Geophysical inversion quantum style

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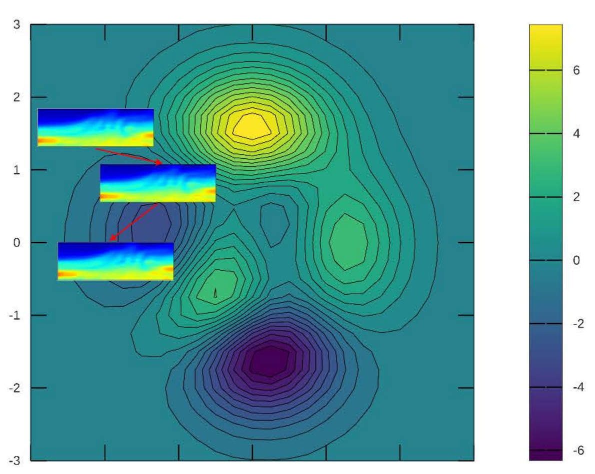
Abstract

Quantum computation is described as a promising paradigm for the future. One of their advantages is the quantum parallelism, that consists in solving many instances of the same problem in a single run. This can be done due to the possibility to set a quantum system in a superposition of states. Although its main limitation is that only one of the states can be read at the end, it is possible to increase the chances of the state we are looking for. We show a framework to pose a geophysical inverse problem, usually solved using gradient methods, as a quantum computing algorithm. This algorithm does a global exploration of the model space using the quantum parallelism. Then it manipulates the quantum phase of their states to increase the chances of reading the model that produces the global minimum. In that way we can read the inversion answer at the end of the quantum computation. Something important to notice is that there is no need to compute gradients or Hessians but only to do forward modelling and residual calculation. The algorithm has been coded and run in a quantum simulator.

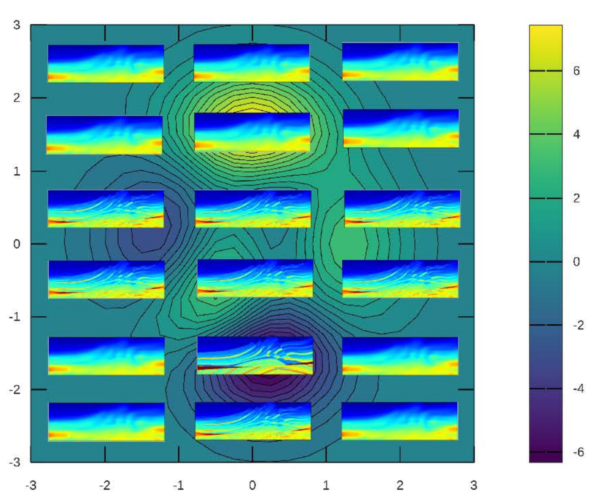
Inversion with gradient methods vs. quantum inversion

Gradient methods:



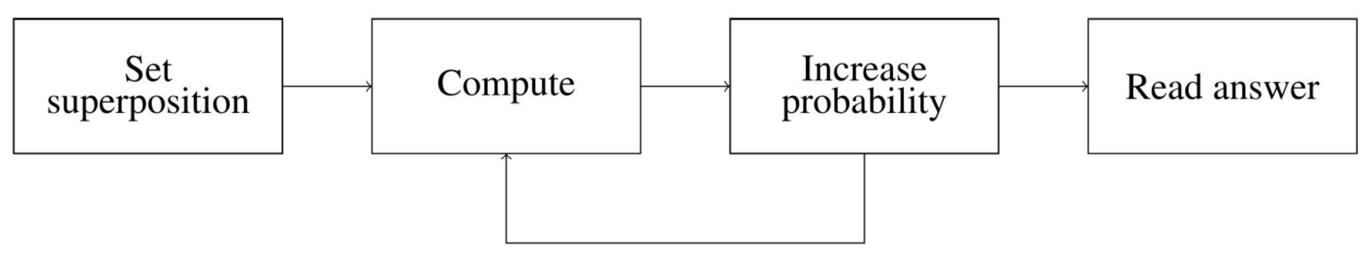
- One initial model.
- Update the model in every iteration.
- Need gradient and sometimes Hessian.
- Explore a part of the model space.

Quantum:



- Evaluates multiple models in a single iteration. ↑
- Explores the whole space. ↑
- No need of gradient or Hessians, only forward modelling. ↑
- Multiple iterations to read answer. ↓

Quantum inversion algorithm diagram flow

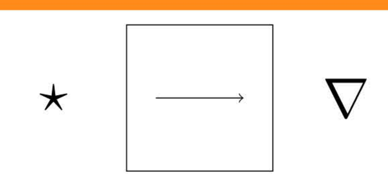


- Set superposition enters the initial velocity models in equal superposition.
- Compute does the forward modelling and residual calculation in superposition. It also flips the sign (phase) of the model with the minimum residual.
- Increase probability applies the mirror circuit to increase the probability of the model with the flipped phase.
- Read answer gets the model with the highest probability.

Quantum algorithm iteration

1. A term is a pair $|residual\rangle|model\rangle$ (model is the picture). All residuals are initially $|0\rangle$:
   $$|0\rangle$$
2. The superposition of states is a weighted summation of all terms:
   $$\alpha_0|0\rangle + \cdots + \alpha_k|0\rangle + \cdots + \alpha_n|0\rangle$$
3. Residuals $|r_i\rangle$ are calculated in superposition.
   $$\alpha_0|r_0\rangle + \cdots + \alpha_k|r_k\rangle + \cdots + \alpha_n|r_n\rangle$$
4. The sign (phase) of the term with the minimum residual, $|r_k\rangle$, is flipped.
   $$\alpha_0|r_0\rangle + \cdots - \alpha_k|r_k\rangle + \cdots + \alpha_n|r_n\rangle$$
5. The *mirror* or *Groover's iteration* quantum circuit increases the probability of the term with the flipped phase, $\alpha'_k > \alpha_k$ and $\alpha'_i < \alpha_i, i \neq k$.
   $$\alpha'_0|r_0\rangle + \cdots + \alpha'_k|r_k\rangle + \cdots + \alpha'_n|r_n\rangle$$
6. Reset the residuals to undo quantum entanglement and return to step 2 if probability is not high enough.

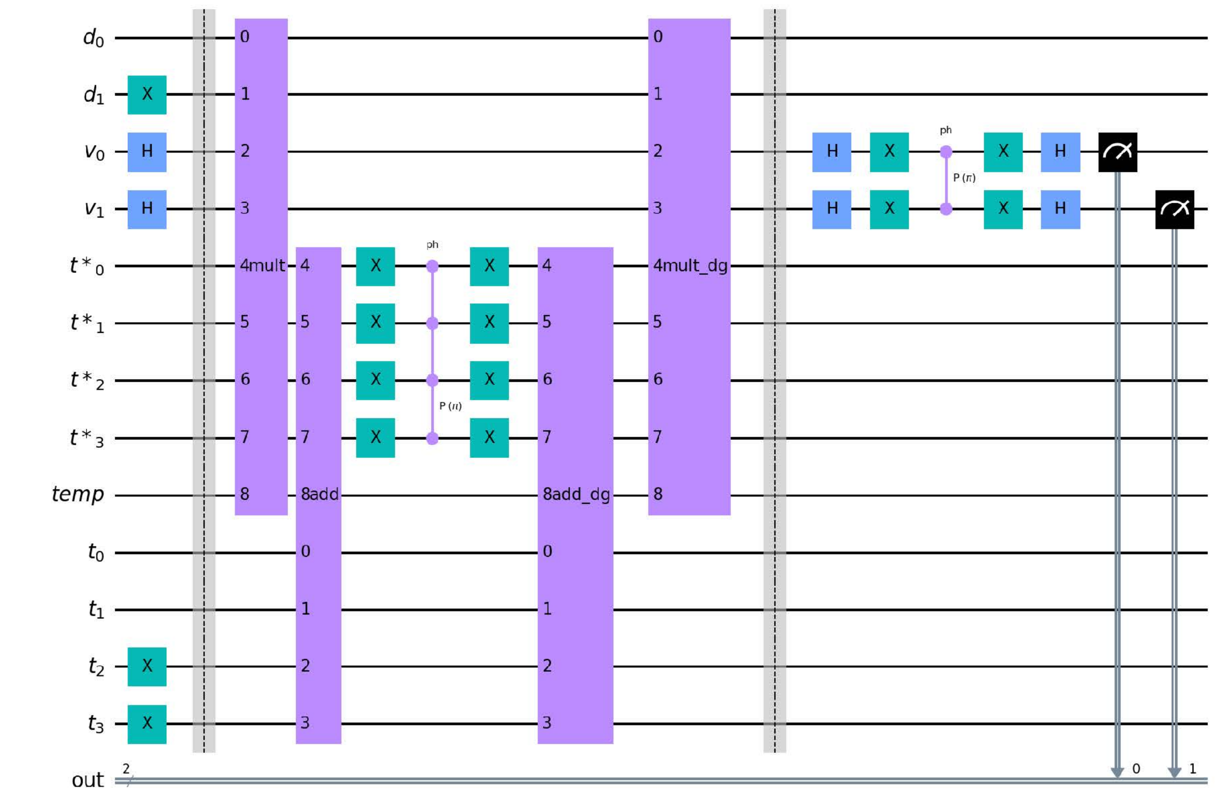
Proof of concept problem



- One slowness cell.
- One pair source-receiver.
  - Raypath length: $d = 2$ distance units.
  - Measured traveltime: $t = 4$ time units.
  - Result should be $v = 2$ slowness units.

Proof of concept quantum circuit and simulation

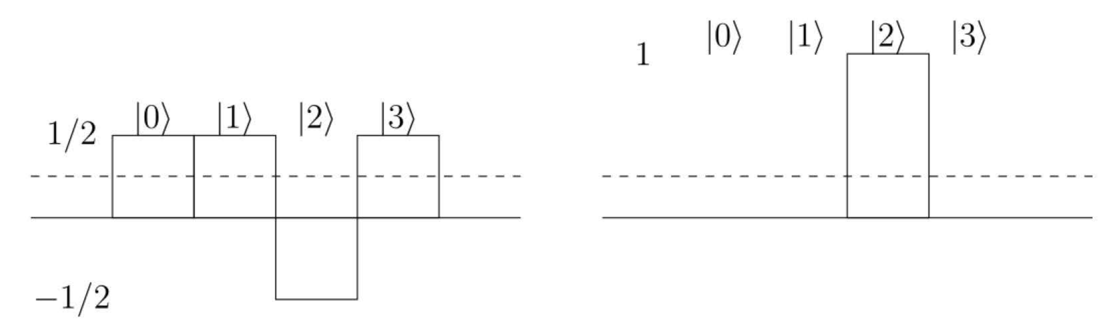
- Register d is distance, v are the velocities in superposition, t^* will be the calculated traveltimes and residuals, and t is the measured traveltimes.



- Step by step simulation (# in circuit):

#	Step	t	t^*	v	t
1	Initial	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
2	Setup	$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle + \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$	$\|2\rangle$
3		$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|01\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|03\rangle$		$\|2\rangle$
4	Mult	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|21\rangle + \frac{1}{2}\|42\rangle + \frac{1}{2}\|63\rangle$		$\|2\rangle$
5	Add	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle + \frac{1}{2}\|-21\rangle + \frac{1}{2}\|02\rangle + \frac{1}{2}\|23\rangle$		$\|2\rangle$
6	Phase $e^{i\pi}$	$\|-4\rangle$	$\frac{1}{2}\|-40\rangle + \frac{1}{2}\|-21\rangle - \frac{1}{2}\|02\rangle + \frac{1}{2}\|23\rangle$		$\|2\rangle$
7	Add†	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|21\rangle - \frac{1}{2}\|42\rangle + \frac{1}{2}\|63\rangle$		$\|2\rangle$
8	Mult†	$\|-4\rangle$	$\frac{1}{2}\|00\rangle + \frac{1}{2}\|01\rangle - \frac{1}{2}\|02\rangle + \frac{1}{2}\|03\rangle$		$\|2\rangle$
9		$\|-4\rangle$	$\|0\rangle$	$\frac{1}{2}\|0\rangle + \frac{1}{2}\|1\rangle - \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$	$\|2\rangle$

- Mirror increases the probability of the model with the flipped phase, $|2\rangle$, by rotating amplitudes around the mean (dashed line):



- Reading stage reads the correct model, $|2\rangle$, with probability 1. Bigger problems need more iterations.

References

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