Elastic full waveform inversion results uncertainty analysis: a comparison between the model uncertainty given by conventional FWI and machine learning methods

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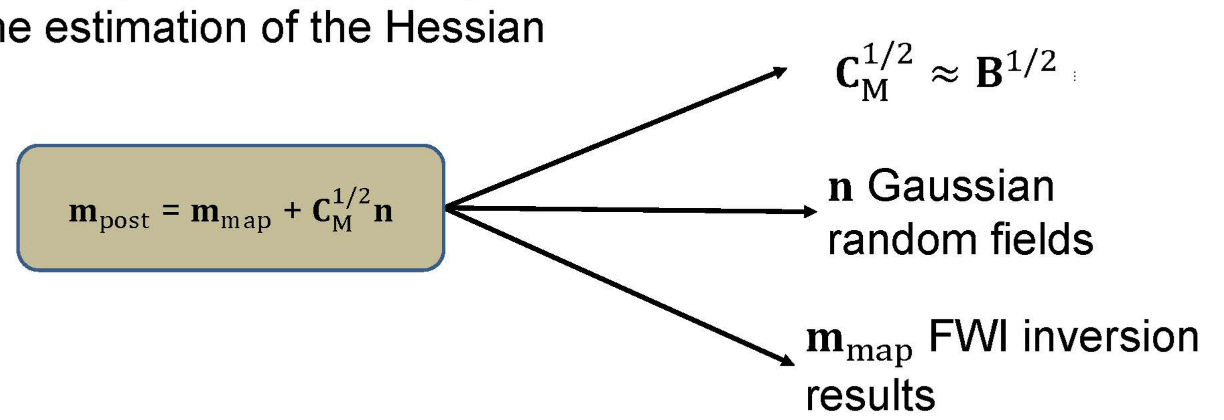
Abstract

Uncertainty analysis is an important aspect of quantifying the results of the inversion problem. In this report, we compare the uncertainty analysis given by two methods for full waveform inversion. The first method is by using the approximation of the inverse Hessian to perform the uncertainty analysis, as the approximation of the inverse Hessian is closely related to the posterior model covariance matrix. The second method is based on a machine learning-based method, which uses the Bayesian neural network (BNN) to generate elastic models and then performs the inversion. In the BNN, each trainable weight is represented as a Gaussian distributed probability distribution function (pdf). When BNN is well trained, we can forward calculate the BNN several times and perform the statistic analysis for the prediction results and give the uncertainty analysis for the generated models. Our numerical results suggested that both methods can generate promising inversion results and reasonable uncertainty quantification when compared with the true model errors.

Introduction

● Uncertainty analysis given by Hessian matrix

In the FWI workflows, Bayesian inference allows us to incorporate the prior information into waveform tomography to estimate posterior uncertainties of the inverted results. The effective quantification of the uncertainty by using the Bayesian inference relies on the efficient way of the estimation of the Hessian



Statistic analysis on collected model set $\mathbf{m}_{\text{post}}$ gives the uncertainty analysis for FWI.

● Uncertainty analysis given by neural network

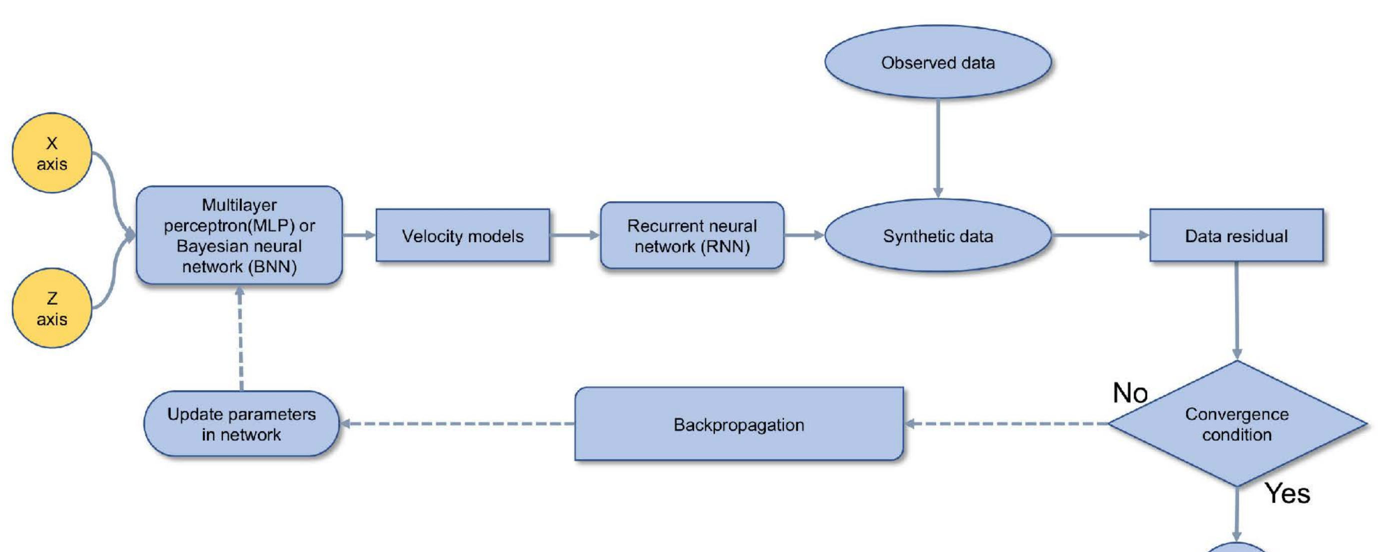


Fig1. BNN-based FWI inversion framework

We perform the forward calculation using the well-trained BNN 1000 times, and obtain three sets for the elastic models, respectively. Each set contains 1000 models. As the weights are drawn from the well-trained posterior probability distribution function, each forward calculation gives relatively different elastic models. If the elastic models in the same set agree with each other at a particular location of the model, the uncertainty for this point is low. If the elastic models in the same set disagree with each other at a particular location of the model, the uncertainty for this point is high.

Numerical tests

● Uncertainty analysis given by Hessian matrix

- The grid length of the model is 20 × 20.
- α is the hyper-parameter for the initialization of B_0
- The grid size of the model is 100 × 200.
- Maximum receiving time 2.6s.
- Ricker's wavelet 10 Hz main frequency.
- Maximum iteration time 500.

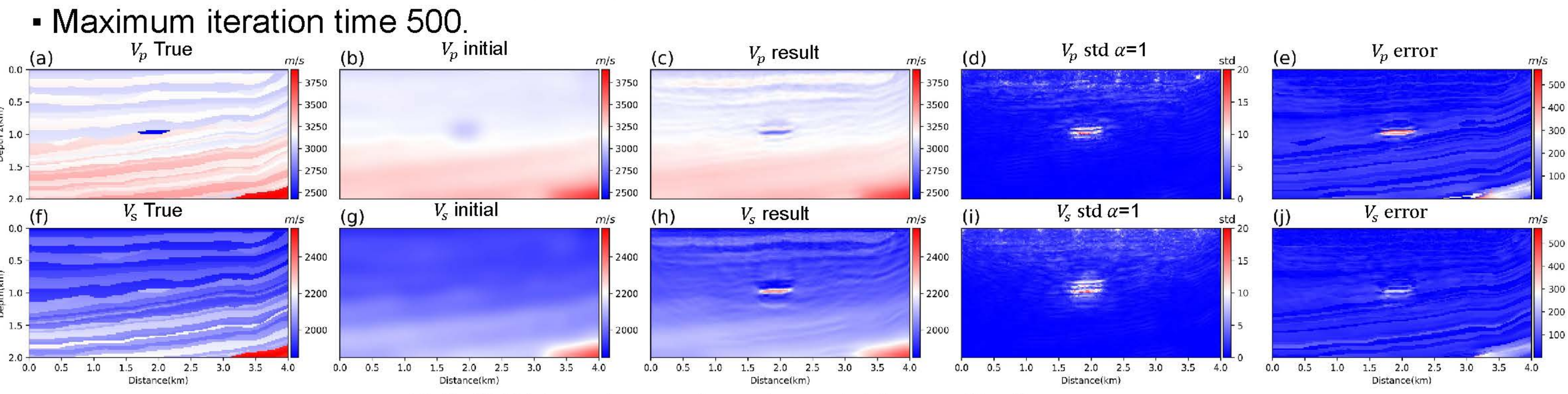


Fig2. CFWI inversion results and uncertainty quantification

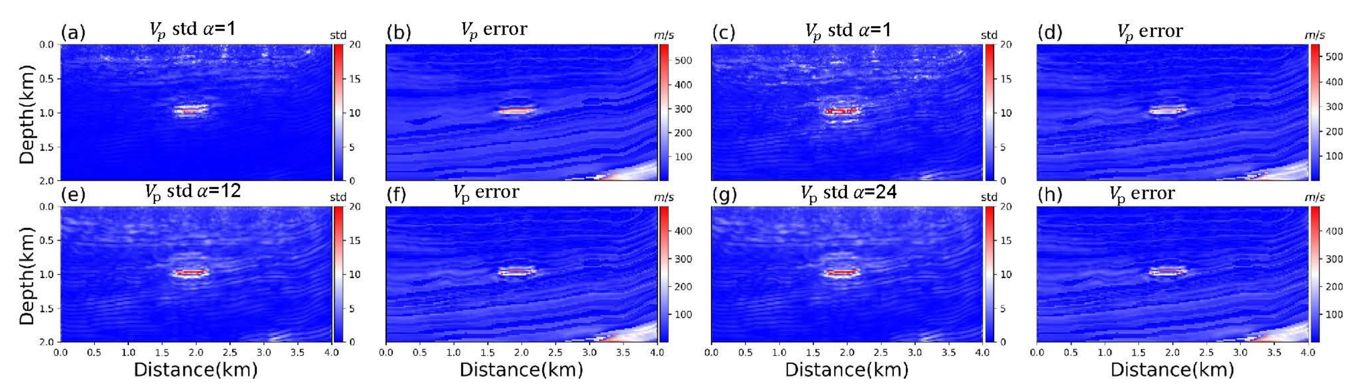


Fig3. CFWI inversion results for V_p and uncertainty quantification using different α

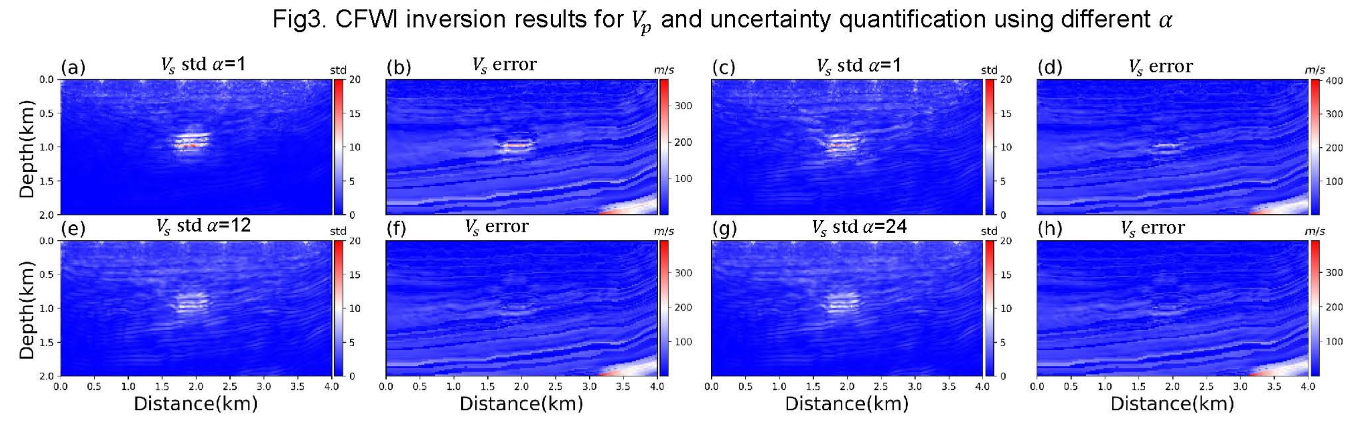


Fig4. CFWI inversion results for V_s and uncertainty quantification using different α

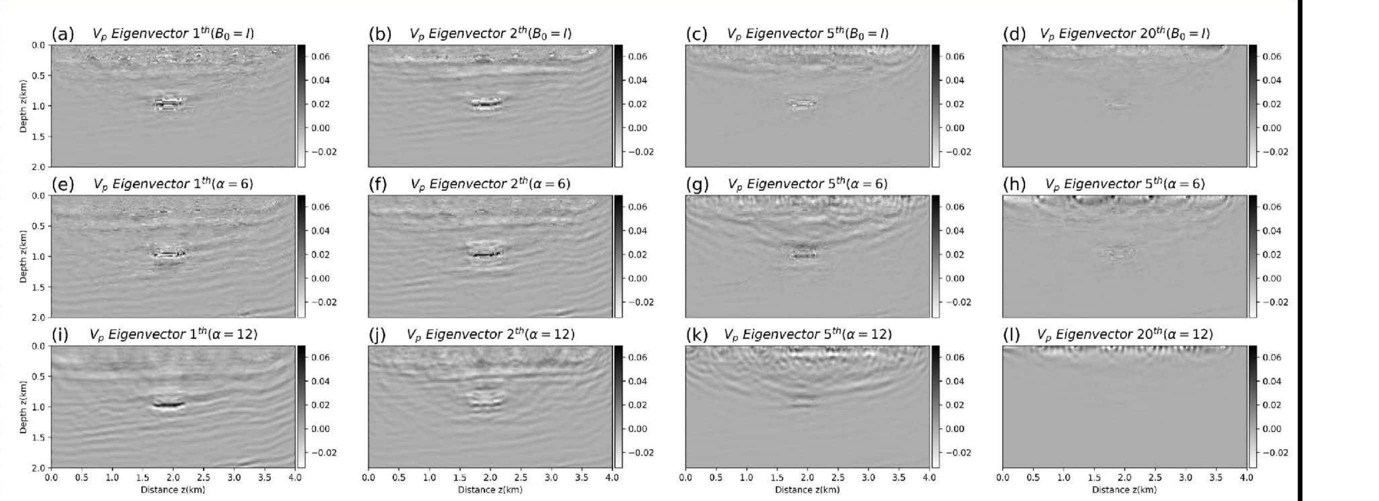


Fig5. Eigenvectors for V_p using different α

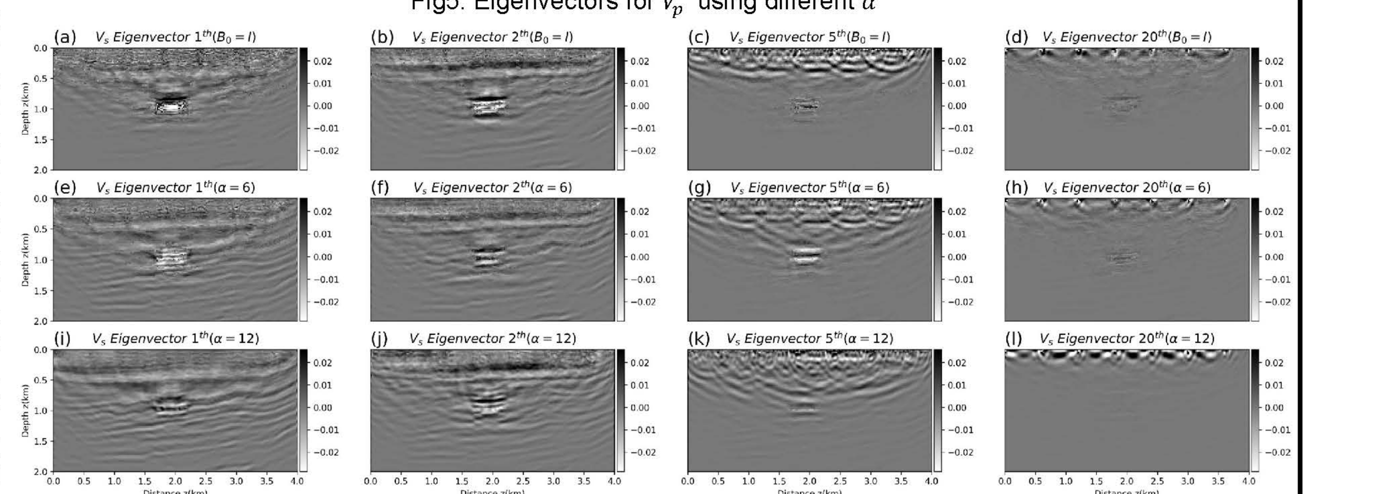


Fig6. Eigenvectors for V_s using different α

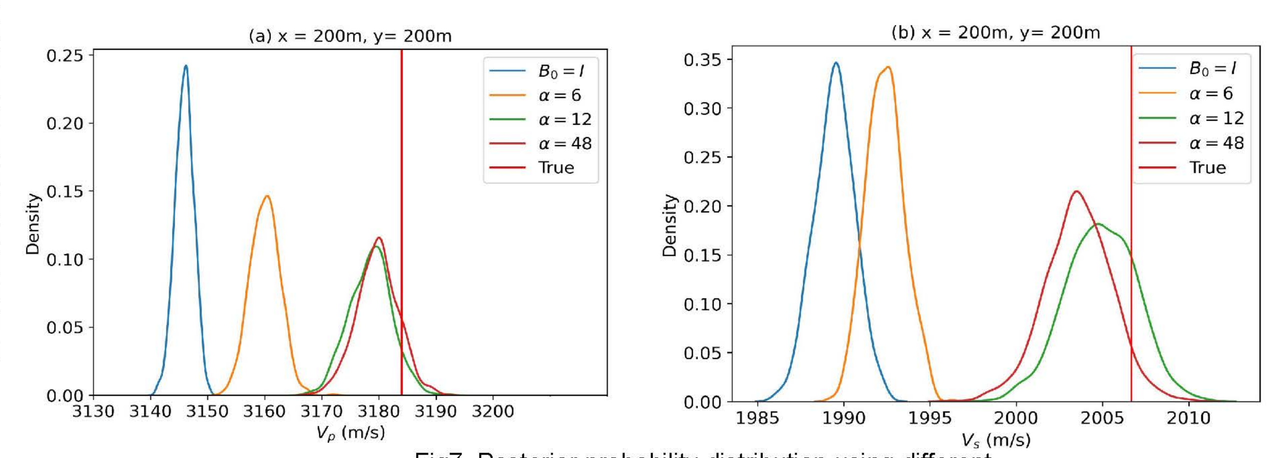


Fig7. Posterior probability distribution using different α

● Uncertainty analysis given by BNN

- The grid length of the model is 20 × 20.
- The grid size of the model is 100 × 200.
- Time interval dt = 0.002s.
- Maximum receiving time 2.6s.
- Ricker's wavelet 10 Hz main frequency.
- Maximum iteration time 1000.
- Adam's algorithm optimization.

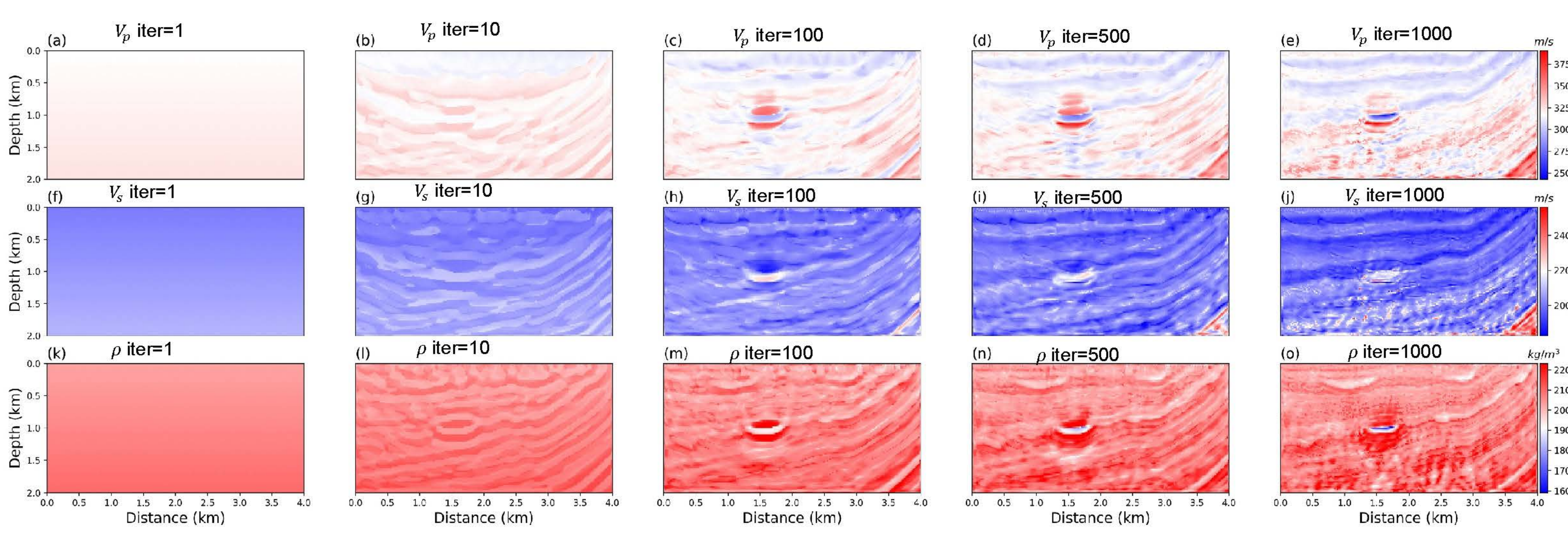


Fig8. BNN inversion results at different iteration

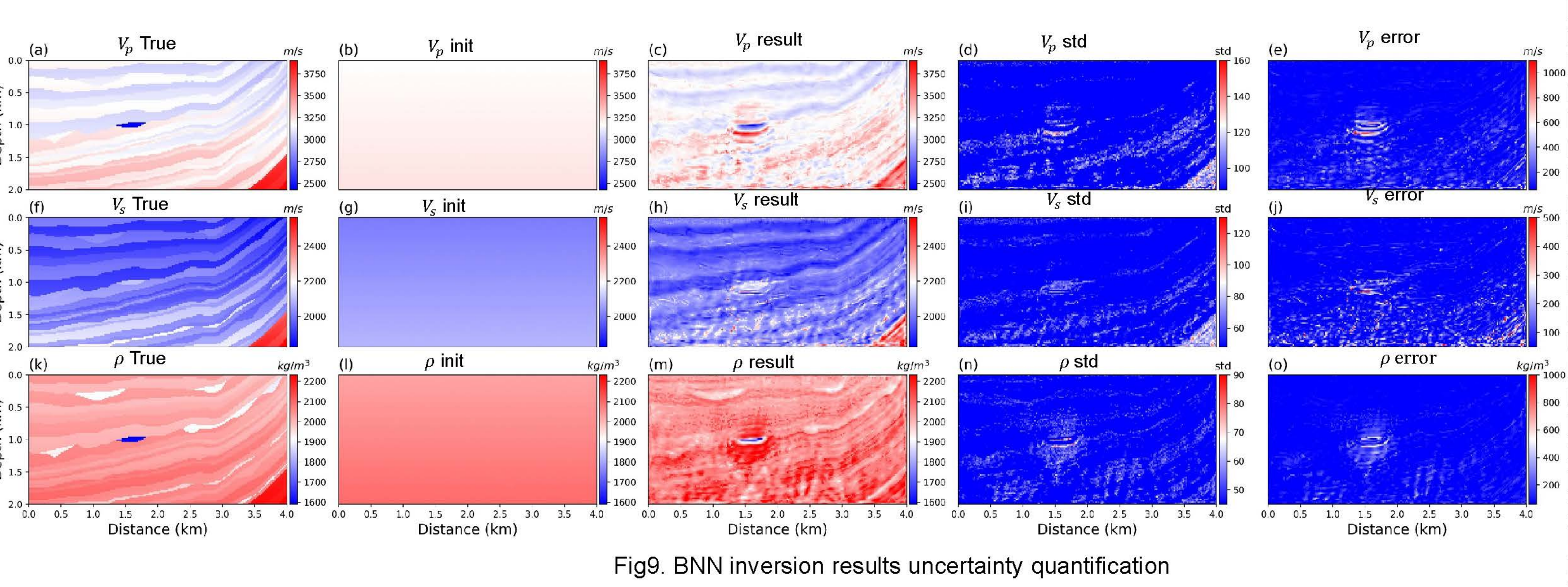


Fig9. BNN inversion results uncertainty quantification

● Uncertainty analysis Comparison between Hessian-based method and BNN-based method

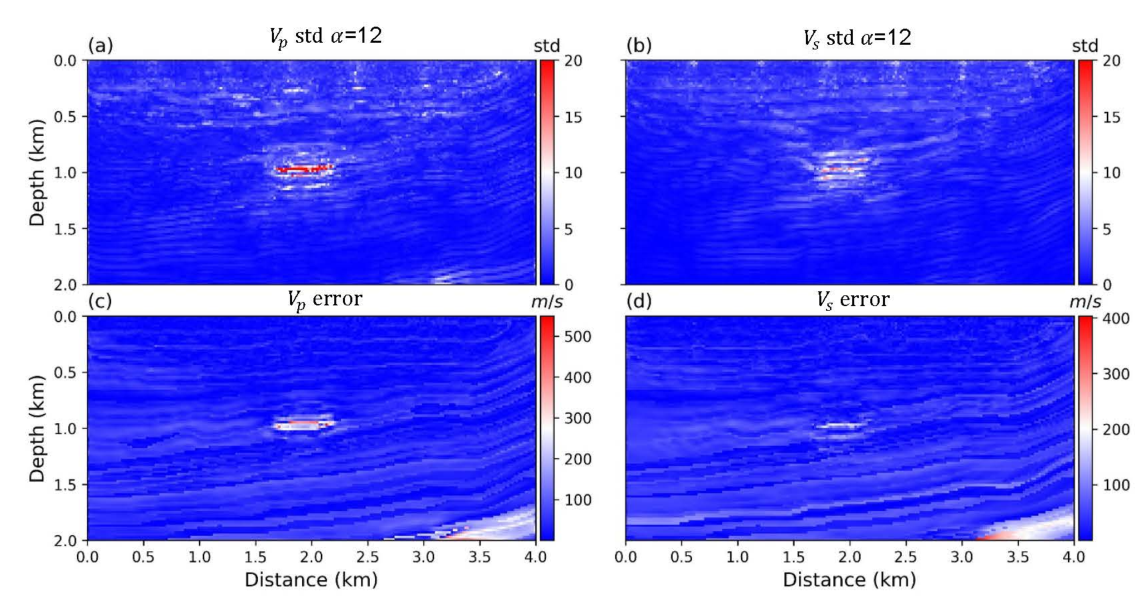


Fig10. Uncertainty quantification given by Hessian matrix

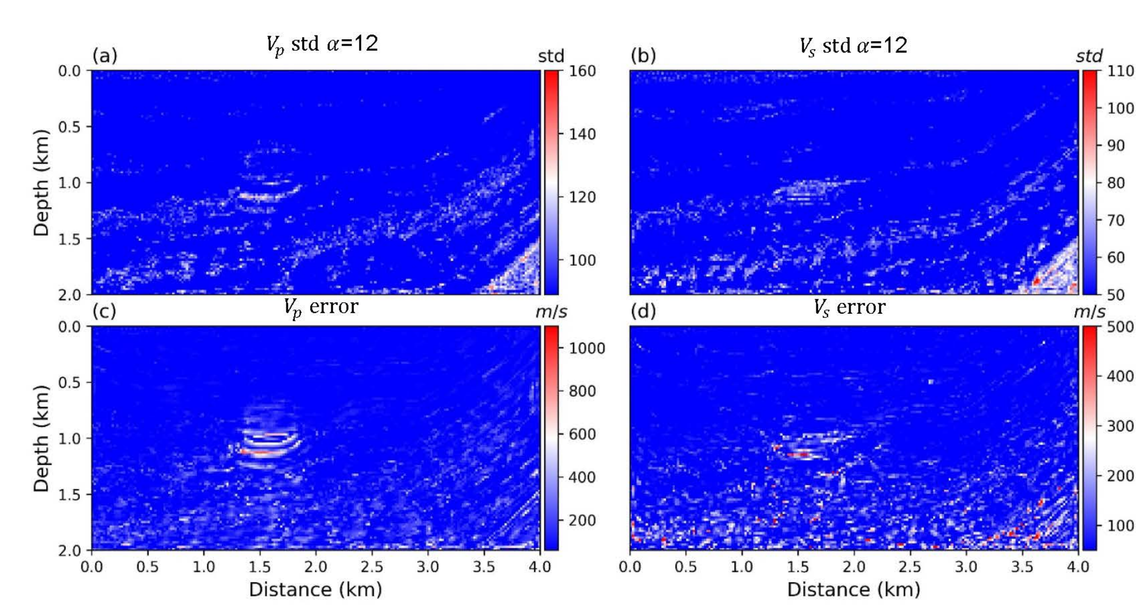


Fig11. Uncertainty quantification given by BNN