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# 3D-FDM for elastic wave modeling in the presence of irregular topography by using unstructured index array representation on a GPU

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## Introduction.

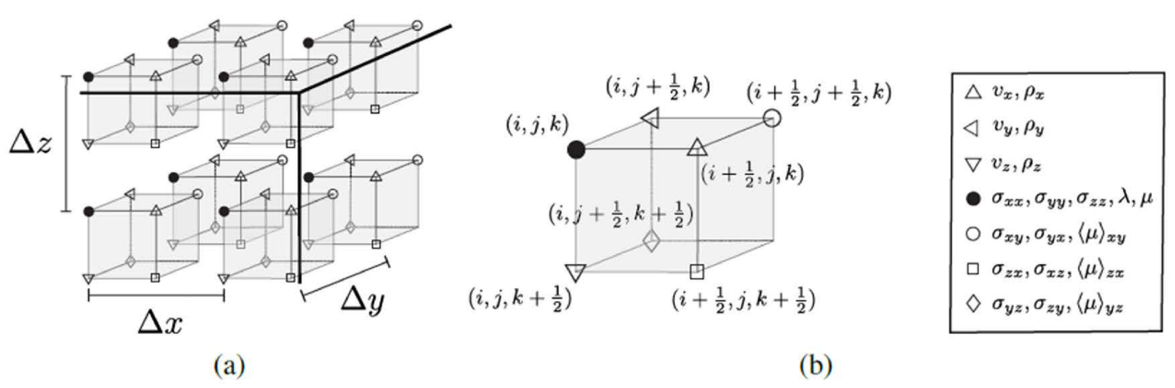
- This study introduces the PMFD3D-GPU, a novel 3D solver optimized for simulating seismic waves in terrains with complex topography. This solver leverages GPU acceleration to enhance performance.
- A unique approach of using an unstructured index array representation was adopted in the PMFD3D-GPU solver to ensure the proper implementation of the free-surface condition when dealing with complex topographical changes, while maintaining a feasible computational performance.
- This tool addressed a gap in the availability of open-source tools for 3D elastic wave propagation in terrains with complex topography.

## Theory

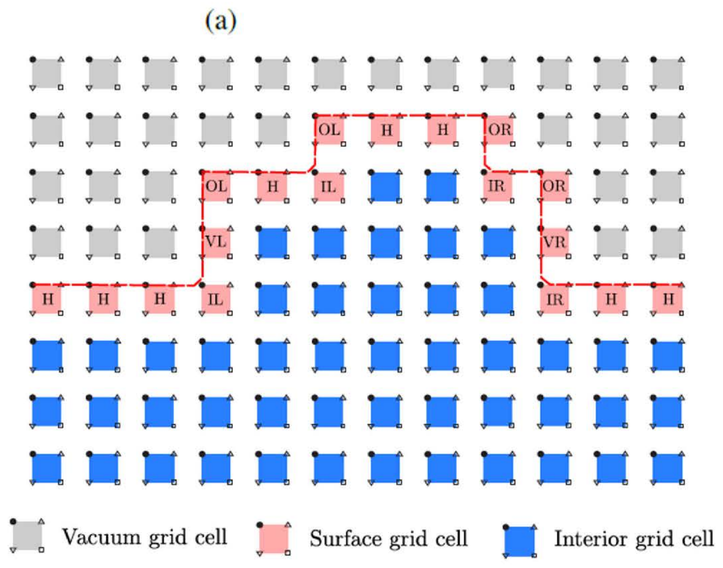
$$D_t^- v_x = b_x\left(D_x^+\sigma_{xx} + D_y^-\sigma_{xy} + D_z^-\sigma_{zx}\right),$$
$$D_t^- v_y = b_y\left(D_x^-\sigma_{xy} + D_y^+\sigma_{yy} + D_z^-\sigma_{yz}\right),$$
$$D_t^- v_z = b_z\left(D_x^-\sigma_{zx} + D_y^-\sigma_{yz} + D_z^+\sigma_{zz}\right),$$
$$D_t^+\sigma_{xx} = (\lambda+2\mu)D_x^- v_x + \lambda D_y^- v_y + \lambda D_z^- v_z,$$
$$D_t^+\sigma_{yy} = \lambda D_x^- v_x + (\lambda+2\mu)D_y^- v_y + \lambda D_z^- v_z,$$
$$D_t^+\sigma_{zz} = \lambda D_x^- v_x + \lambda D_y^- v_y + (\lambda+2\mu)D_z^- v_z,$$
$$D_t^+\sigma_{xy} = \langle\mu\rangle_{xy}\left(D_x^+ v_y + D_y^+ v_x\right),$$
$$D_t^+\sigma_{yz} = \langle\mu\rangle_{xy}\left(D_y^+ v_z + D_z^+ v_y\right),$$
$$D_t^+\sigma_{zx} = \langle\mu\rangle_{xy}\left(D_z^+ v_x + D_x^+ v_z\right),$$

The elastic wave equation can be expressed as a system of first-order partial differential equations by using the velocity-stress formulation (Virieux, 1986)...

$D_i^+$ and $D_i^-$ are the forward and backward differencing operators with respect to the variable $i = \{x, y, z, t\}$. We use 4[th] order of approximation in space and 2[th] order of approximation in time.




The 3D space domain is discretized into a numerical grid with spatial steps $\Delta x, \Delta y, \Delta z$ . using a staggered grid scheme

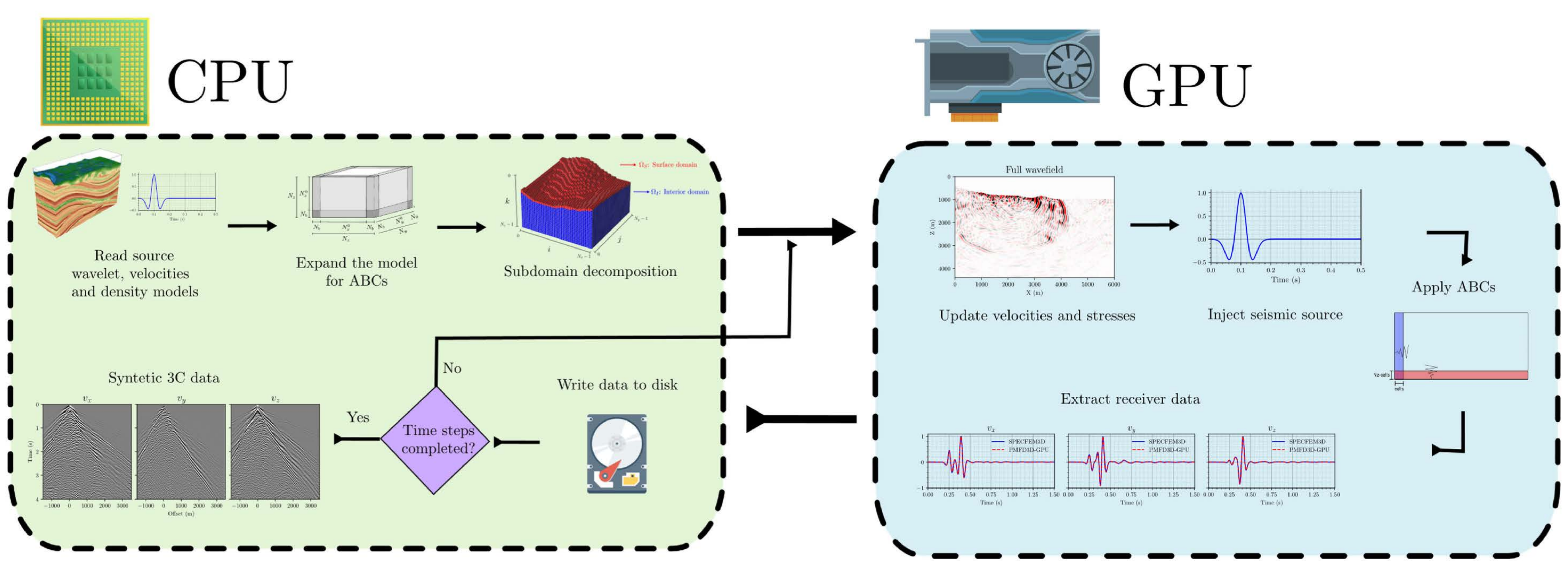



The Parameter Modified method categorizes the grid cells on the surface based on their relative position to the air (Cao and Chen, 2018). There are five basic classes:

- H: grid cell with air above,
- VR: grid cell with air to the right,
- VL: grid cell with air to the left
- VF: grid cell with air to the front
- VB: grid cell with air to the back

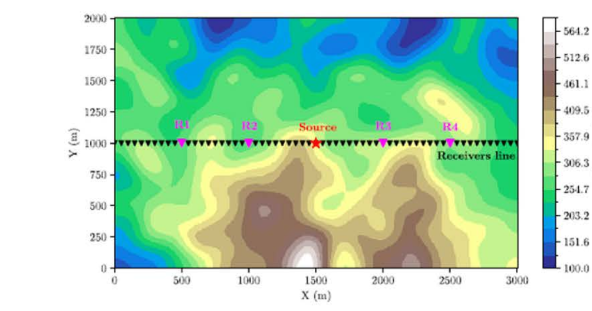
## Implementation Workflow

Several steps of the workflow are executed on the CPU, including model expansion, domain decomposition, and the calculation of medium parameters in the surface subdomain. Subsequently, the GPU is set up, and model data is transferred from the CPU to the GPU. Once this transfer is complete, the GPU conducts the FDM computations for both the surface and interior subdomains..



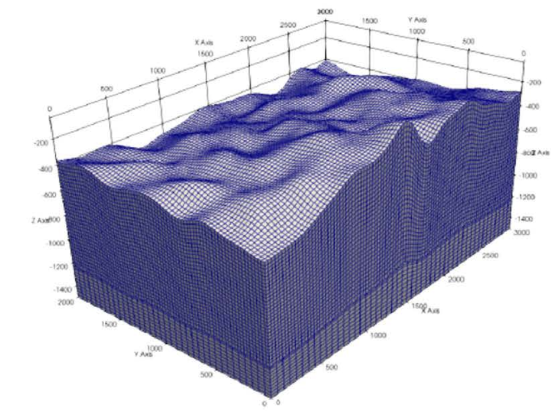

## Accuracy validation with SPECFEM3D

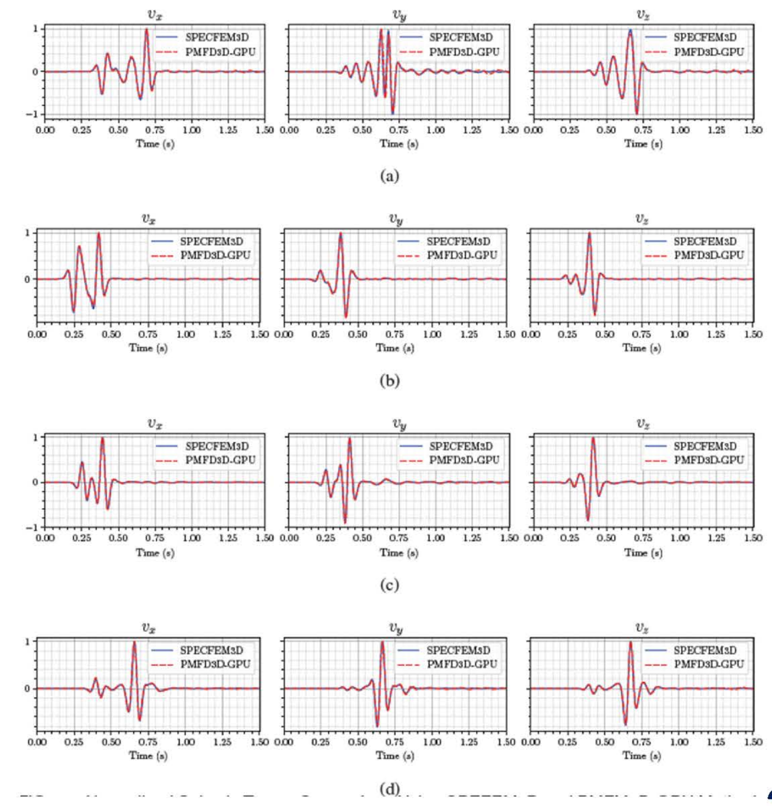
we validate the accuracy of PMFD3D-GPU by conducting elastic wave modeling in a homogeneous model with rough topography, using both PMFD3D-GPU and SPECFEM3D for comparison.



$$T(x,y) = 350 + 250\sin\left(\frac{x}{1000}\right)\cos\left(\frac{y}{1000}\right)$$

| $V_P$ (m/s) | $V_S$ (m/s) | $\rho$ (Mg/m$^3$) | $V_P/V_S$ | Dimension (m) | Grid cell size (m) |
|---|---|---|---|---|---|
| 3550 | 2050 | 2.0 | 3.5 | $3000 \times 2000 \times 2000$ | $5 \times 5 \times 5$ |





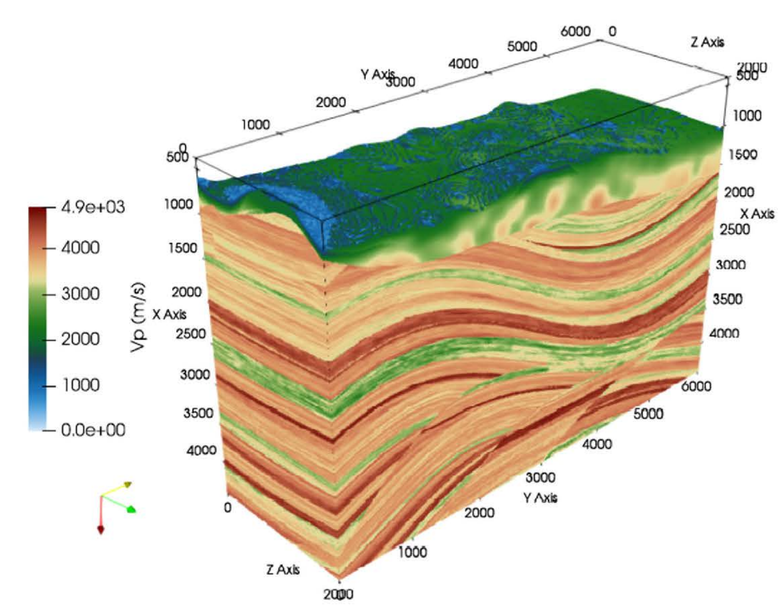

we extracted the traces from four receivers along the receiver line. These receivers, labelled R1, R2, R3, and R4, are positioned at distances of 500 m, 1000 m, 2000 m, and 2500 m along the $x$-axis. Visually, the traces appear identical across all components and receivers.

$$L_2 = \frac{\|\mathbf{u} - \mathbf{u}^0\|^2}{\|\mathbf{u}^0\|^2},$$

| Receiver | $v_x$ (%) | $v_y$ (%) | $v_z$ (%) |
|---|---|---|---|
| R1 | 1.15 | 7.77 | 0.80 |
| R2 | 0.88 | 0.97 | 0.51 |
| R3 | 0.27 | 0.42 | 0.11 |
| R4 | 1.24 | 2.25 | 0.37 |

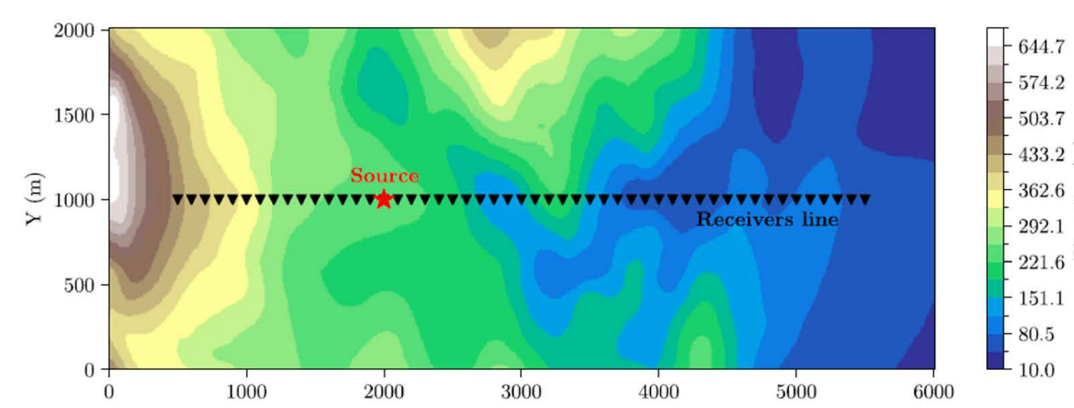
we computed the $L_2$ misfit error between the shot gathers from PMFD3D-GPU and SPECFEM3D. Most of the errors were under 1%.

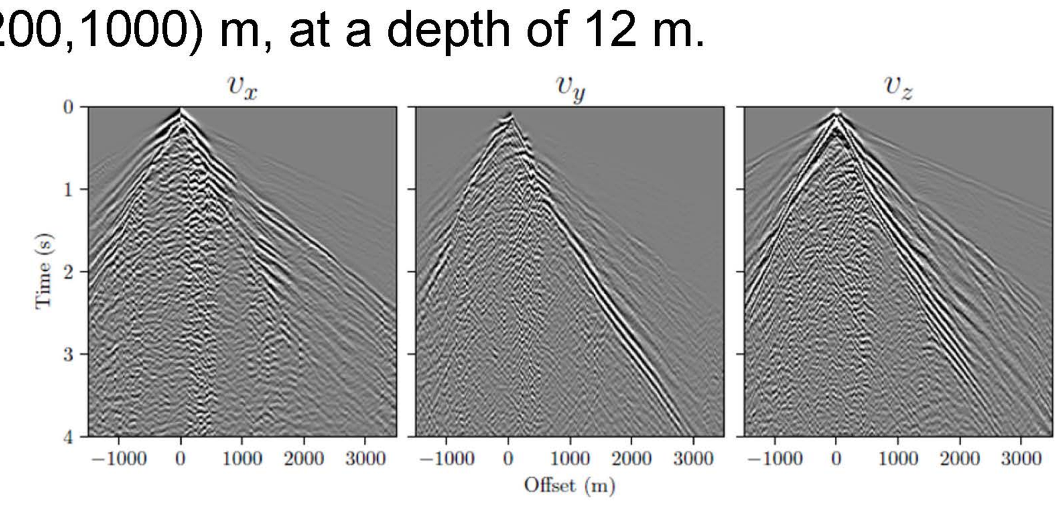
## 3D SEAM Foothills model example



We aim to generate realistic seismic synthetic data by performing elastic wave modeling in a portion of the SEAM Foothils Phase II model using PMFD3D-GPU. The chosen portion measures 6000 m x 2000 m x 4400 m. The model comes discretized uniformly across all axes with $\Delta h$ = 10 m.

Receivers were placed on the surface along a line stretching from 500 to 5500 m at Y=1000 m, with an inter-receiver distance of 10 m and the explosive source at coordinates (200,1000) m, at a depth of 12 m.




## Computational performance

| Model | Dimension ($N_x \times N_y \times N_z$) | Time samples | CPU (min) | GPU (min) |
|---|---|---|---|---|
| **CPU:** Intel Core i7-12700 processor, 12 Core, 2.1GHz to 4.9GHz, 16GB of memory using C and OpenMP<br>**GPU:** NVIDIA GeForce RTX 3060 12 GB of memory using CUDA-C | | | | |
| Homogeneous topography | $(681 \times 481 \times 340)$<br>$410,341$ surface points<br>$86,335,041$ interior points | 2256 | 115.65 | 5.47 |
| SEAM portion | $(641 \times 241 \times 461)$<br>$177,235$ surface points<br>$56,195,513$ interior points | 5003 | 166.61 | 7.53 |

When comparing the time spent by PMFD3D-CPU on a uniform grid with that of PMFD3D-GPU on the SEAM model, the GPU implementation is found to be approximately 20 times faster.

## Conclusions

We have presented PMFD3D-GPU, a finite-difference solver designed for 3D elastic wave modeling in heterogeneous models with irregular topography. The solver leverages a GPU implementation subdomain decomposition strategy to satisfy the free-surface condition by managing of indices of grid cells within irregular subdomains shape, improving the computational efficiency. This solver addressed a gap in the availability of open-source tools for 3D elastic wave propagation in terrains with complex topography.

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