Projective Geometric Algebra enabling FWI with sparse acquisitions & targeted updating

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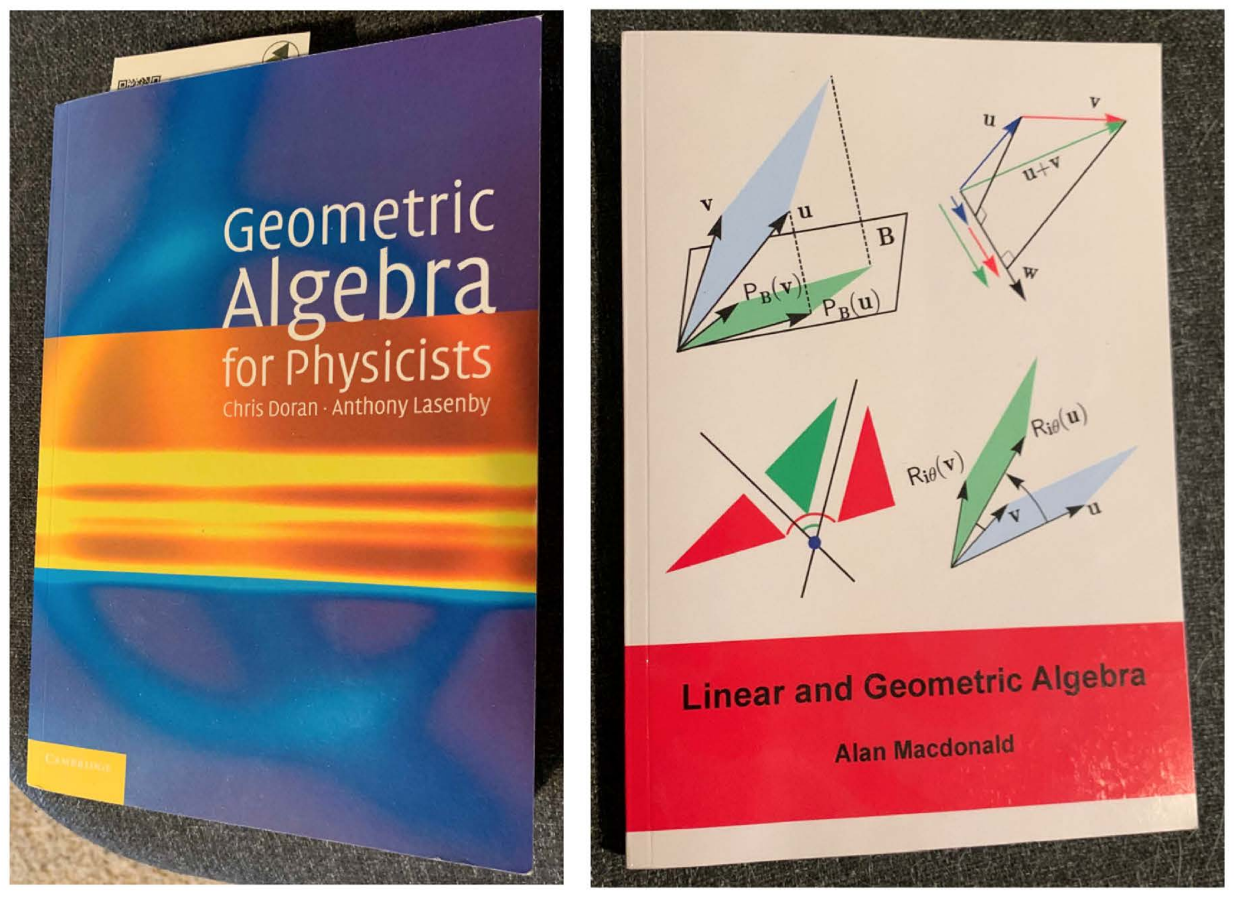
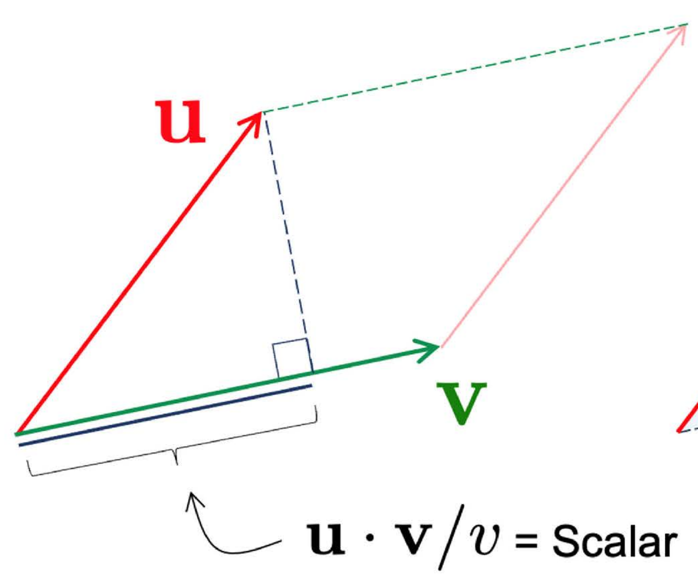
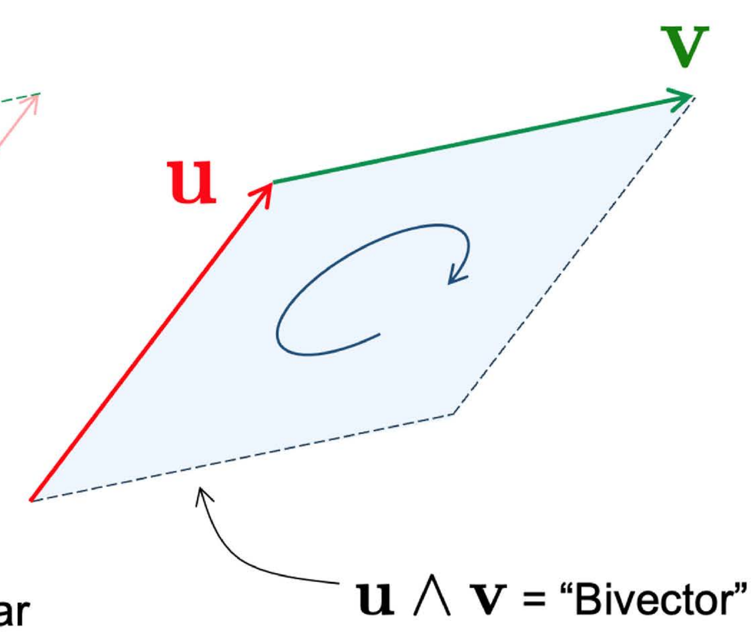
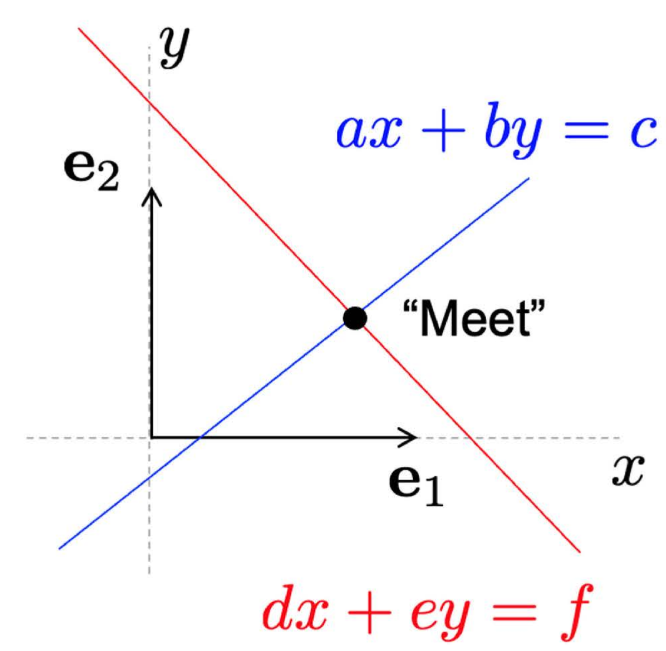


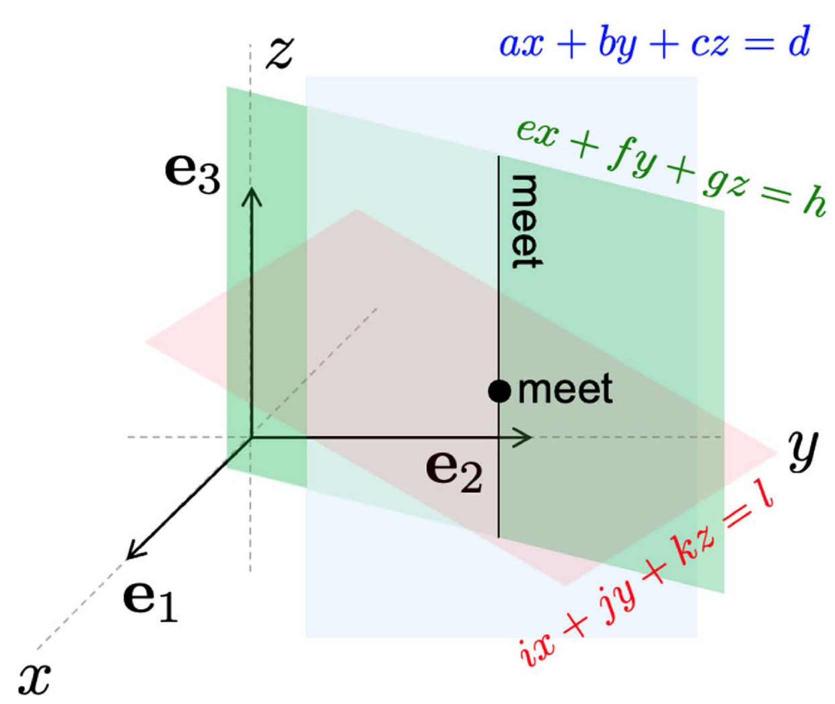
Projective Geometric Algebra (PGA) is a special case of Geometric Algebra (GA), which is currently attracting interest across the breadth of theoretical science, for the simple and elegant way theories can be expressed within it. PGA is concerned with the linear space of lines, planes, and hyperplanes, each of which object is accorded a vector or multivector (e.g., the bivectors pictured below). Common algebraic operations, such as products between vectors, then correspond to geometric relationships, such as intersections between planes, etc. To the extent that the solutions of linear systems (**Ax** = **b**) can be understood in terms of the intersection points of hyperplanes, we can examing PGA for its natural approach to generating such solutions, and whether these offer anything new, especially in application to sparse-acquisition, target oriented waveform inversion. The answer seems to be yes!

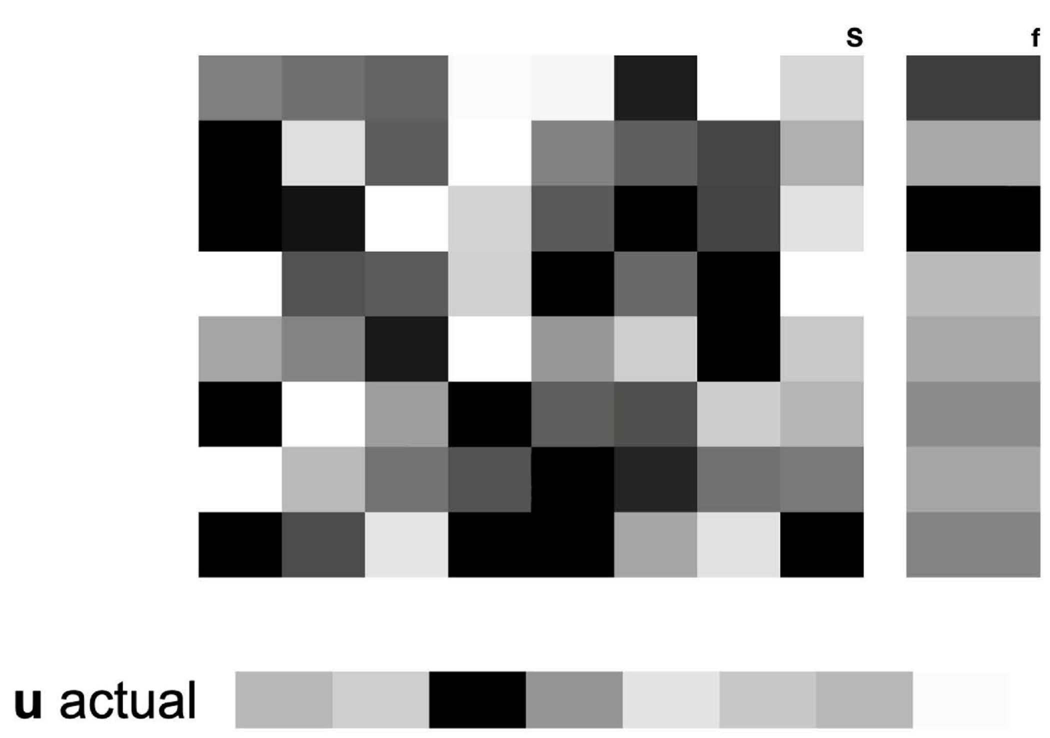
GA and PGA are based on an extension of the types of products and objects we are familiar with from vector algebra. In particular, the wedge product and the geometric product are seen to give rise to, and relate, a range of multivectors. If we assign the rows of A and b to vectors in a PGA, their wedge products successively find their intersections, solving the system **Ax** = **b**.

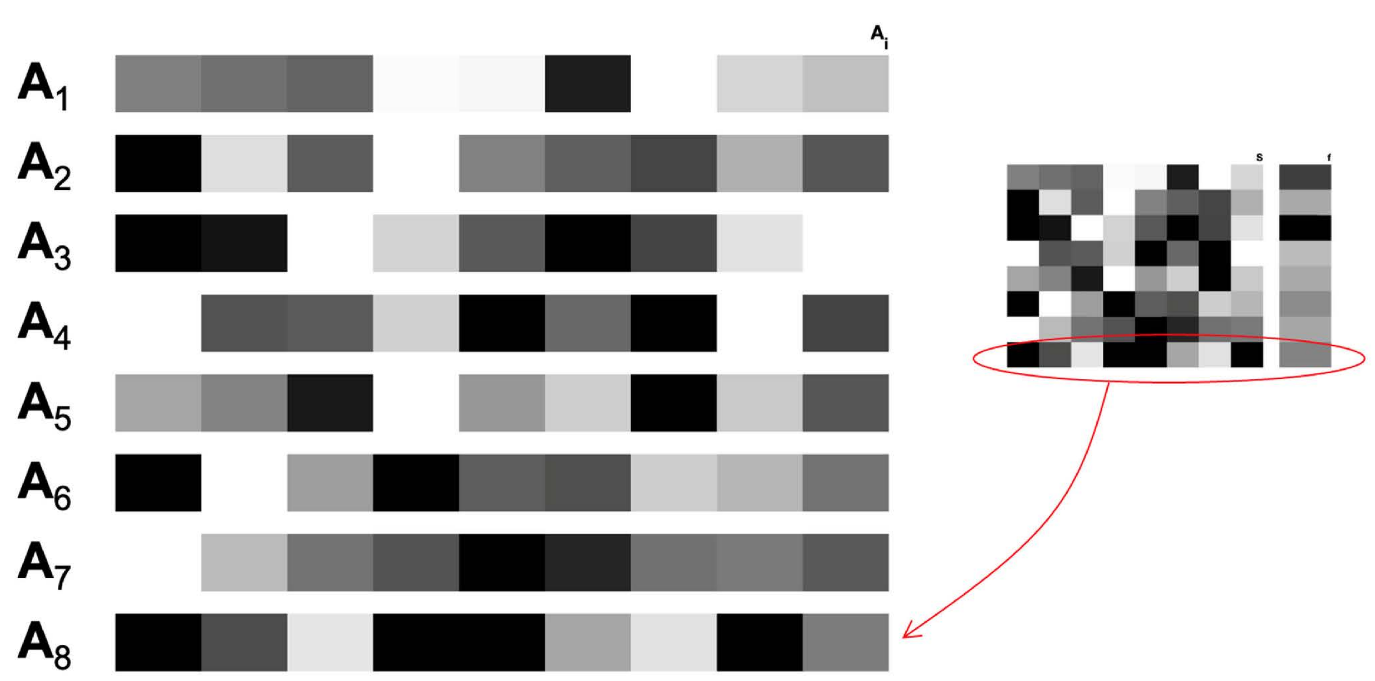


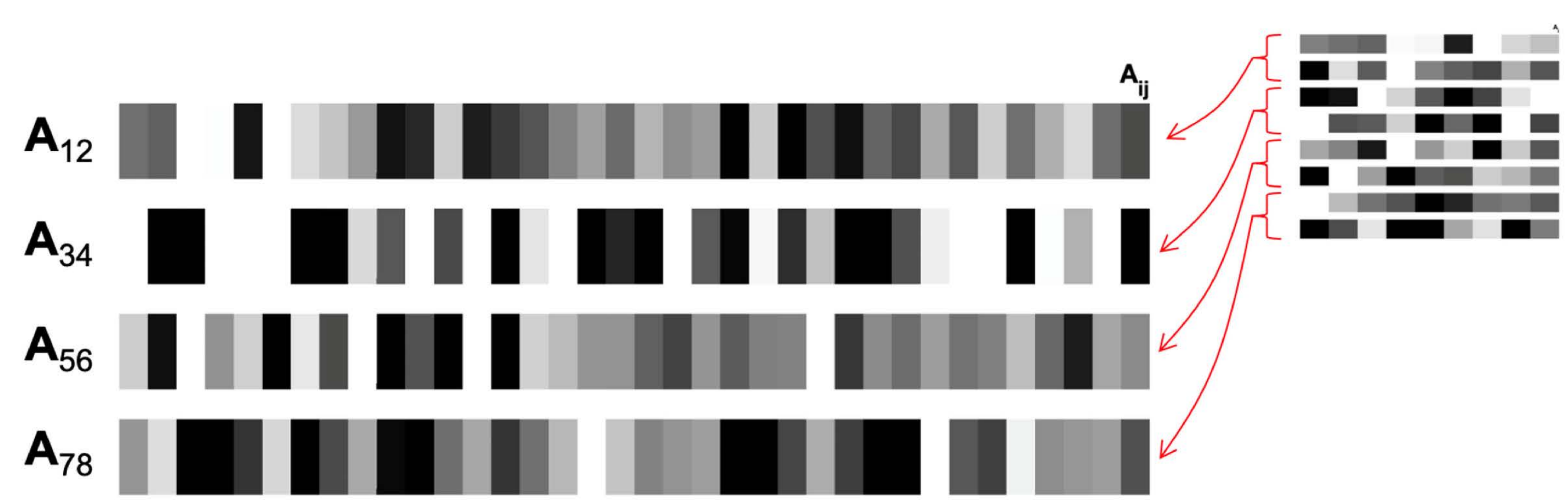


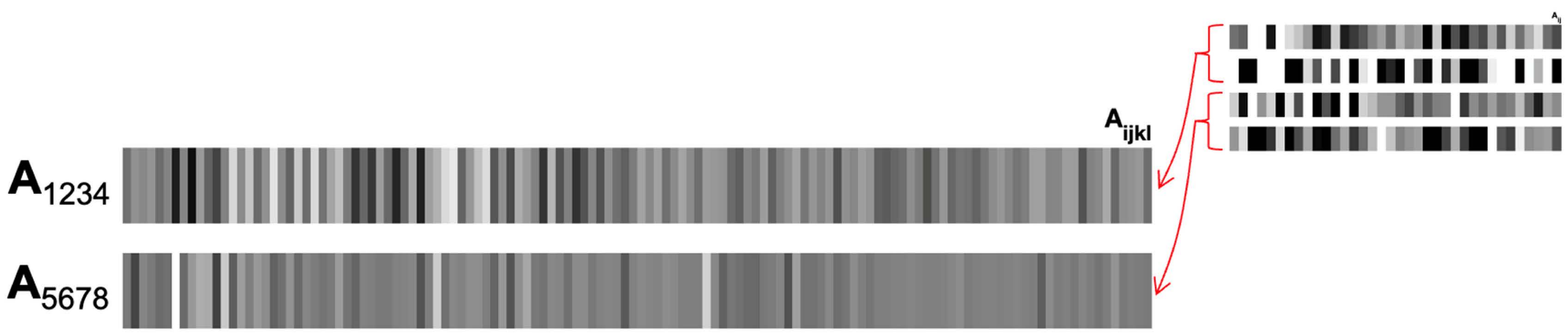


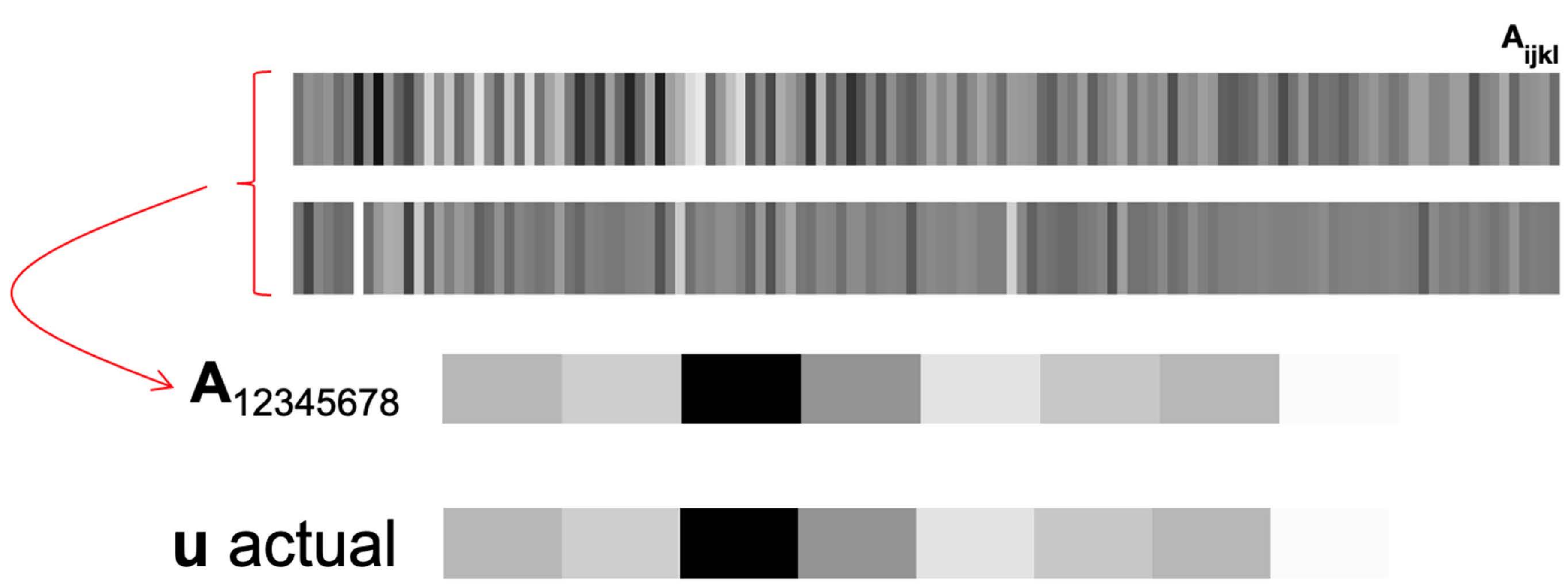












Here a random 8 x 8 matrix is inverted through successive wedge products, and we confirm that the correct solution is determined.

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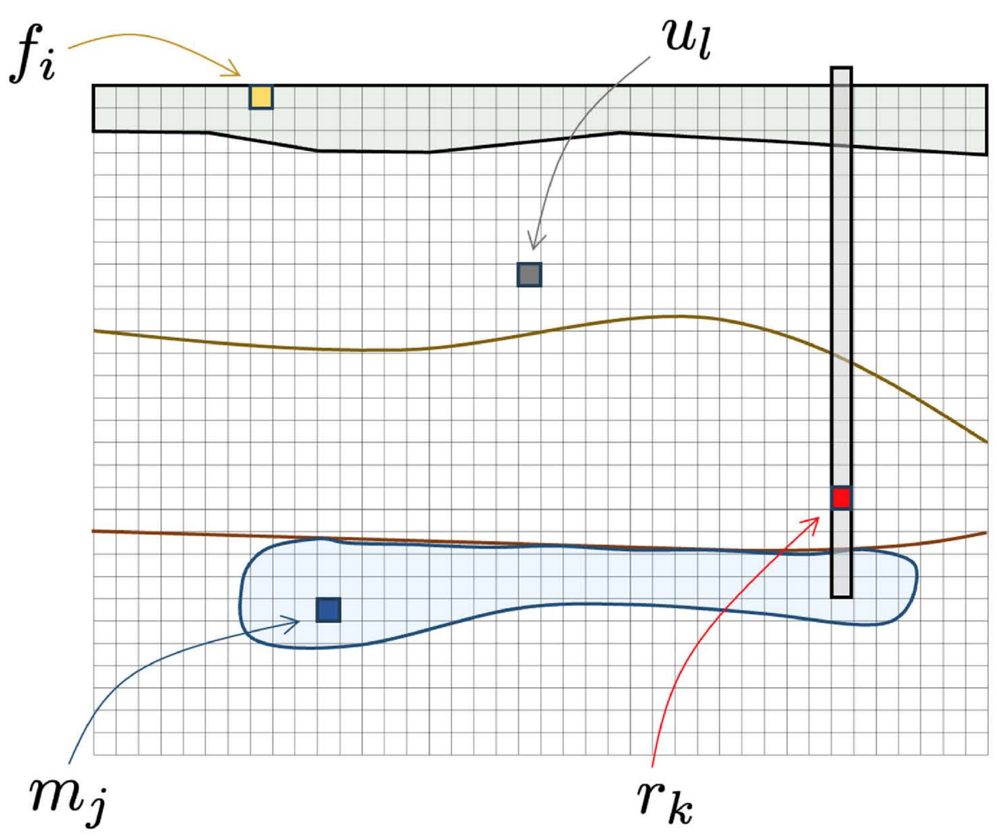
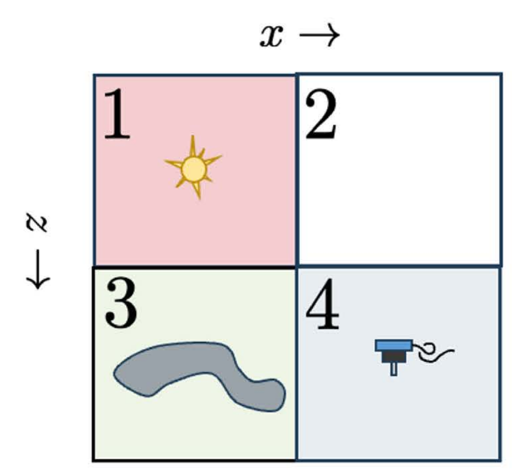


$$\mathbf{S}^T\boldsymbol{\lambda} = \mathbf{r} \qquad \mathbf{S}\mathbf{u} = \mathbf{f}$$

$$\boldsymbol{\lambda}^T \frac{\partial \mathbf{S}}{\partial \mathbf{m}} \mathbf{u}$$

$$\frac{d\phi}{dm_3} \propto \left\{ \begin{bmatrix} s_{11} & x & x & 0 \\ x & s_{22} & x & x \\ x & x & s_{33} & x \\ 0 & x & x & s_{44} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ r_4 \end{bmatrix} \right\}^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \left\{ \begin{bmatrix} s_{11} & x & x & 0 \\ x & s_{22} & x & x \\ x & x & s_{33} & x \\ 0 & x & x & s_{44} \end{bmatrix}^{-1} \begin{bmatrix} f_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

impedances — residuals — sensitivities — impedances — sources

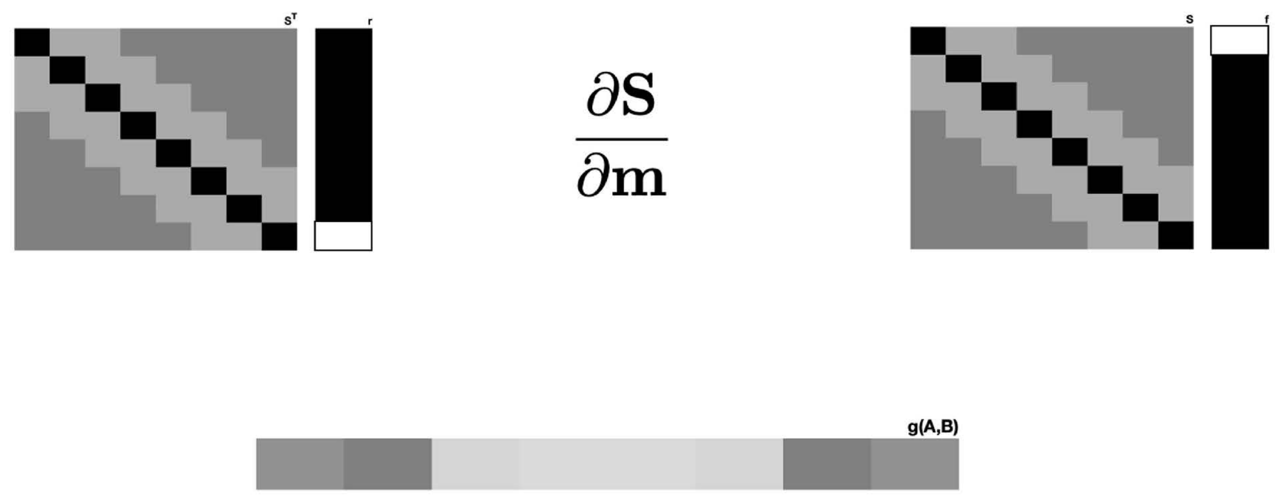


- If certain model components are targeted…
- If source positions are sparsely occupied…
- If receiver positions are sparsely occupied…
- Then certain rows in the calculation are focused on
- Info from most rows incorporated again & again

The gradient in FWI computed via adjoint-state techniques is the product of solutions of several related systems, combining model vector, source vector, data/residuals vector, and wavefield vector. When acquisition is sparse, and we are targeting only some parts of the model, this includes some very repetitive calculations and matrix inversions. We might ask if the intersection / row picture of PGA allows us to avoid this duplication of computations, and speed up FWI.

STEP 0: 2 linear systems

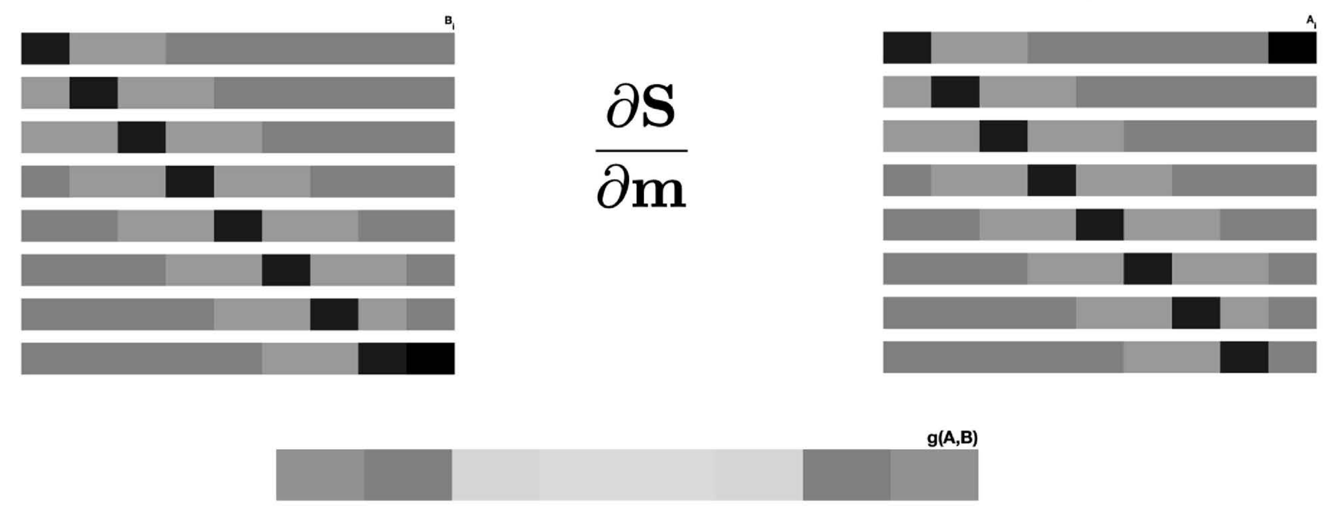
$\mathbf{B}\boldsymbol{\lambda} = \mathbf{r}$ $\qquad \frac{\partial \mathbf{S}}{\partial \mathbf{m}} \qquad$ $\mathbf{A}\mathbf{u} = \mathbf{f}$



ground truth

STEP 1: organize into PGA vectors

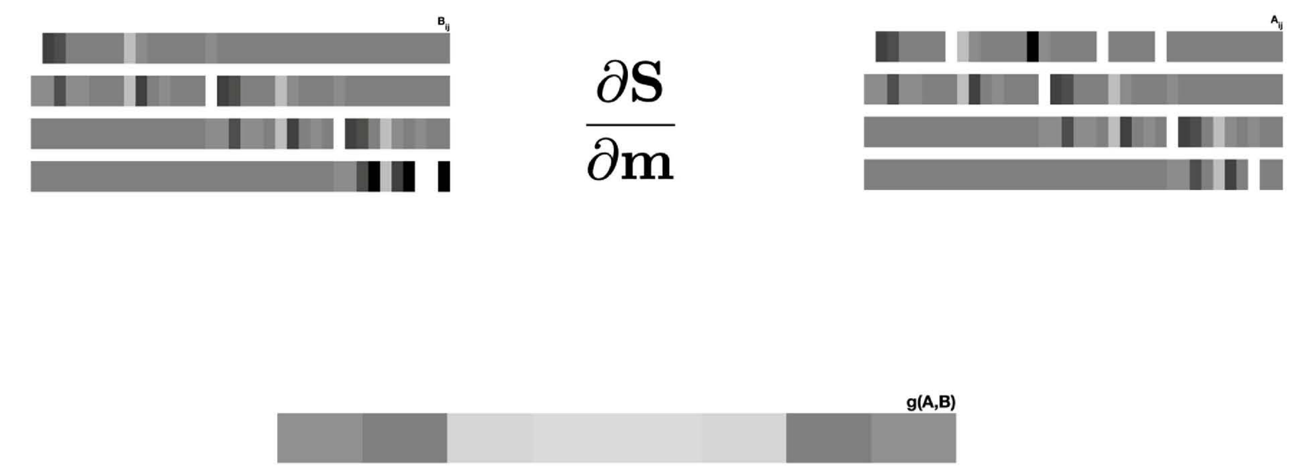
$\mathbf{B}\boldsymbol{\lambda} = \mathbf{r}$ $\qquad \frac{\partial \mathbf{S}}{\partial \mathbf{m}} \qquad$ $\mathbf{A}\mathbf{u} = \mathbf{f}$



ground truth

STEP 2: compute "meets"

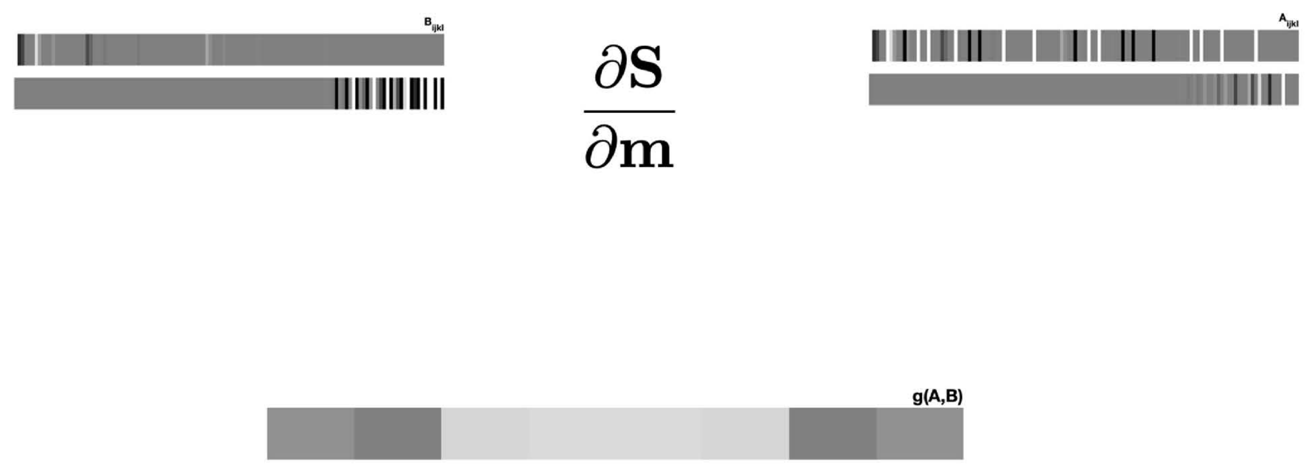
$\mathbf{B}\boldsymbol{\lambda} = \mathbf{r}$ $\qquad \frac{\partial \mathbf{S}}{\partial \mathbf{m}} \qquad$ $\mathbf{A}\mathbf{u} = \mathbf{f}$



ground truth

STEP 3: compute "meets"

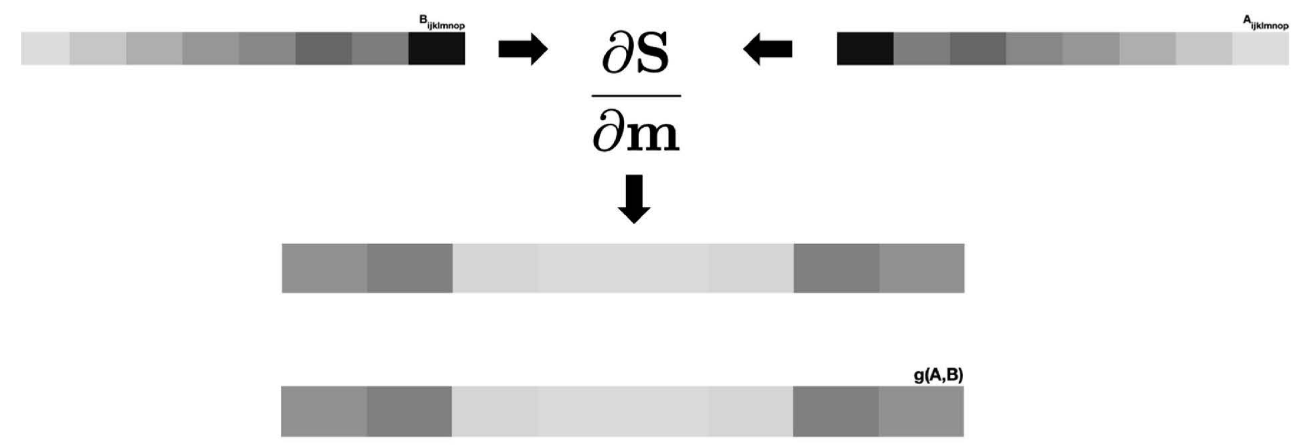
$\mathbf{B}\boldsymbol{\lambda} = \mathbf{r}$ $\qquad \frac{\partial \mathbf{S}}{\partial \mathbf{m}} \qquad$ $\mathbf{A}\mathbf{u} = \mathbf{f}$



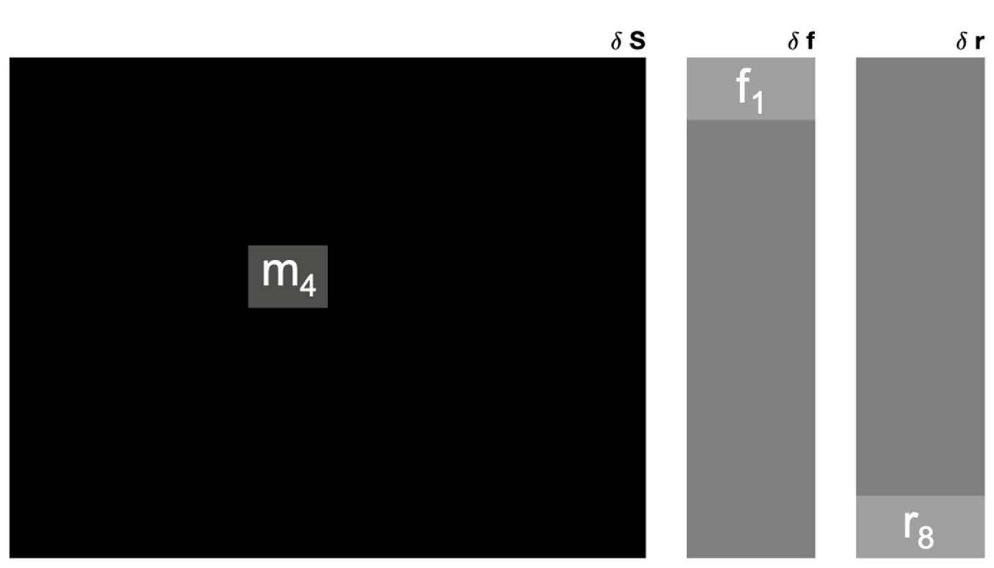
ground truth

STEP 4: final "meets" match ground truth

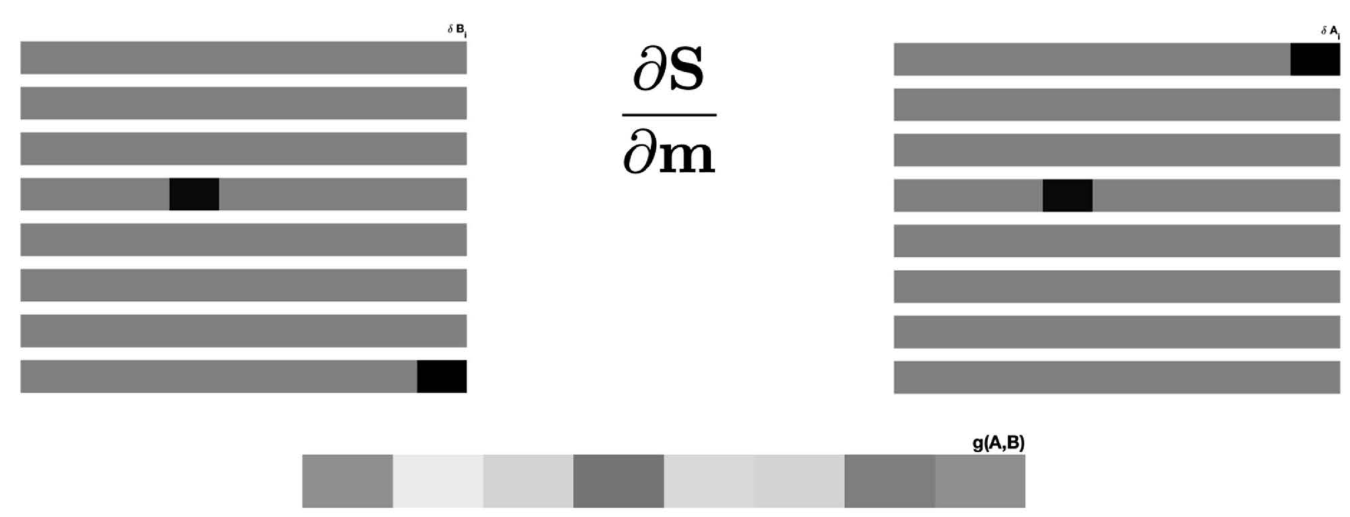
$\mathbf{B}\boldsymbol{\lambda} = \mathbf{r}$ $\rightarrow \frac{\partial \mathbf{S}}{\partial \mathbf{m}} \leftarrow$ $\mathbf{A}\mathbf{u} = \mathbf{f}$



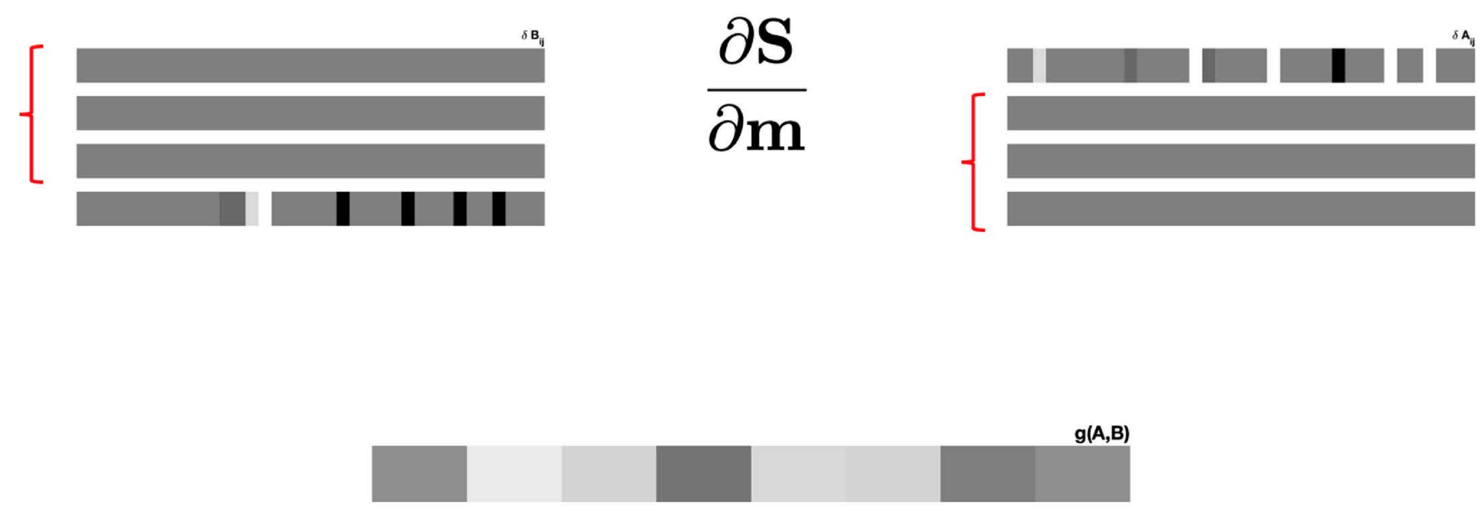
ground truth



$\mathbf{B}\boldsymbol{\lambda} = \mathbf{r}$ $\qquad \frac{\partial \mathbf{S}}{\partial \mathbf{m}} \qquad$ $\mathbf{A}\mathbf{u} = \mathbf{f}$



$\mathbf{B}\boldsymbol{\lambda} = \mathbf{r}$ $\qquad \frac{\partial \mathbf{S}}{\partial \mathbf{m}} \qquad$ $\mathbf{A}\mathbf{u} = \mathbf{f}$



With the correctness of the gradient calculation confirmed, note that IF f and r are sparse (sparse acquisition), and if only a small number of rows of the A and B matrices are different across iterations, only a very small fraction of the multivectors / hyperplanes in the meets are changing.