Bayesian approaches to estimating rock physics properties from seismic attributes

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Abstract

We have examined three Bayesian approaches using the well-log data at the Carbon Management Canada Newell County Facility, assuming Gaussian, Gaussian mixture, and non-parametric distributions of the rock physics variables. The solution is represented by the posterior distribution of the porosity and lithology parameters conditioned on the elastic data. In this application, because the nonlinearity of the rock physics model is not strong and the data are approximately Gaussian distributed, the three results are similar. However, owing to its efficiency and its ability to recognize the multimodality of the data, the Gaussian mixture model might be more suitable to complex geological conditions.

Methods

General problem

$$\mathbf{d} = g(\mathbf{m}) + \mathbf{e}, \tag{1}$$

Bayesian framework

$$P(\mathbf{m}|\mathbf{d}) = \frac{P(\mathbf{m},\mathbf{d})}{\int P(\mathbf{m},\mathbf{d})d\mathbf{m}} = \frac{P(\mathbf{d}|\mathbf{m})P(\mathbf{m})}{\int P(\mathbf{d}|\mathbf{m})P(\mathbf{m})d\mathbf{m}}, \tag{2}$$

1. Linear problem with Gaussian prior

$$\text{if} \quad \mathbf{m} \sim \mathcal{N}(\mu_m, \Sigma_m), \quad \mathbf{e} \sim \mathcal{N}(\mathbf{0}, \Sigma_e), \quad g \text{ is linear}$$

$$\text{then} \quad \mathbf{m}|\mathbf{d} \sim \mathcal{N}(\mu_{m|d}, \Sigma_{m|d}) \tag{3}$$

$$\mu_{m|d} = \mu_m + \boldsymbol{\Sigma}_m \mathbf{G}^T(\mathbf{G}\boldsymbol{\Sigma}_m\mathbf{G}^T + \boldsymbol{\Sigma}_e)^{-1}(\mathbf{d} - \mathbf{G}\mu_m) \tag{4}$$

$$\boldsymbol{\Sigma}_{m|d} = \boldsymbol{\Sigma}_m - \boldsymbol{\Sigma}_m\mathbf{G}(\mathbf{G}^T\boldsymbol{\Sigma}_m\mathbf{G}^T + \boldsymbol{\Sigma}_e)^{-1} - \mathbf{G}\boldsymbol{\Sigma}_m \tag{5}$$

2. Nonlinear problem with Gaussian-mixture prior

$$\mathbf{m} \sim \sum_{k=1}^{N} \lambda_k \mathcal{N}(\mu_m^k, \Sigma_m^k)$$

$$\mathbf{m}|\mathbf{d} \sim \sum_{k=1}^{N} \lambda_{k|d} \mathcal{N}(\mu_{m|d}^k, \Sigma_{m|d}^k)$$

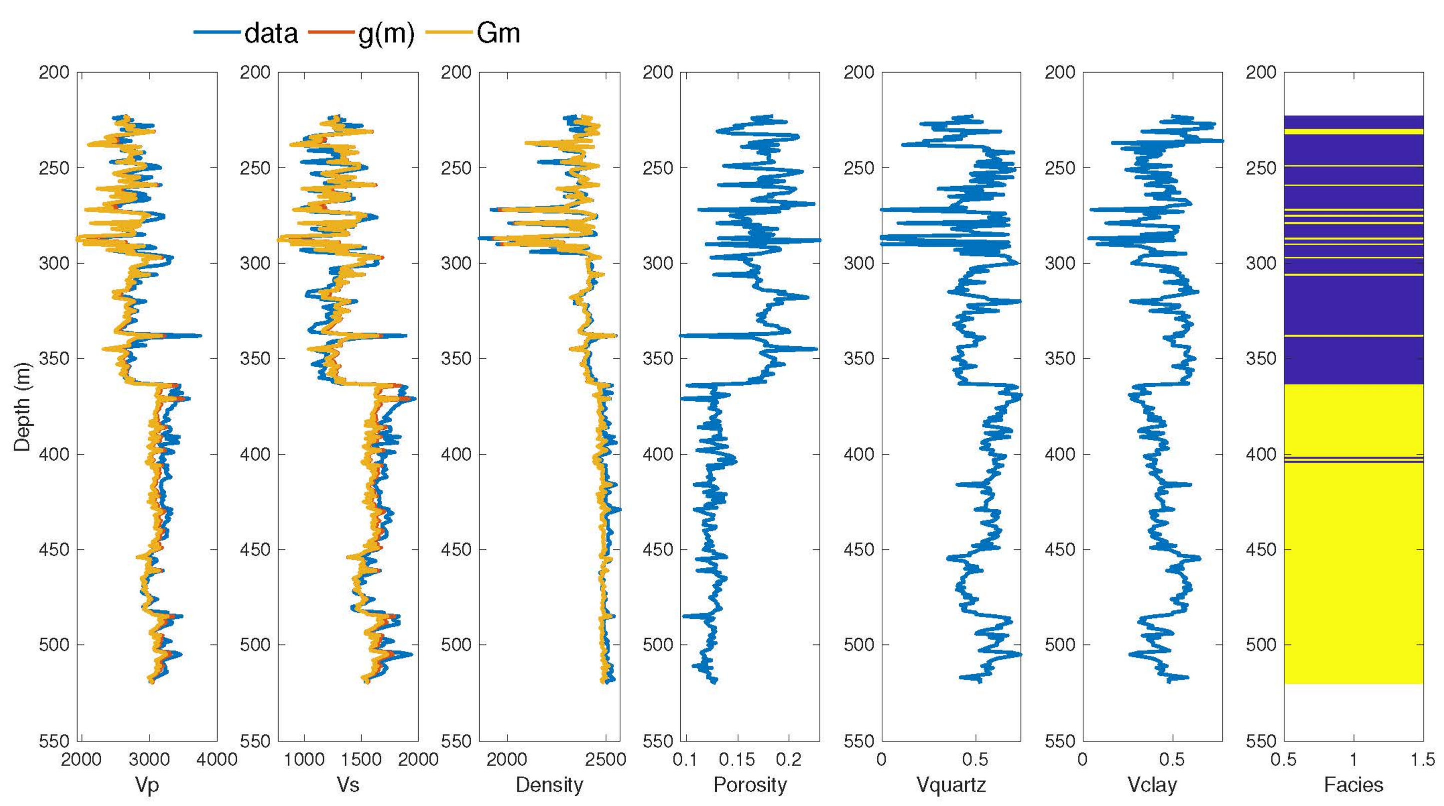
$$\begin{aligned} \mu_{m|d}^k &= \mu_m^k + \Sigma_{m,d}^k(\Sigma_{d,d}^k)^{-1}(\mathbf{d} - \mu_d^k) \\ \Sigma_{m|d}^k &= \Sigma_{m,m}^k - \Sigma_{m,d}^k(\Sigma_{d,d}^k)^{-1}\Sigma_{d,m}^k. \end{aligned} \tag{6}$$

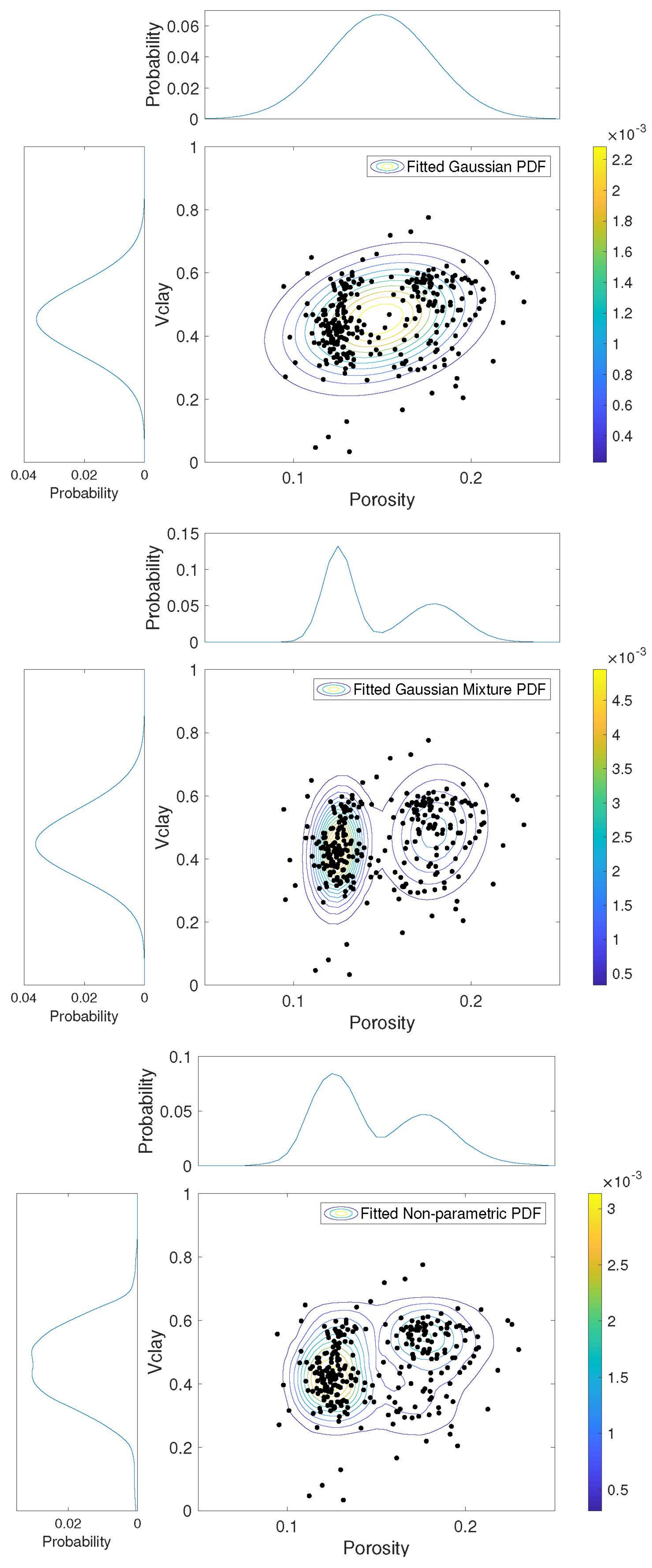
3. Nonlinear problem with non-parametric prior

$$P(\mathbf{m},\mathbf{d}) = \frac{1}{N_s \prod_{u=1}^{M} h_{m_u} \prod_{v=1}^{D} h_{d_v}} \sum_{i=1}^{N_s} \prod_{u=1}^{M} K\left(\frac{m_u - m_{u_i}}{h_{m_u}}\right) \prod_{v=1}^{D} K\left(\frac{d_v - d_{v_i}}{h_{d_v}}\right), \tag{7}$$

where $\mathbf{m} = (m_1, ..., m_M)$ and $\mathbf{d} = (d_1, ..., d_D)$. K is the kernel function (e.g., Gaussian or Epanechnikov), $\mathbf{h}_m$ and $\mathbf{h}_d$ are kernel bandwidths.

Analysis of well-log data





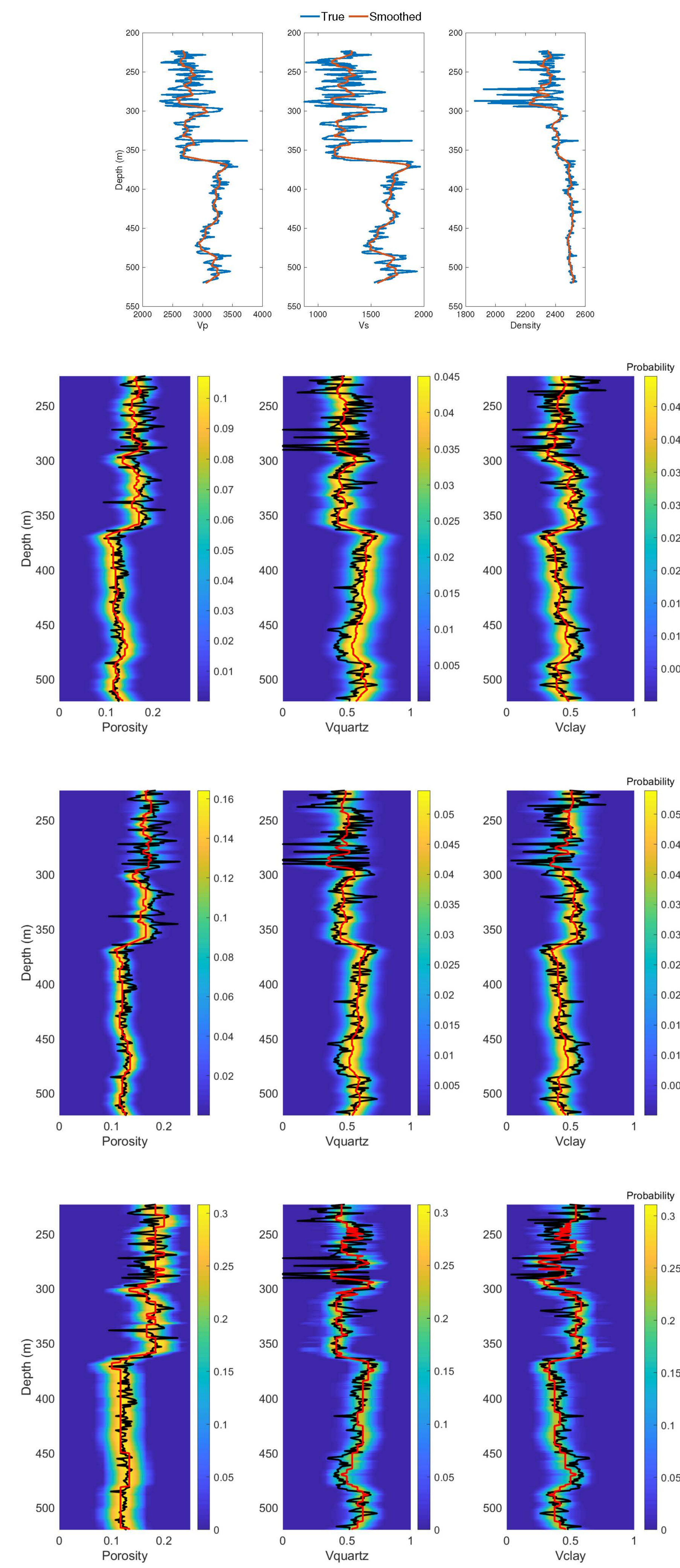


Fig. 1: Top to bottom: input data (smoothed logs) and the results of Gaussian, Gaussian mixture, and non-parametric.

Acknowledgments

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