# Clifford Neural Operators for learning the horizontal and vertical elastic wavefield displacements

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## Abstract

Neural operators are extensions of neural networks which in supervised training learn how to map complex relationships, such as classes of PDE. Recent literature reports efforts to develop one type of these, the Fourier Neural Operator (FNO), such that it learns to create relatively general solutions to PDEs such as the Navier-Stokes equation. Clifford algebra is very useful for representing multidimensional data. In this study, we use the Clifford Fourier Neural Operator (CFNO) be trained to learn the elastic wave equation from a synthetic training data set. CFNO attempts to find a manifold for elastic wave propagation. On that manifold, wave fields are represented in lower dimensions than those needed for standard solutions, and the calculations for wave propagation are correspondingly simpler. The CFNO combines a linear Clifford fully connected transform, the Clifford Fourier transform, and a non-linear local activation to produce a network with sufficient freedom to map from a general parameterization of a forward wave problem to its solution. Post-training, the CFNO is observed to generate accurate elastic wave fields.

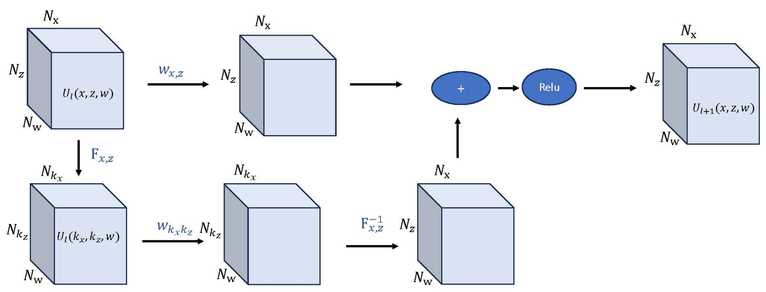


Figure 1: The architecture of the Fourier neural layer. including the Fourier transform $\mathrm{F}_{\mathrm{x,z}}$, the wavenumber-domain multiplication with filter $w_{k_x,k_z}$ , the inverse Fourier transform $\mathrm{F}^{-1}_{\mathrm{x,z}}$, the space-domain filter $w_{x,z}$, and the ReLU activation function.

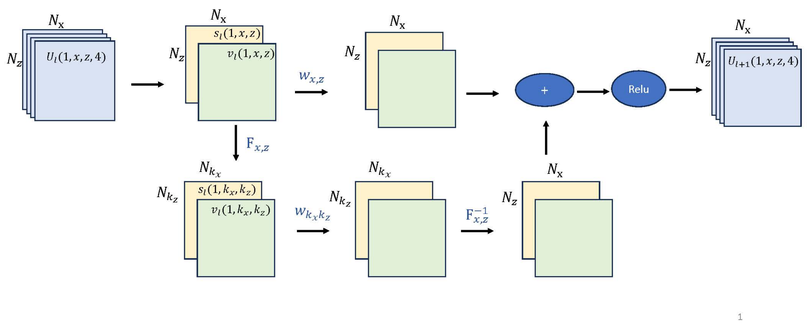


Figure 2: The architecture of the Clifford Fourier neural operator, using one Clifford multivector $U_l$, with a dimension of $\mathbb{C}^{1\times \mathrm{N_z},\times \mathrm{N_x}\times 4}$ for the input as an exmaple. The Clifford multivector $U_l$ is firstly being viewed as dual parts, which means that it will be changed into the spinor part $s_l$ and vector part $v_l$. The complex-valued spinor part $s_l$ and vector part $v_l$, each of which has the dimension of $\mathbb{C}^{1\times \mathrm{N_z},\times \mathrm{N_x}\times 1}$, will undergo the 1D convolution, and the Fourier domain multiplication just like Figure **??**. Finally the spinor part $s_l$ and vector part $v_l$ are transformed back into the Clifford multivector with dimension $\mathbb{C}^{1\times \mathrm{N_z},\times \mathrm{N_x}\times 4}$, which will be regarded as the updated data $U_{l+1}$.

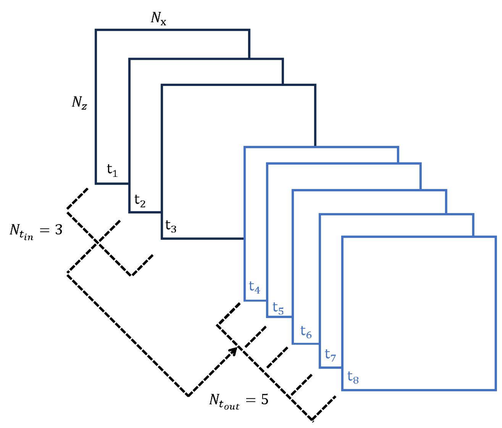


Figure 3: Roll-out training method. Supposing that we have a maximum of 8 time steps for training. The roll-out training method uses several time steps of the wavefield as input (3 time steps in this Figure), and is trained to generate the following time steps of the wavefield(5 time steps in this Figure)

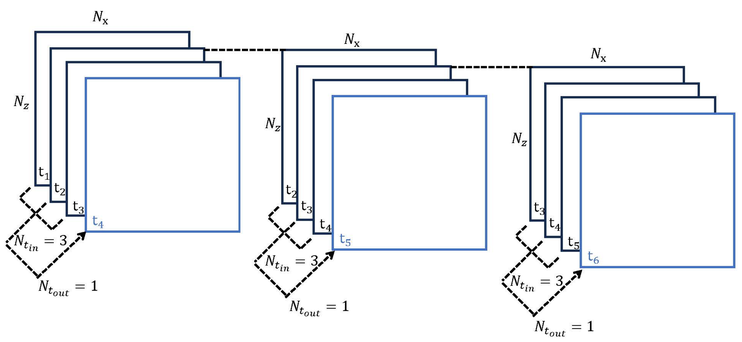


Figure 4: One-step training method training stage. In the training stage of the one-step method, the network uses several time steps of the wavefield as input (3-time steps in this Figure) and is trained to generate the wavefield of the next one time step.

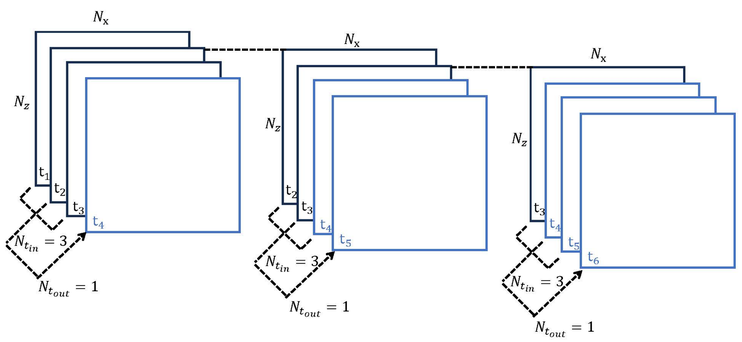


Figure 5: One-step training method validation stage. In the validation stage of the one-step training, the network uses several time steps of the wavefield as input (3-time steps in this Figure) and is trained to generate the wavefield of one time step, but the wavefields generated by the neural network will be used as the input for generating the next time step.

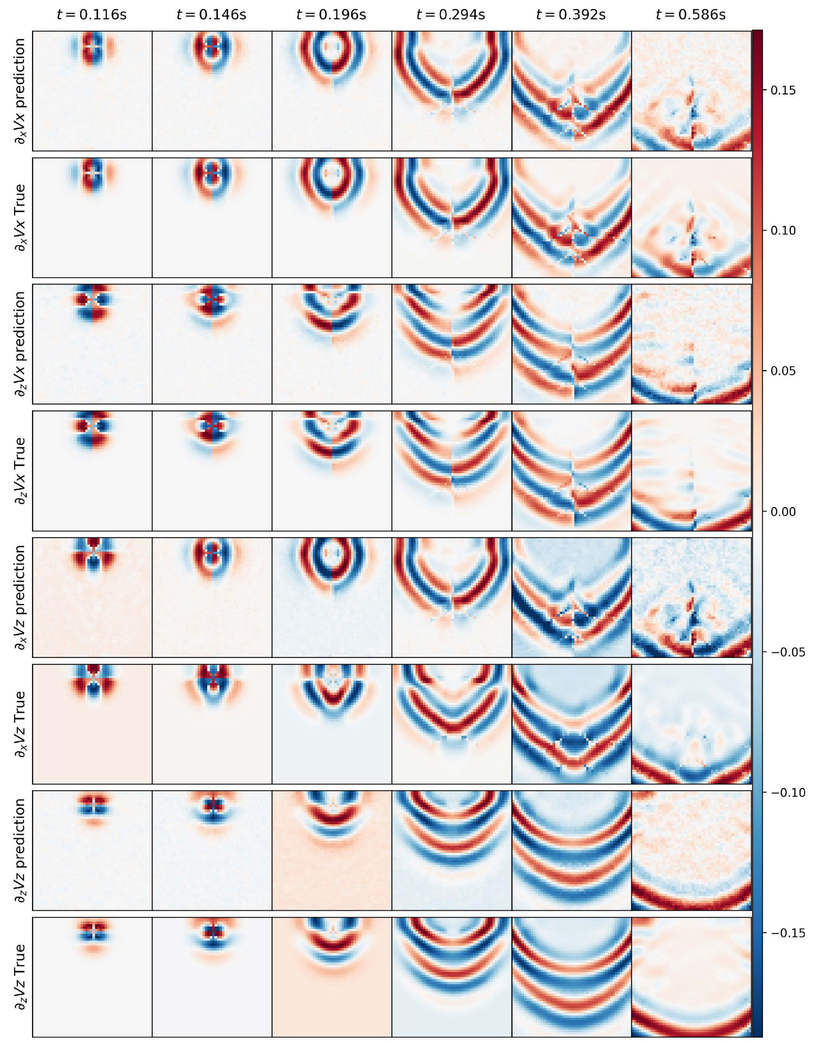


Figure 6: A example of the snapshots of the wavefields at six-time steps, $\partial_x V_x$, $\partial_z V_x$, $\partial_x V_z$, $\partial_z V_z$, generated by the Clifford FNO. The Clifford FNO is trained by using the roll-out training methodology.

## Conclusions

- (In this study, we train a fast-forward modeling method with the Clifford Fourier Neural Operator(FNO). The network consists of three parts: two-dimensional projection layers that operate on time and several Fourier layers that learn the spatial partial derivatives. The power of the Fourier Neural operator comes from the combination of the Clifford linear operation, operators that resemble partial differential calculation (via the Clifford Fourier transform), and the non-linear local activation. The numerical tests suggest that FNO could generate promising wavefields. ).

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