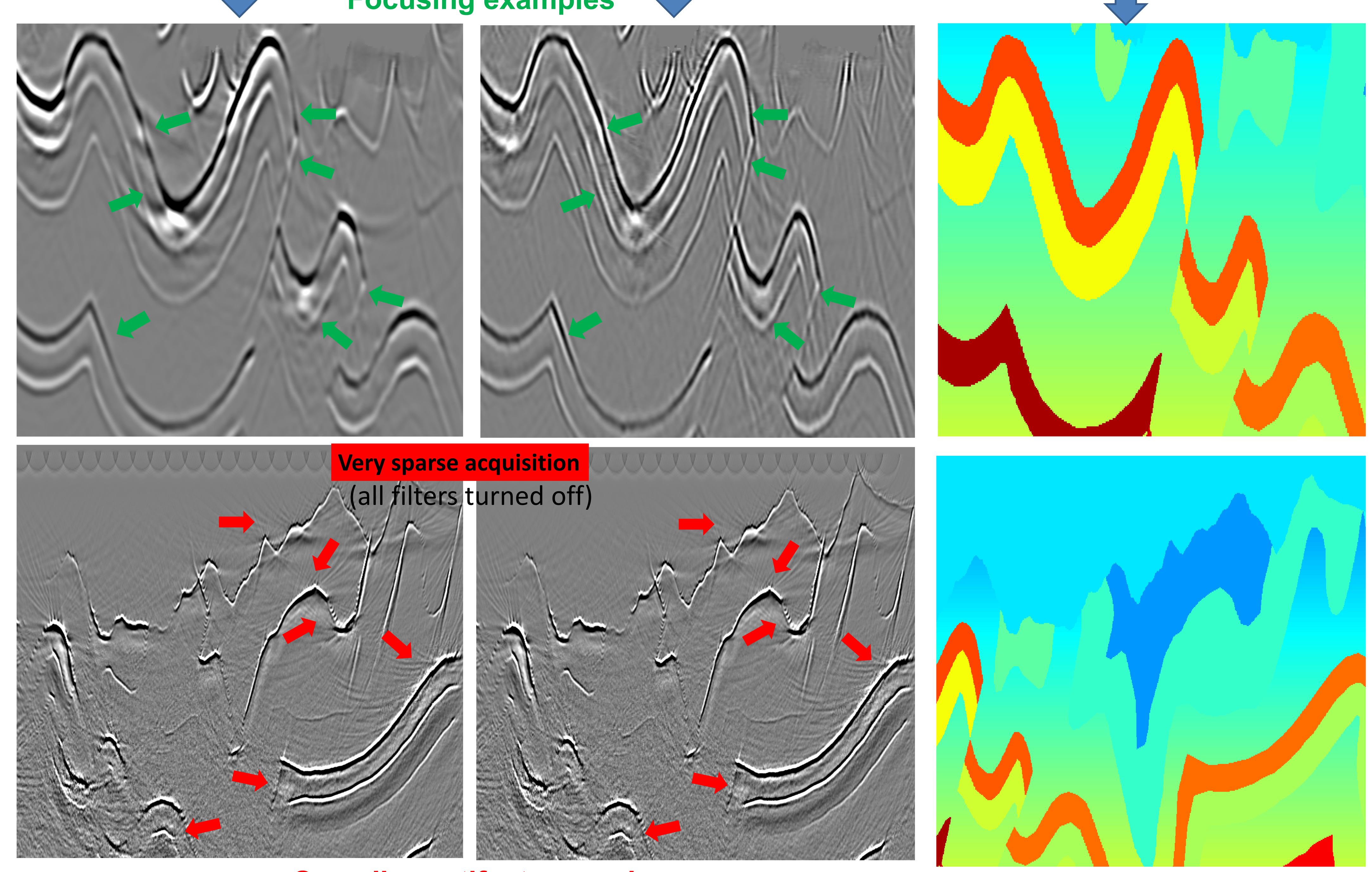
LSMIG GOALS: a) focusing, b) sampling artifacts, c) illumination compensation

LSRTM examples





LSMIG KIRCHHOFF



Sampling artifact examples