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Robust reconstruction via Group Sparsity with Radon operators

Ji Li*, Daniel Trad
li.ji1@ucalgary.ca

Abstract

Sparse solutions of linear systems play an essential role in seismic data processing. An additional structure called group sparsity can be used to improve the performance of the sparse inversion. We propose a robust group sparse inversion algorithm based on Orthogonal Matching Pursuit with the Radon operators in the frequency slowness $(w-p)$ domain. In each iteration, The proposed algorithm first picks the dominant slowness group. Then, all the Radon coefficients within the currently selected slowness groups fit the data in the time-space $(t-x)$ domain via a robust solver, which is a $\ell_1 - \ell_1$ ADMM solver (Wen et al., 2016). We prove that the proposed algorithm is resistant to erratic noise, making it attractive to applications such as simultaneous source deblending and reconstruction of noisy onshore datasets.

Introduction

Sparse representation is an essential tool for seismic data processing. An additional group structure can be added to the problem, often referred to as group sparse or group Lasso (Huang and Zhang, 2010), to improve the performance of the sparse estimation. One exciting goal is to develop a robust sparse solver to solve the inversion problem for seismic data corrupted by erratic noise. In this poster, we cooperate the robust inversion with group sparsity to get a better sparse estimation. We first develop a robust algorithm combining robust inversion and group sparsity. The algorithm works on the linear Radon transform in the frequency-slowness $(f-p)$ domain. We use the orthogonal Matching Pursuit to build the estimated results iteratively. OMP picks the group $\mathbf{m}_p$ with the maximum norm in each iteration. Then we use a robust solver to fit the Radon coefficients.

Methods

By using the linear Radon operators in the $f-p$ domain, the robust strong group sparse inversion problem can be summarized as follow.

$$\text{argmin}\ \|\mathcal{L}^*\mathbf{m}-\mathbf{d}\|_1 + \lambda\|\mathbf{m}\|_1 + \beta\sum_{i=1}^{N}\|\mathbf{m}[i]\|_2. \tag{1}$$

where $\mathbf{L}^*$ is the forward Radon operator, which transfers the Radon coefficients in the $f-p$ domain to the $t-x$ domain. $\mathbf{m}$ is the Radon coefficients in the $f-p$ domain. The $\|\mathcal{L}^*\mathbf{m}-\mathbf{d}\|_1$ make the cost function robust to the erratic noise. $\beta\sum_{i=1}^{N}\|\mathbf{m}[i]\|_2$ is use to promote the group sparsity, and $\lambda\|\mathbf{m}\|_1$ can enhance the sparsity within the groups. We use the orthogonal Matching Pursuit (OMP) to solve this problem.

Input: $\mathbf{d}$, $\mathcal{L}$,and k
Output: $\widehat{\mathbf{m}}^{[k]}$
Initialization: $\mathbf{r}^{[0]} = \mathbf{d}$,$\widehat{\mathbf{m}}^{[k]} = 0$, and $T^{[0]} = \emptyset$
for $k = 1, 2, ..., K$ **do**
$\mathbf{M} = \mathcal{L}\mathbf{r}$
Pick group p_k with maximum $\|\mathbf{m}[p]\|_2$
$T^k = T^{k-1} + p_k$
$\widehat{\mathbf{m}}^k_{T^k} = \underset{\widetilde{\mathbf{m}}_{T^k}}{\text{argmin}}\ \|\mathbf{d} - \mathcal{L}^*_n(\widetilde{\mathbf{m}}_{T^k})\|_1 + \lambda\|\mathbf{m}\|_1$
$\mathbf{r}^k = \mathbf{d} - \mathcal{L}^*\widehat{\mathbf{m}}^k_{T^k}$
end for

Figure 1 shows how the $f-p$ domain and norm of the groups look like with different kinds of data.

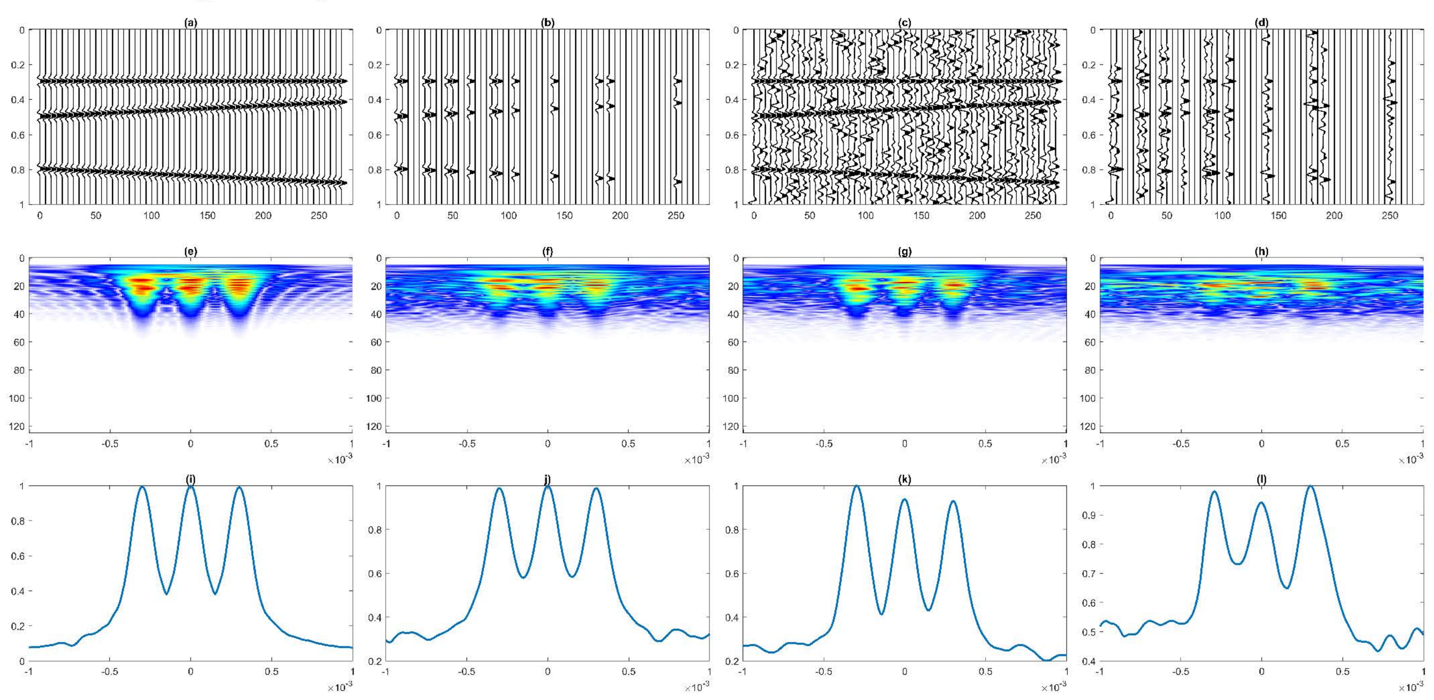

Figure 1: Method explanation

Results

Figure 2 shows the 2D synthetic denoised results with different methods. Figure 3 shows the 3D synthetic denoised and interpolated result.

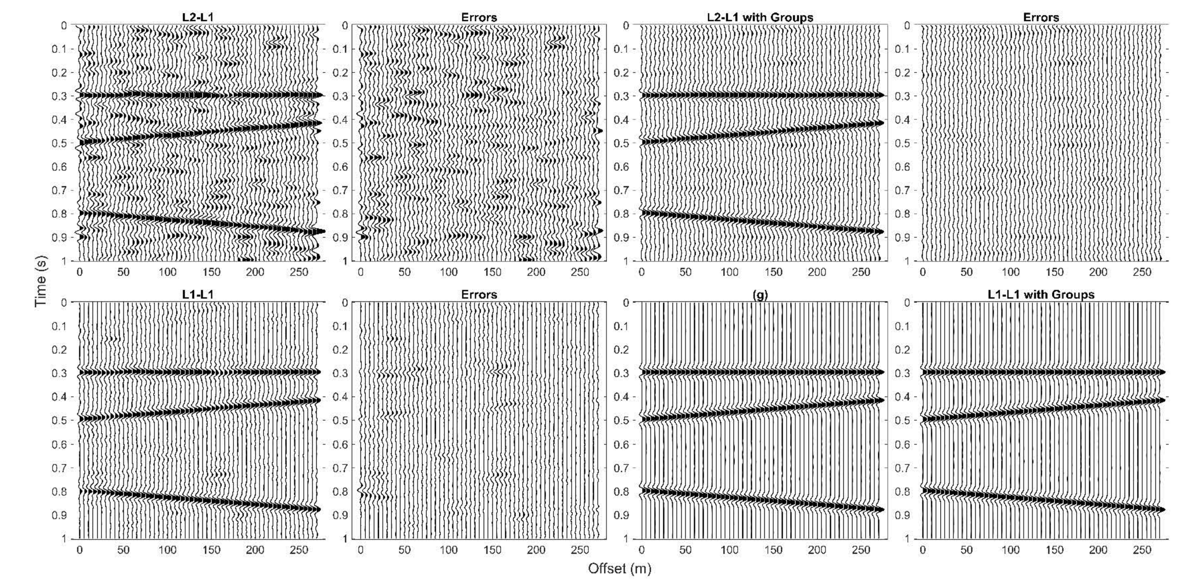

Figure 2: 2D Example

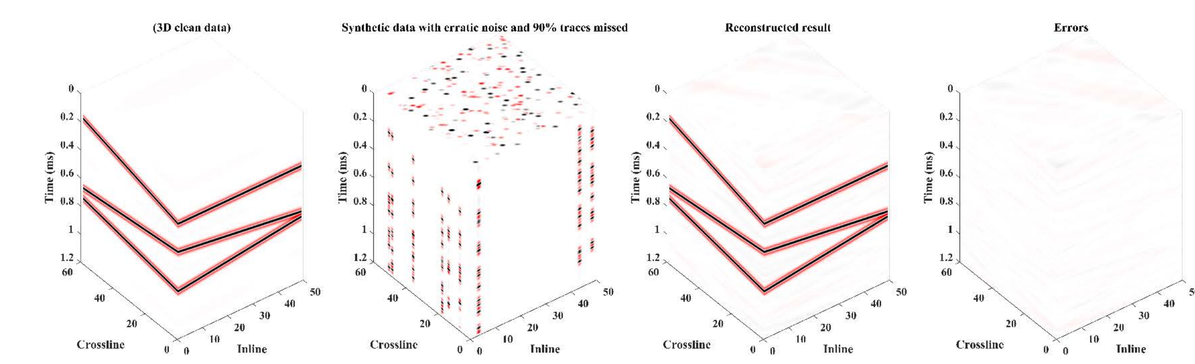

Figure 3: 3D Example

Conclusions

We proposed a robust group sparse inversion algorithm which can provide a better sparse estimation and be robust to the erratic noise simultaneously. The core of the proposed method is based on the orthogonal Matching Pursuit. The tests on both 2D and 3D synthetic examples prove the effectiveness and robustness of the proposed method. Furthermore, compared with the traditional MP and OMP algorithm, the proposed algorithm can save the total costs a lot since it selects one group with multiple coefficients instead of picking just one best-correlated coefficient like MP and OMP.

References

- Wen, F., Liu, P., Liu, Y., Qiu, R. C., and Yu, W., (2016) *Robust sparse recovery in impulsive noise via $\ell_p - \ell_1$ optimization.* IEEE Transactions on Signal Processing.

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UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
Department of Geoscience