Depth-variant mapping and moveout correction of converted-wave data

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ABSTRACT

Converted-wave imaging is more complex than compressional-wave imaging because of the nonsymmmtery of the raypath geometry of converted-wave (P to Sv) data and the variance of the offset location of the conversion point with depth. P to S raytracing through a southern Alberta model shows that this depth variance is most significant at the shallow depths where the offset-to-depth ratio is greater than one, and that the conversion-point trajectory tends to an asymptote value with depth. The nonsymmetry of the raypath geometry implies that midpoint gathering cannot be used, and the depth variance of the offset location of the conversion point implies that for correct imaging, gathering must be done on a depth-variant basis. The traveltime curves of converted-wave data are different than those of compressional-wave data and therefore the compressional-wave hyperbolic approximation cannot be used. A review of literature revealed that most converted-wave images to date have been constructed by gathering and stacking in a manner analogous to conventional compressional-wave gathering and stacking, except some value other than the midpoint value is used. This value is usually the asymptote value. Work done shows that depth variance is important. Depth-variant mapping is proposed to properly reconstruct converted-wave images. Application of various methods to synthetic data show that depth-variant mapping gives superior results to other methods which tend to smear the image. An improved expression for removing the moveout of converted wave data in a single isotropic layer is given, and it is shown to be accurate and stable for large offset-to-depth ratios.

INTRODUCTION

The recognition of the benefits of having measurements of both the compressibility and the rigidity of the earth has helped to foster the growing interest in elastic-wave seismology as an exploration tool. Acquiring converted-wave (P to Sv - compressional wave converted to shear wave) data has certain technical advantages that makes it attractive as a method of acquiring shear-wave information of the subsurface (eg. a P-Sv survey can be acquired simultaneously with a P-wave survey using the same P-wave source). It is thus compelling that unresolved issues of converted-wave image reconstruction be addressed. This paper deals with the issues moveout correction and the gathering and stacking of converted-wave data.

The gathering and stacking of multichannel seismic data introduced by Mayne (1956) is a firmly established fundamental step of reflection-seismic data processing. Gathering converted-wave data is more complicated than gathering compressional-wave data because of its non-symmetrical raypath geometry and the depth-variance of the offset location of the conversion points. Other workers have focussed on the important problem of raypath nonsymmetry (Chung et al., 1985; Fromm et al., 1985; Garotta, 1985). This paper focusses on the issue of the depth variance of the offset location of the conversion point. The objectives of this paper are to show that current methods tend to smear the events from the shallow depths, and to propose depth-variant mapping and show that it gives superior results than regular gathering. A new formulation for removing the moveout

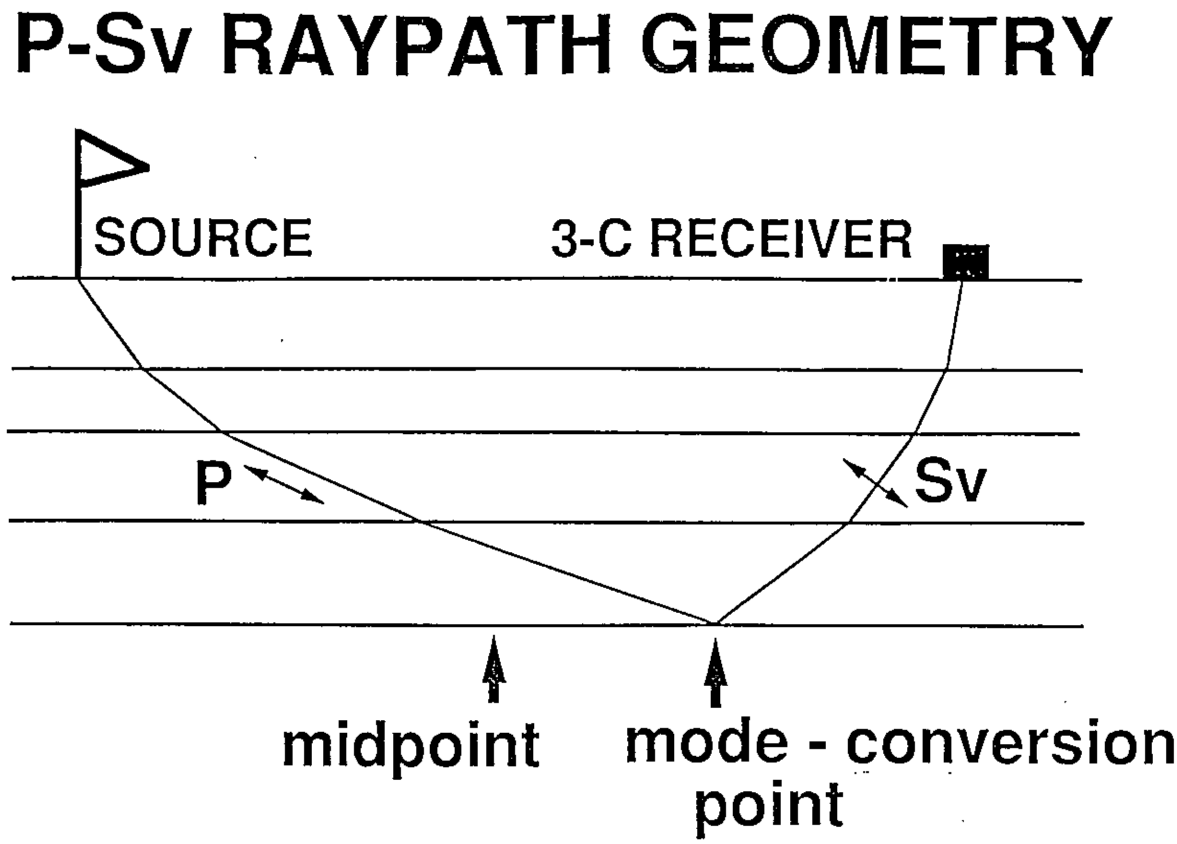

Fig. 1a. The raypath geometry of converted-wave data is nonsymmetrical in a flat and isotropic earth.

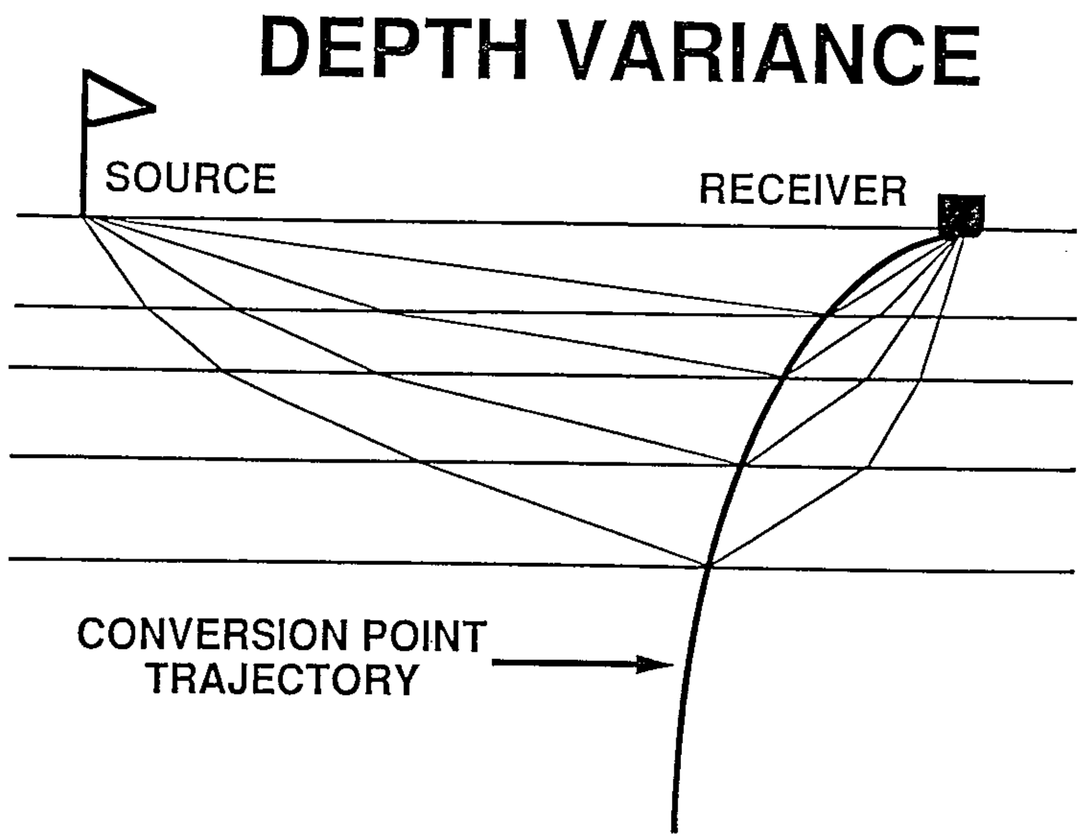

Fig. 1b. The offset locations of the mode-conversion points vary with depth.

of converted-wave data from a single isotropic layer is presented and it is shown to be very accurate and stable.

NONSYMMETRY AND DEPTH VARIANCE

The raypath geometry of a converted wave is nonsymmetrical in a flatly layered isotropic earth (Figure 1a). Shear waves propagate with velocities that are generally about half that of compressional waves; so that upon conversion from P to S the ray bends toward the normal, as predicted by Snell's law. Midpoint gathering therefore cannot be used. Furthermore, for a given source-receiver offset, the offset location of the conversion point varies with depth (Figure 1b). This variance is greatest at the shallow depths where the offset-to-depth ratio is less than one. This implies that to gather converted-wave data properly, it should be done on a time (or depth) - variant basis.

To illustrate these general characteristics of converted-wave data, P to Sv ray tracing was performed on a model based on P and S velocities determined by VSP measurements made in southern Alberta (Geis et al., 1989). The input model is shown in Figure 2a. For a source-receiver offset of 1600 metres, the offset location of the conversion point was determined for each horizon. The conversion point locations (plotted with depth in Figure 2b) are offset from the midpoint. Furthermore, these conversion-point locations show a clear bias toward the receiver at the shallow depths. It can also be observed that the offset locations of the conversion points tend toward an asymptote at the deeper depths. Thus to correctly construct converted-wave images some value other then the midpoint should be used to estimate the conversion-point locations and gathering should be done on a depth-variant basis.

CONVERTED-WAVE MOVEOUT CORRECTION

As the seismic wave propagates downwards through the earth with a different velocity (P-wave) than the velocity with which it returns to the surface (S-wave, about half that of P-wave), the conventional compressional-wave moveout equation cannot be used. The moveout of converted-wave data is greater than that of corresponding compressional-wave data. Tessmer et al.(1988) derived coefficients that they substituted into the traveltime series of Taner et al. (1969) to give the traveltime expression for converted-wave data from a single layer. It is shown below:

$$T(x)^2 = T^2_{o(ps)} + \frac{X^2}{V_p V_s}, \tag{1}$$

where T(x) is the traveltime for a source-receiver offset X, $T_{o(ps)}$ is the p to s two-way traveltime, and V_p and V_s the compressional-wave and shear-wave velocities. This expression is valid for moderate offsets. The following is a more stable expression developed from the formula of Tessmer et al. (1988):

$$T_{o(ps)} = T(x) - \frac{V_p X^2}{2V_s T(X) V^2_{pp}}, \tag{2}$$

where V_{pp} is the compressional-wave root-mean-square (rms) velocity. A brief development of this expression is given in Appendix A.

SOUTHERN ALBERTA VELOCITY PROFILE

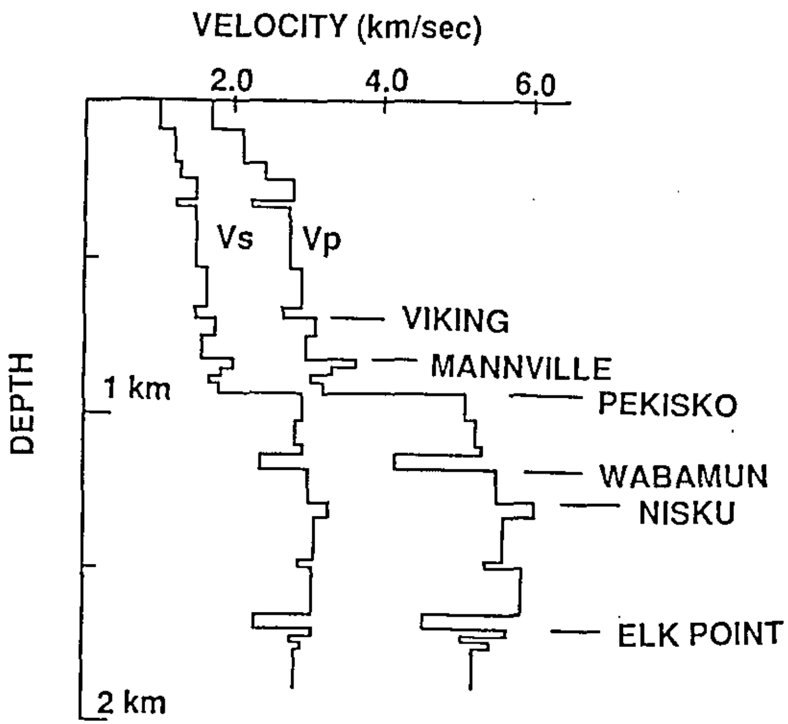

Fig. 2a. Southern Alberta velocity profile. Based on VSP measurements (after Geis et al., 1989)

LOCATION OF P-Sv CONVERSION POINTS

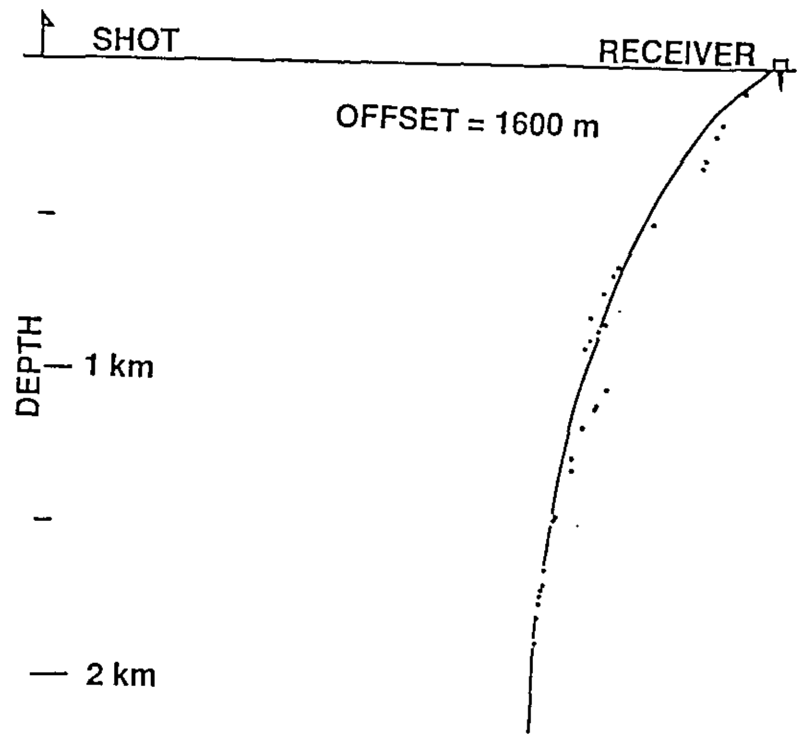

Fig. 2b. Offset locations of mode-conversion points determined by raytracing through southern Alberta velocity profile shown in Fig. 2a.

The second term of this equation is used to estimate the moveout of a time sample given its offset, its offset traveltime, the velocity ratio of P to S, and the P-wave rms velocity. The velocity ratio serves to adjust the expression for the greater moveout of converted-wave data while the T(x) term in the denominator stabilizes the expression at the far offsets. Figure 4 gives the error with offset of these two equations for a single isotropic layer of thickness 250 metres. Error was determined by comparing the true normal-incidence time with the moveout-corrected time given by the two formulae. It can be seen that Equation (2) is more stable.

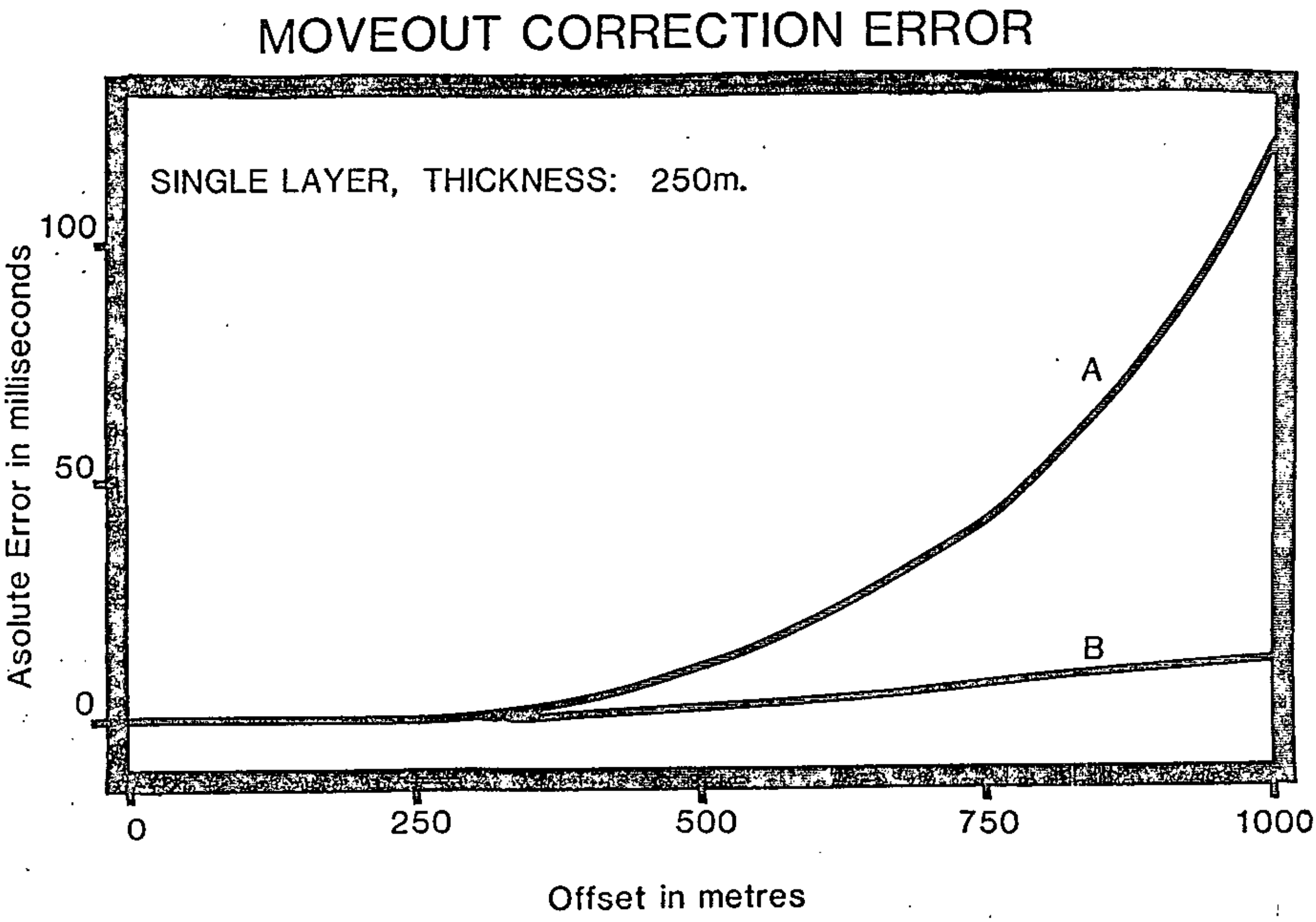

Fig. 3. Absolute error in milliseconds incurred by using equations (1) (Curve A) and (2) (Curve B) to moveout correct converted wave data from a single isotropic layer of thickness 250 metres. True taveltime was computed using raytracing.

CURRENT TECHNIQUES

Nonsymmetry has been recognised by other authors (Chung et al.,1985;Garotta, 1985; Frasier et al., 1986) and a number of formulae that estimate the location of the conversion point have been published. Although arrived at in different manners, most of the published formulae give the asymptote value of the conversion point trajectory and are of the general form:

$$X_p = \frac{X}{1 + V_s/V_p}, \qquad (3)$$

where Xp is the source-conversion point offset, X is the source-receiver offset, and Vs/Vp the shear-to-compressional wave velocity ratio in the area. This formula or some variation thereof has been used by others to gather converted-wave data in a manner analogous to conventional compressional-wave gathering.

Recently a formula has been published that takes into account depth variance (Tessmer and Behle, 1988). This formula gives for a single isotropic layer of a given thickness the analytic solution of the offset location of a conversion point that would be obtained by ray tracing. The formula is the solution to a quartic equation, is complex to solve and is thus not given here. The formulation allows for a method of gathering that takes into account depth variance. However, this formula has as its limitation the assumption that the earth can be approximated by a single layer.

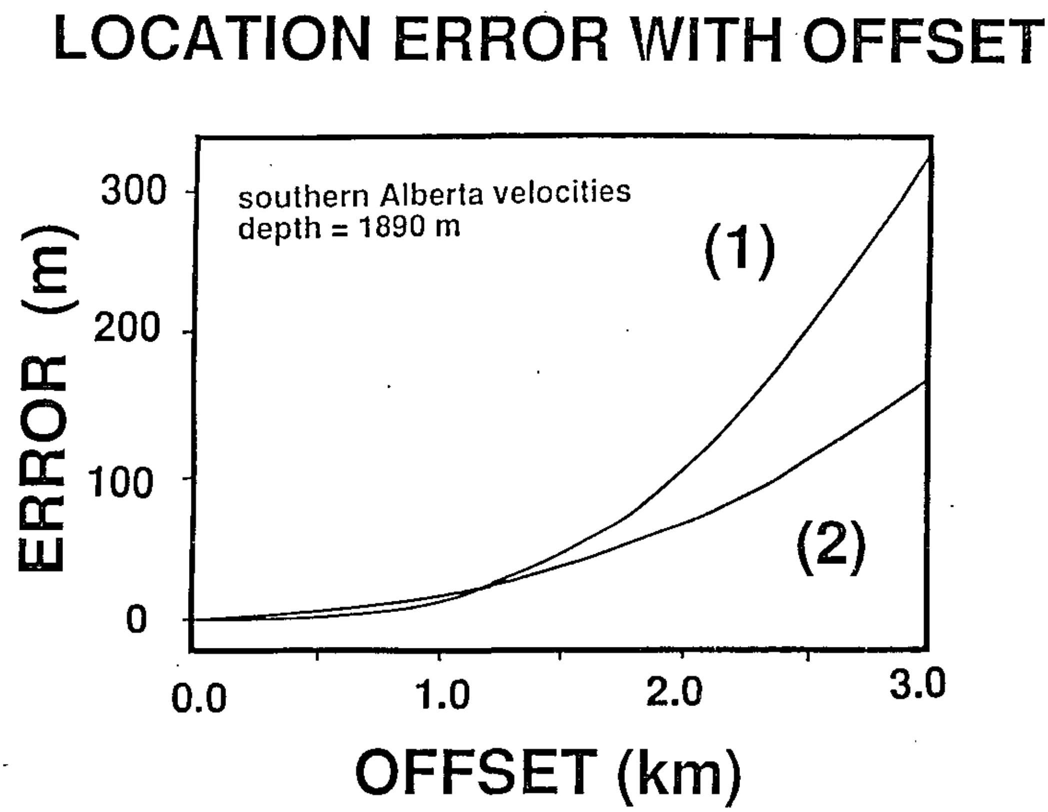

Fig. 4. Location error with offset of approximating formulae. Curve 1 is error incurred by using asymptote formula (equation 3). Curve 2 is error incurred by using algorithm of Tessmer et al.(1988) focussed for depth of 1890 metres. Error is defined as absolute difference in metres of location given by formulae and raytraced location.

To test the sensitivity of this assumption raytracing was performed on the model shown in Figure 2a to determine to offset location of the conversion points for a depth of 1890 metres and an offset range from zero to three kilometres. The differences between the ray-traced location and the location given by the formulation of Tessmer et al. (1988) are plotted in Figure 4 as a function of offset. Also plotted are the differences between the raytraced location and the asymptote location. What this figure shows is that for this relatively deep reflector where the ray has travelled through many layers, the asymptote location is actually more accurate at the near offsets where the offset-to-depth ratio is high. However even at this depth the asymptote approximation quickly tends to large errors at the far offsets. The single-layer approximation is clearly more stable at the far offsets and this would be more evident if the testing had been done for a shallower reflector. What is important to note, however, is that although an improvement, this formula does have errors that build with offset as more layers are approximated with a single layer. An improved formulation that accounts for vertically varying velocities has yet to be derived.

DEPTH-VARIANT MAPPING

At present converted-wave images are generally constructed by gathering and stacking in a manner analogous to conventional compressional-wave midpoint gathering and stacking, except the data are gathered using some value (such as the asymptote) other than the midpoint. For many interpretation objectives that are relatively deep (i.e. offset-to-depth ratio is greater than unity even for far offsets), asymptote gathering may be adequate and the mispositioning of the shallow events can be accepted as a compromise of the method. However when imaging the shallow section is important, this compromise is not acceptable. With the advent of more sophisticated formulae that honour depth variance and with some knowledge of the velocity ratio in the area it is possible to construct converted-wave images that are focussed for specific depths i.e. gathering can be done using some value other than the asymptote value. However the resulting image is still not ideal because events from other depth levels may be mispositioned. A superior solution is to correctly position data from all the depths by depth-variant mapping.

The schematic diagrams of Figures 5a and 5b illustrate the difference between gathering and depth-variant mapping. Rather than shifting the entire trace to a gather location (determined by the asymptote value or some other value), each sample point is mapped to its own offset location. A key element to the success of this technique is the ability to estimate the conversion point trajectory (Figure 1b) within reasonable limits. One way to establish this trajectory is to use the formulations of Tessmer et al . (1988) or Taylor (1989). Another possibility is to ray trace through a velocity model of the area. Such a model could be estimated using information from full waveform sonic logs, VSP measurements or other techniques.

The schematic diagram of Figure 5b shows the mapping procedure for a trace already moveout corrected. A possible procedure for depth-variant mapping is to do an initial gather using the asymptote or some other approximation so that velocity analysis and moveout corrections can be made. The moveout corrected traces would then be mapped and simultaneously stacked to construct the final image. However, in concept traces need not be initially moveout corrected before mapping eg. moveout correction and mapping could be executed simultaneously. This has been successfully implemented by Stewart (1988) for VSP data.

Asymptotic Gather

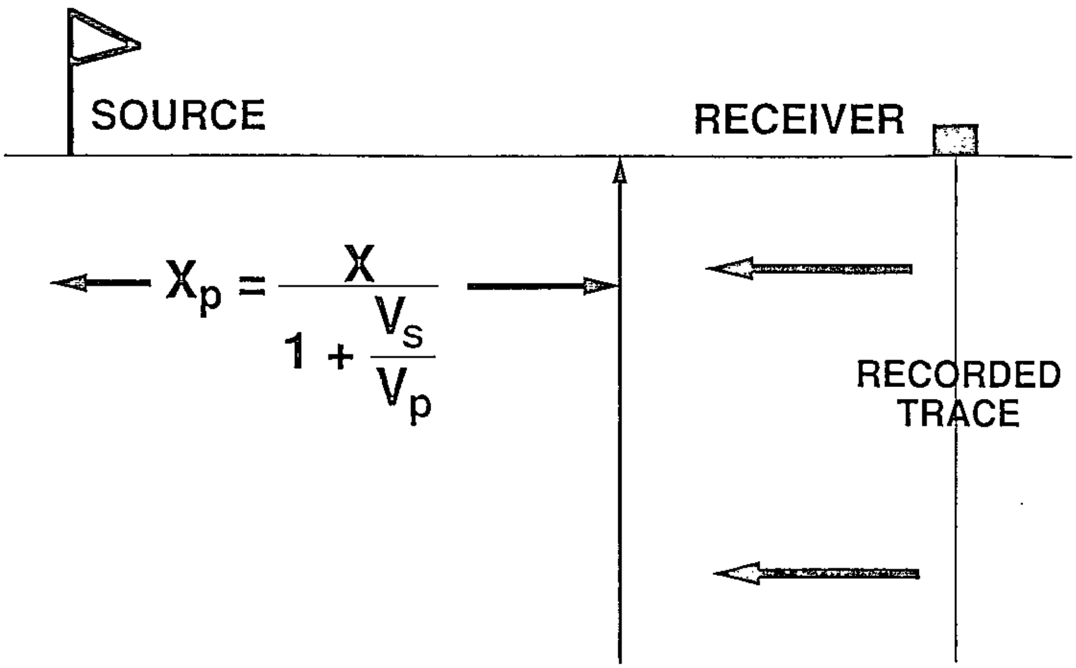

Fig. 5a. Schematic diagram showing gathering of converted-wave data in procedure analagous to compressional-wave gathering except some value (eg. asymptote value of conversion-point trajectory shown in Fig. 1b) is used in place of the midpoint.

Depth-Dependent Map

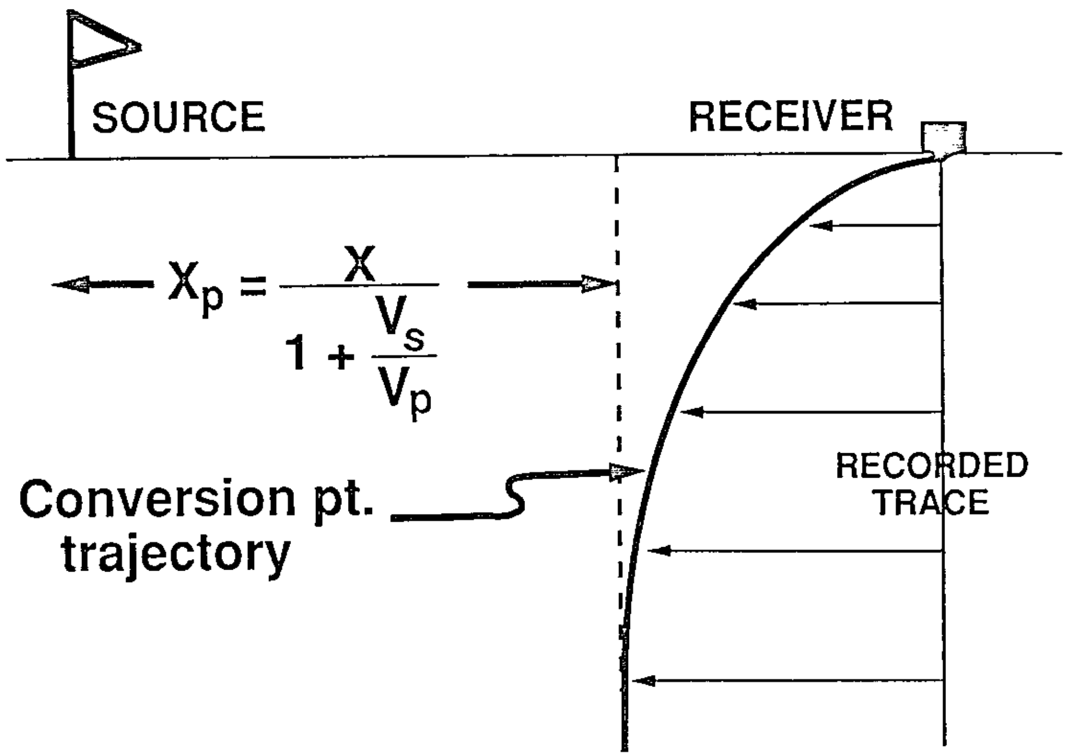

Fig. 5b. Schematic diagram illustrating depth-variant mapping. Each sample point of the moveout-corrected trace is mapped to its appropriate offset location.

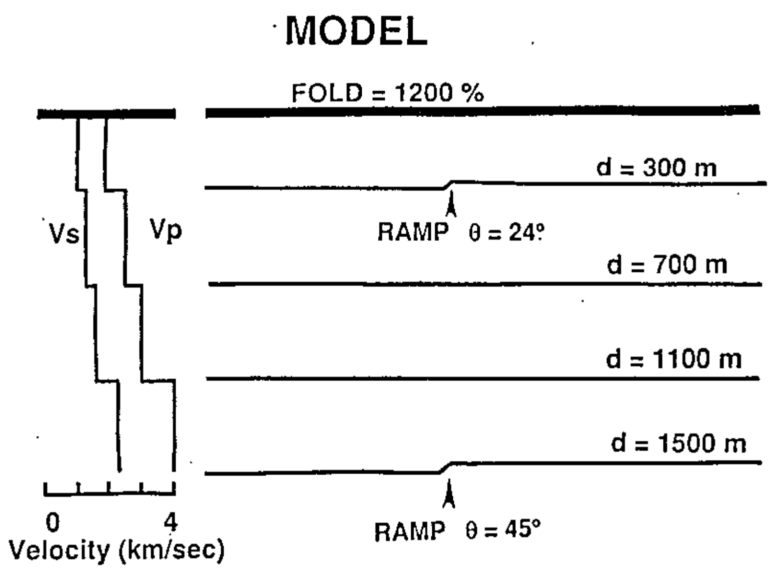

Fig. 6. Velocity model used to generate raytraced synthetic data.

EXPERIMENT - SYNTHETIC DATA

To test the efficacy of the different techniques described above a synthetic dataset was created and processed using these methods. A simple model (Figure 6) with constant Vp/Vs and four interfaces, two with small ramps, was used to generate the synthetic data. Travel times and the locations of the conversion points were determined by P to Sv raytracing. As the objective of the experimentation was to see how well the various methods reconstructed the ramps the model data were kept simple: amplitude variation with offset effects were ignored and diffractions were not included. Data were created to a maximum offset to depth ratio of 2.5 for the shallow interface.

Asymptotic gathering and stacking was the first method to be tested. Figure 7a shows the stacked version of the data gathered using the asymptotic approximation. Even for this simple model the shallow reflectors are clearly smeared while the deepest reflector has been reasonably well imaged. Depth-variant mapping was then carried out. Raytracing was used to establish the conversion point trajectory for each offset. The results are shown in Figure 7b. The model image has now been properly reconstructed without compromise to any particular interface.

DISCUSSION

All of the previously discussed methods assume a flatly layered isotropic earth. The mapping procedure is a more expensive technique than gathering and stacking. It also requires some prior knowledge of the velocity structure. Although these are similar constraints to a full prestack elastic-wave migration the mapping procedure is still preferable to this for two main reasons: stacked sections are desired for interpretation purposes, and mapping and post-stack migration is still considerably more economic than prestack migration.

The formula for removing the moveout from converted-wave data is useful because it allows converted-wave moveout corrections to be made with a knowledge of the P-wave velocity structure and an estimate of the Vp/Vs in the area.

RESULTS: Asymptotic Gather and Stack

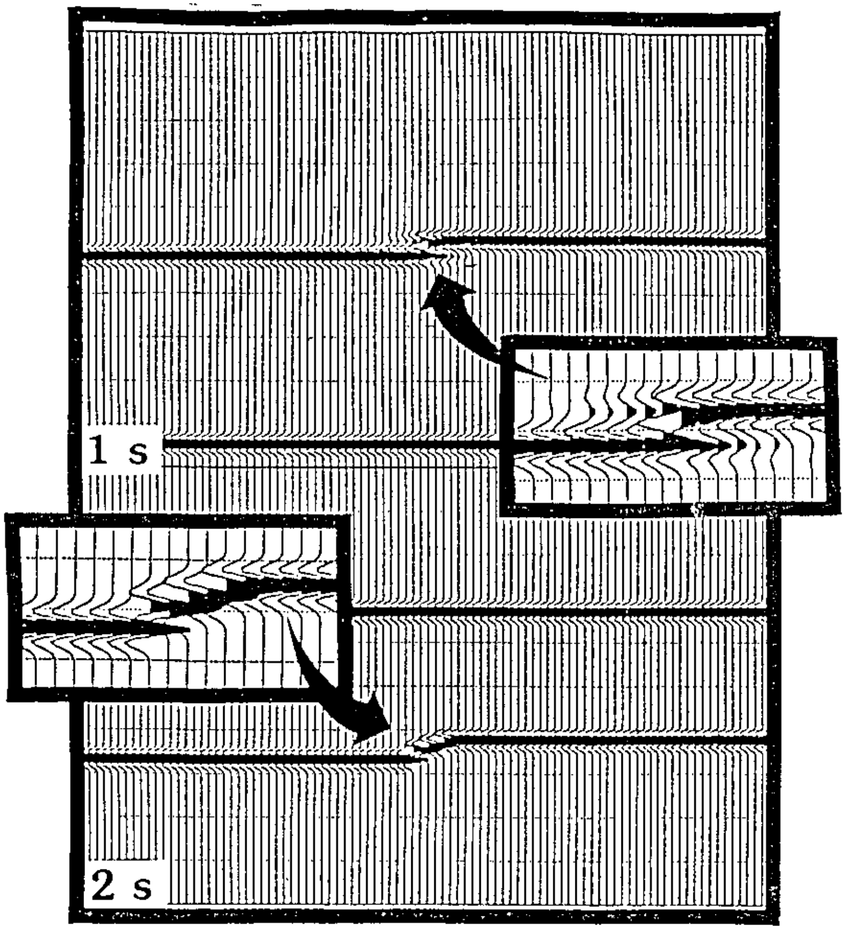

Fig. 7a. Results of stacking the model data gathered using the asymptote formula (equation 3). Note the smearing of the shallow event.

RESULTS: Depth-Variant Map and Stack

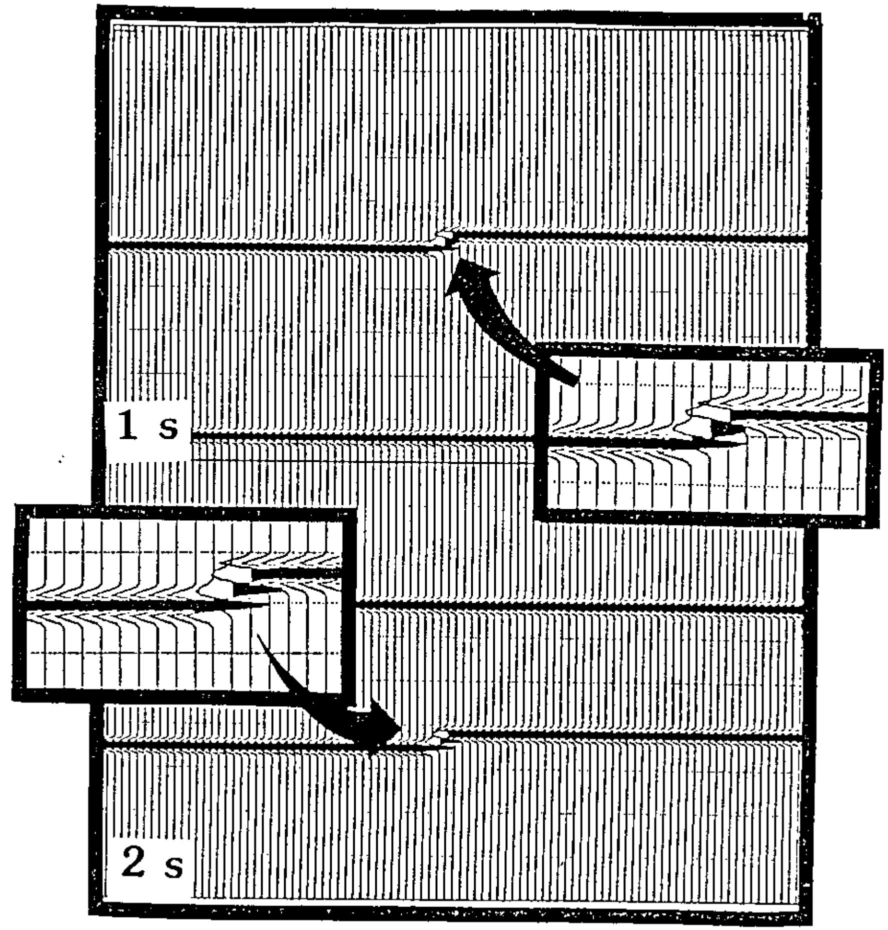

Fig. 7b. Results of depth-variant mapping. Note both ramps have been well reconstructed.

CONCLUSIONS

It has been shown that constructing images of the earth using converted-wave data is more complex than using compressional-wave data because of nonsymmetry of the raypath geometry and depth-variance of the offset location of the conversion point. It has been shown that depth-variant mapping gives better results than asymptotic gathering which tends to smear the shallow section. An improved formulation for estimating the moveout of converted-wave data in a single isotropic layer has been presented and it has been shown that it is accurate and very stable.

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APPENDIX A

Equation (2) is developed from Equation (1) of Tessmer et al. (1988), restated below:

$$T(x)^2 = T^2_{o(ps)} + \frac{X^2}{V_p V_s}. \tag{1}$$

For estimating moveout it is desirable to have an expression of the general form:

$$T_o = T(x) - T_{NMO}. \tag{4}$$

This expression gives the corrected traveltime (T_o) as the total (offset) traveltime ($T(x)$) less the normal moveout (T_{NMO}). Tessmer et al. (1988) showed that Equation (2) can also be expressed in the following form:

$$T(x)^2 = T^2_{o(ps)} + \frac{V_p X^2}{V_s V^2_{pp}}, \tag{5}$$

where V_{pp} is the compressional-wave root-mean-square velocity. To solve the square root and get an expression for $T(X)$, the right half of (5) is expanded using the binomial expansion. Neglecting higher order terms the following is obtained:

$$T(x) = T_{o(ps)} + \frac{V_p X^2}{2V_s T_{o(ps)} V^2_{pp}}, \tag{6}$$

which is then rearranged to give a general expression of the form of (4)

$$T_{o(ps)} = T(x) - \frac{V_p X^2}{2V_s T_{o(ps)} V^2_{pp}}. \tag{7}$$

Testing of this expression using traveltime data obtained by P to S raytracing in a single flat isotropic layer shows that at far offsets the moveout is consistently overestimated, resulting in data that are overcorrected. Thus, to stabilize the moveout term, it is desirable to have a variable in the second term of the right hand side of (7) that progressively decreases the term as the source-receiver offset (X) increases, eg. as shown below:

$$T_{o(ps)} = T(x) - \frac{V_p X^2}{2V_s T(X) V^2_{pp}}. \tag{2}$$

$T(X)$ increases as the offset increases and so $T(X)$, a known when doing moveout corrections, is substituted for $T_{o(ps)}$ in the denominator of (7). By numerical experimentation on a single flat isotropic layer this result proves accurate and very stable at the far offsets. It is a useful result because knowledge of the compressional-wave velocity structure (eg. from P-wave velocity analysis) and Vp/Vs of an area can be used to do converted-wave moveout corrections.