# Simultaneous prediction of velocity and angle-dependent reflectivity in time domain FWI

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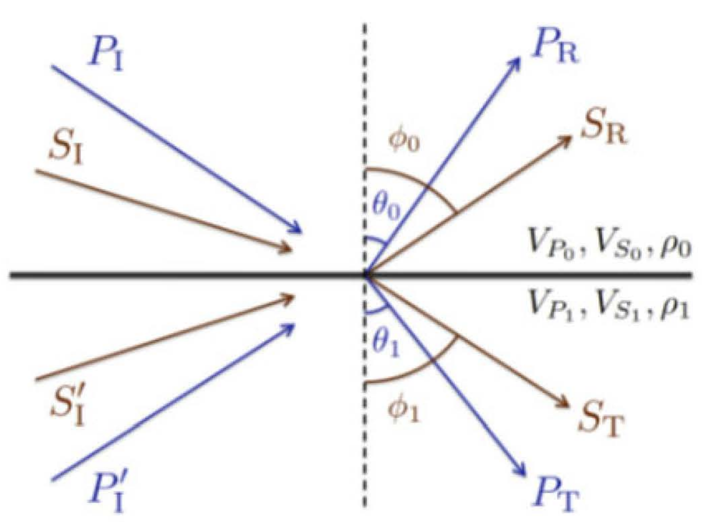
- Abstract

Common-image gathers (CIGs) are essential for migration-based velocity analysis and amplitude-versus-angle (AVA) analysis, which could be utilized to predict lithology and fluid properties. Velocity is necessary for migration-based velocity analysis, and angle-dependent reflectivity is necessary for AVA analysis. To determine velocity and reflectivity for subsurface models, seismic inversion has been a conventional approach, followed by attribute calculation to aid interpretation. We present an iterative non-linear inversion method to simultaneously predict both velocity and amplitude-preserved angle-domain-common-image-gathers (ADCIGs). The key aspect of our technique is the extraction of angle information from the solution of a direction-vector-based wave equation in acoustic with density media.Because the extraction is solely dependent on the direction of wave propagation and is not dependent on acquisition coordinates, it can be applied to blended acquisition, which is another name for simultaneous seismic source acquisition. Our iterative inversion method is based on the time domain FWI using the nonlinear conjugate-gradient method. Throughout each iteration, the velocity model is sequentially updated at each reflection angle.


- AVA response in elastic and acoustic media

The AVA response in elastic media can be described as Knott-Zoeppritz equations

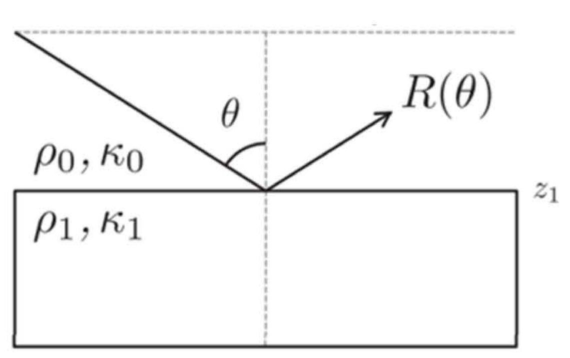
$$\begin{cases}\sin\theta_0\,(P_I+P_R)+\cos\phi_0\,(S_I+S_R)=\sin\phi_1\,(P_T+P_I')+\cos\phi_1\,(S_T+S_I'),\\ \cos\theta_0\,(P_I-P_R)-\sin\phi_0\,(S_I-S_R)=\cos\phi_1\,(P_T-P_I')-\sin\phi_1\,(S_T-S_I'),\\ 2\rho_0\left(V_{S_0}^2/V_{P_0}\right)\sin\theta_0\cos\theta_0\,(P_I-P_R)-\rho_0 V_{S_0}\left[1-2\left(V_{S_0}^2/V_{P_0}^2\right)\sin^2\theta_0\right](S_I-S_R)\\ =2\rho_1\left(V_{S_1}^2/V_{P_1}\right)\sin\theta_0\cos\theta_0\,(P_T-P_I')+\rho_1 V_{S_1}\left[1-2\left(V_{S_1}^2/V_{P_0}^2\right)\sin^2\theta_0\right](S_T-S_I'),\\ \rho_0 V_{P_0}\left[1-2\left(V_{S_0}^2/V_{P_0}^2\right)\sin^2\theta_0\right](P_I-P_R)-2\rho_0\left(V_{S_0}^2/V_{P_0}\right)\sin\theta_0\cos\phi_0\,(S_T+S_I')\\ =\rho_1 V_{P_1}\left[1-2\left(V_{S_1}^2/V_{P_0}^2\right)\sin^2\theta_0\right](P_T+P_I')-2\rho_1\left(V_{S_1}^2/V_{P_0}\right)\sin\theta_0\cos\phi_1\,(S_T+S_I').\end{cases} \tag{1}$$



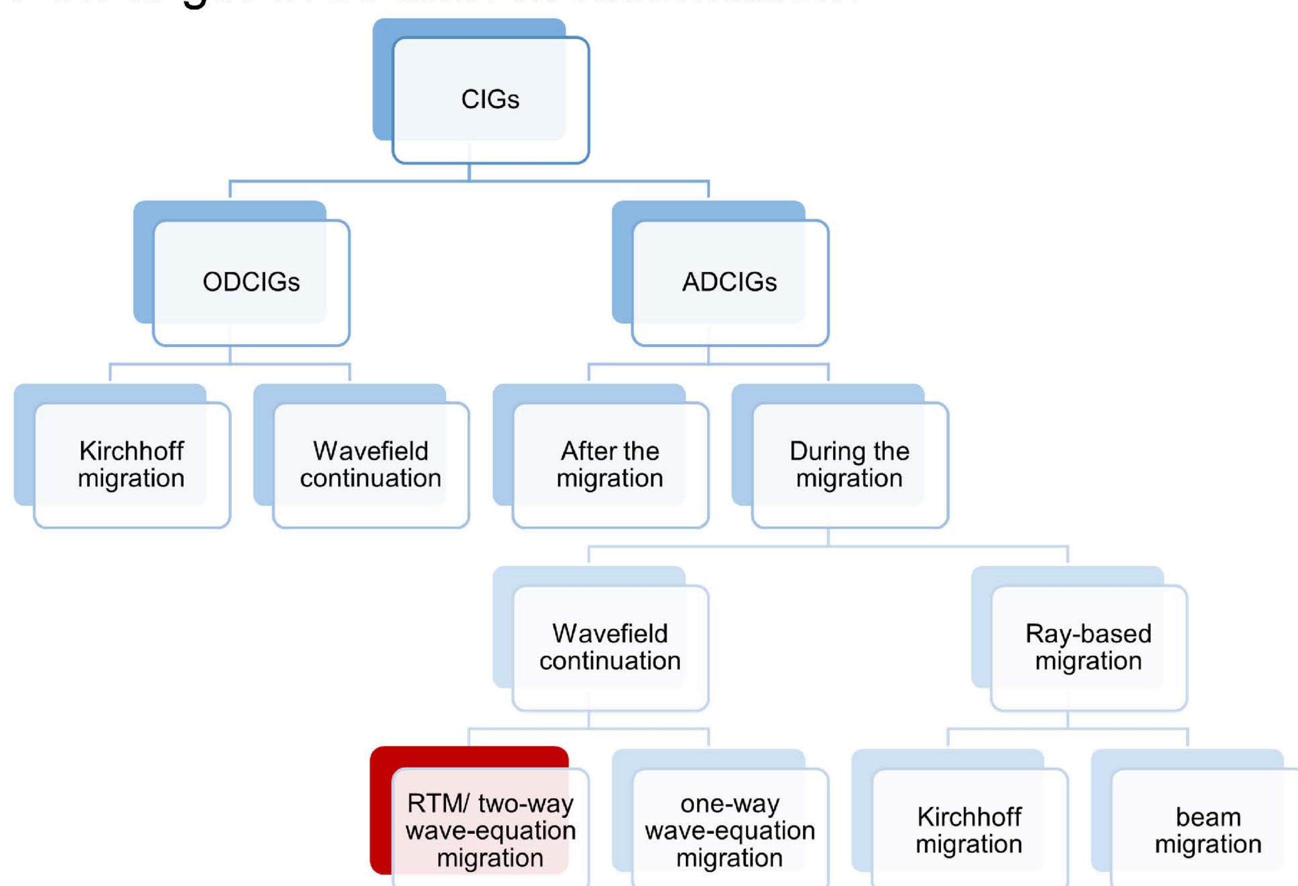

- The AVA response in acoustic media can be described as

$$R(\theta)=\frac{1-\Omega(\theta)}{1+\Omega(\theta)}$$

$$\Omega(\theta)=\left(\frac{\rho_0}{\rho_1}\right)\sqrt{\frac{\kappa_0}{\kappa_1}\frac{\rho_1}{\rho_0}}\left(\frac{1}{\cos\theta}\right)\sqrt{1-\frac{\kappa_1}{\kappa_0}\frac{\rho_0}{\rho_1}\sin^2\theta}.$$




- How to get CIGs and AVA information?




Because the Reverse time migration/RTM / two-way wave-equation migration is based on the direct solutions of the wave equation, it requires fewer assumptions and approximations than others. Itself naturally carries the correct propagation amplitude, amplitude if the shot record is well approximated by solving the wave equation. Because the RTM is based on the direct solutions of the wave equation, energy associated with multiple scatter events, steep drops and a broad range of wavenumbers will be preserved.

- ADCIGs from RTM / two-way wave-equation migration

The ADCIGs IC adds the angle dimension to the output.

Angle calculated from wave propagation direction.

$$R(\vec{x})=\int p_B(\vec{x};t)p_F^{-1}(\vec{x};t)dt \qquad cos2\theta=\frac{\mathbf{S}_{source}\mathbf{S}_{receivers}}{|\mathbf{S}_{source}||\mathbf{S}_{receivers}|}$$

$$R(\vec{x},\theta)=\int p_B(\vec{x},\theta;t)p_F^{-1}(\vec{x},\theta;t)dt \qquad \theta=\frac{1}{2}arccos\frac{\mathbf{S}_{source}\mathbf{S}_{receivers}}{|\mathbf{S}_{source}||\mathbf{S}_{receivers}|}$$

- Migration with velocity and reflectivity simultaneously

| Media type | Acoustic media | Acoustic with reflectivity media | Elastic media |
|---|---|---|---|
| Wavefield | P | P, Vx, Vz | Stress and velocity |
| Model property | Vp | Vp, density | Vp, Vs, Density |
| Polarization direction | ∇P | Direction: Vx, Vz | Vx, Vz |
| Poynting vector | | Amplitude: P | |

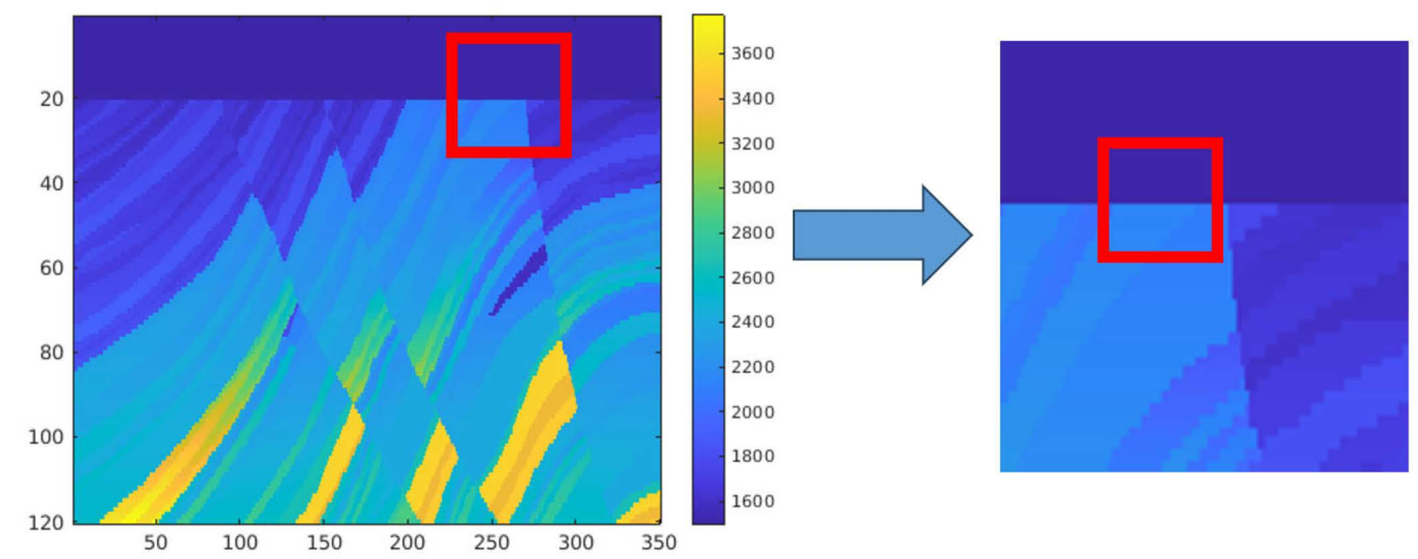
$$\begin{cases}\dfrac{\partial P}{\partial t}=-\rho c^2\nabla\cdot\vec{v}\\ \dfrac{\partial \vec{v}}{\partial t}=-\dfrac{1}{\rho}\dfrac{\partial P}{\partial x}\end{cases} \qquad Z=\rho V, \qquad \boldsymbol{R}=\frac{1}{2}\frac{\nabla Z}{Z}=-\frac{1}{2}Z\nabla\left(\frac{1}{Z}\right)$$

- Pseudo code for angle domain FWI

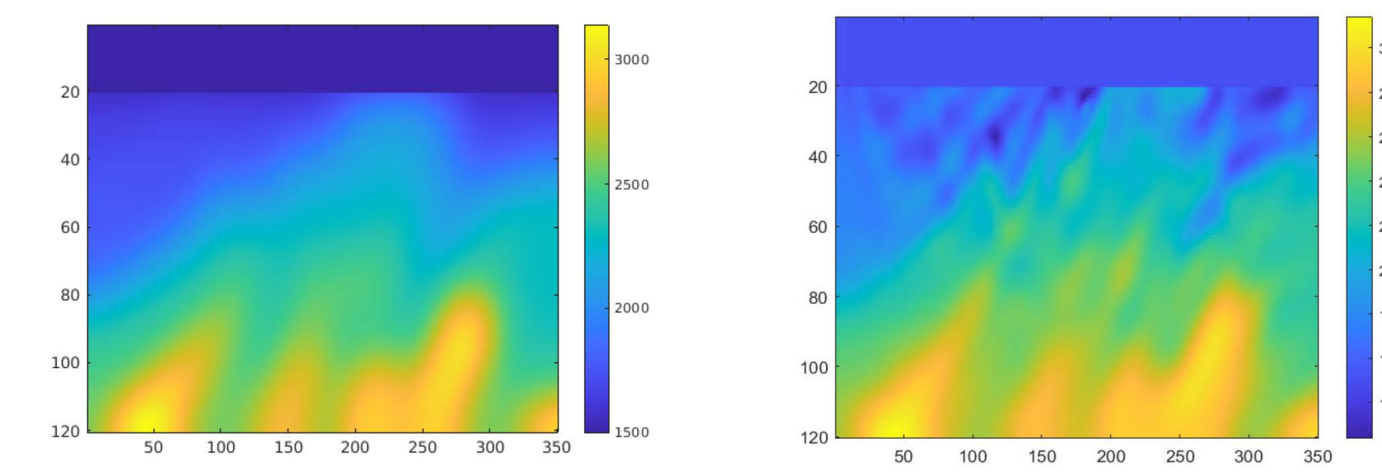
```
For iter=1:niter
    For is=1:ns
        For it=1:nt
            D_cal=stepforward(S,v,wavelet)
            Δd= D_cal -D_obs
        end
        For it=nt:1
            G+= stepforward(G,v Δd)
            Δm (x,z,Θ) = ADCIG(∇²S,G)
        end
    End
    For i Θ =1:n Θ
        grad= Δm(i Θ) % read from Δm
        cg(i Θ)=-grad+β × cg_old(i Θ)
        α=line search(v)
        v=v+ α × cg
    end
end
```

Sequential FWI, update model with small angle gradient first, use the updated model for large angle.
Different stepsize for each angle

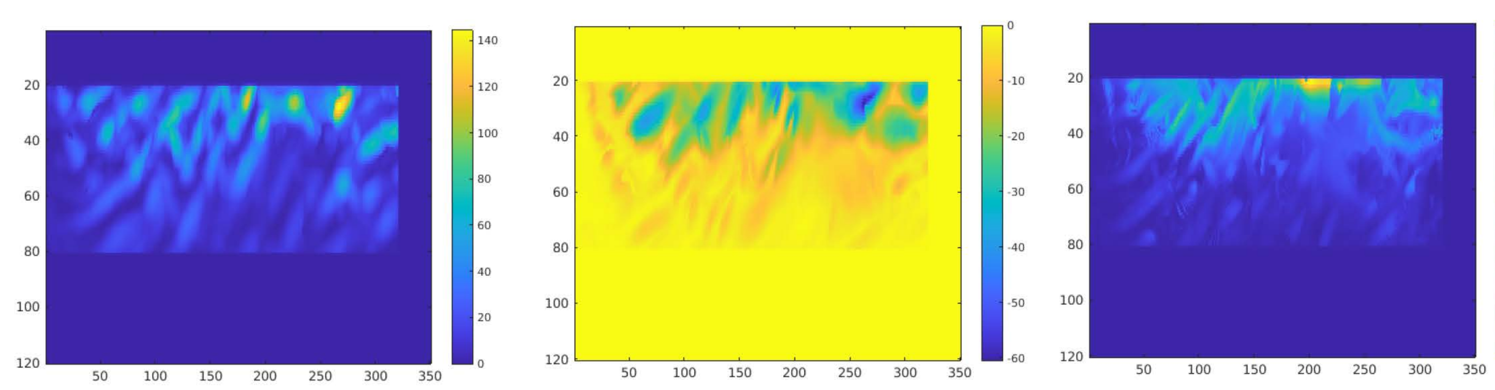
- True velocity model while density is homogenous



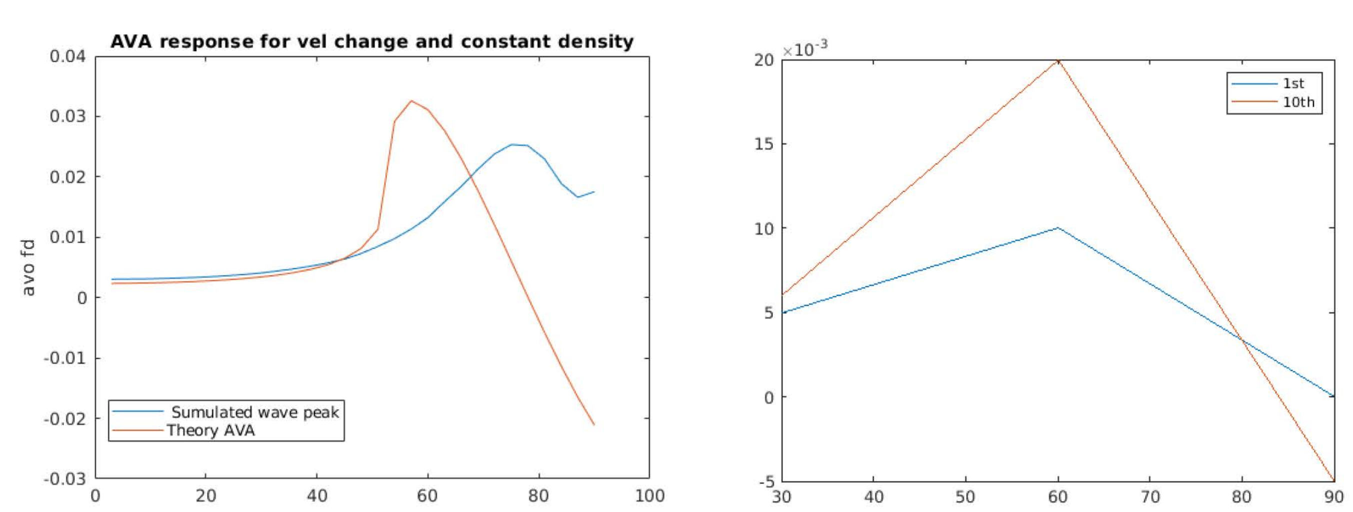
- Initial velocity and updated velocity model in iteration 10



- Gradient for 0-30, 30-60, 60-90 angle in iteration 1



- AVA response in acoustic media: simulated and ADCIGs




- Conclusion

Amplitude variation with angle /AVA information can be extracted from ADCIGs . RTM / two-way wave-equation migration is used for ADCIGs extraction. Wave-equation migration with velocity and reflectivity are updated simultaneously in acoustic media with density.

Future: The simultaneous prediction of velocity and angle-dependent reflectivity in time domain FWI could be used in blended data.

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