Automatic velocity analysis of crosswell seismic data

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INTRODUCTION

Velocity plays an important role in exploration seismology. It is widely used in various areas in seismic data processing(such as multifold seismic data stacking and migration) and seismic stratigraphic interpretation, as well as determination of information regarding structural geology and lithology. Seismic velocity is also used to obtain information about anisotropy and heterogenity of the elastic medium.

Recently, considerable research in inversion of seismic tomographic data for velocity structure has been accomplished(Ivansson, 1985; Chiu and Stewart, 1987; Iverson, 1988; Zhu and McMechan, 1988; Abdalla and Stewart, 1990; Lines and Tan, 1990; Stewart, 1990). A different method for velocity analysis is proposed in this project; it is similar to those commonly used in conventional velocity analysis of CMP gathers. This method is to derive velocity information from the first breaks of crosswell data. Common shot or receiver gatherts of either P- or S-wave data can be used for velocity analysis by the method described here. Assuming that the subsurface is a continuous isotropic medium in which the velocity varies linearly as the depth increases, a farmiliar formula is derived. To obtain the velocities which best fit the subsurface, a semblence analysis is made. Based on this, a computer program is then written to implement this algorithm. Both synthetic and real crosswell shot or receiver gathers will be used to test the validity of the method.

A PROPOSED METHOD

Shown in Figure 1 is the typical configuration of a crosswell seismic survry. Sources are placed at different levels of depth in a well and many receivers are positioned at various depths, usually with a constant spacing interval, in another well. Seismic waves(direct arrivals or first breaks, reflections, transmitted waves and some other wave modes) are generated by each excitation of the source, and these waves propagate along various paths to the receivers where they are recorded. According to Fermat's Principle, in an isotropic continuous elastic medium, the seismic waves travel along the paths which are time minimal. Figure 1 shows some raypaths of the direct arrivals and their traveltime curve. The recorded first breaks are the data which we will use for velocity analysis in this project.

As illustrated in Figure 2, the traveltime for the ray of a seismic wave to travel from the source S at depth Z_S to the receiver R with a depth of Z_R, along a curved raypath SR, is given by

$$t = \int_{\widehat{SR}} \frac{dl}{v(z)} = \int_{z_S}^{z_R} \frac{\left(1 + \tan^2\theta\right)^{1/2}}{v(z)} dz \quad , \tag{1}$$

and the offset X is

$$X = \int_{\widehat{SR}} dx = \int_{z_S}^{z_R} \tan\theta dz \quad , \tag{2}$$

where the velocity V(z) and angle θ are a function of depth. A detailed derivation of the formulae is given in Appendix 1. Through minimizing the traveltime t and some other simplifying procedures, an approximate traveltime equation for the direct arrival is obtained:

$$t^2 = \left(\int_{z_S}^{z_R} \frac{dz}{v(z)}\right)^2 + \frac{\int_{z_S}^{z_R} \frac{dz}{v(z)}}{\int_{z_S}^{z_R} v(z)dz} X^2 \quad . \tag{3}$$

If an assumption is made that there is a linear velocity relation with depth, i.e.,

$$V(Z) = V_0 + \eta Z, \tag{4}$$

where V_0 is the starting velocity, and η is the velocity gradient, the above equation will be further simplified. Jain(1987) inspected sonic logs from western Canada and concluded that most logs in the western Canadian basin justify a linear increase in velocity with depth down to the Paleozoic unconformity. He gave the values of the constant η ranging from 0.25 to 1.0 in the Cretaceous section. With the linear velocity relation, the above traveltime equation is simplified to

$$t^2 = \left[\frac{1}{\eta} \ln\left(\frac{v_0 + \eta z_R}{v_0 + \eta z_S}\right)\right]^2 + \frac{\frac{1}{\eta} \ln\left(\frac{v_0 + \eta z_R}{v_0 + \eta z_S}\right)}{v_0(z_R - z_S) + \frac{\eta}{2}(z_R^2 - z_S^2)} X^2 \quad , \tag{5}$$

and in a special case when the source is at the same depth level as the receiver, that is, $Z_S = Z_R$, the equation has the form of

$$t = \frac{X}{v_0 + \eta z_R} \quad . \tag{6}$$

In these two equations, traveltime, t, can be calculated, the depths of the source and receiver, Z_S and Z_R, and the offset X are known, and the parameters (unknowns here) η and V_0 are to be found through semblence analysis.

In deriving equation (6), we assume that when the source and receiver are at the same level, the wave propagates along a straight raypath at that depth in the medium. Grant and West(1965) point out, however, that surface-to-surface refraction will occur if velocity increases continuously with depth. But here the refraction effect is not considered to simplify the discussion.

A semblence analysis is done in order to find the final suitable velocity model parameters. A useful measure of coherency of signals is their energy. The output energy is defined as

$$E_{out} = \sum_{t=t(i)-\Delta t}^{t(i)+\Delta t} \left(\sum_{i=1}^{N} f_{i,t} \right)^2 , \qquad (7)$$

and the input energy as

$$E_{in} = \sum_{t=t(i)-\Delta t}^{t(i)+\Delta t} \left(\sum_{i=1}^{N} f_{i,t}^2 \right) , \qquad (8)$$

where

i - the ith seismic trace;
$t(i)$ - the traveltime corresponging to the ith trace;
$f_{i,t}$ - the amplitude of the ith trace at a time t within the window$[-\Delta t, \Delta t]$; and
N - the total number of traces involved in the analysis.

Therefore the semblence is given by

$$S_c = \frac{E_{out}}{N E_{in}} , \qquad 0 \le S_c \le 1.$$

ALGORITHM IMPLEMENTATION

Our purpose is to derive the velocity function best fitting the given data, assuming the linear velocity model. Particularly, given the recorded amplitudes of first breaks of crosswell data, and known geometric parameters including offset and depths of the sources and receivers, and when the traveltime is calculated, we need to obtain the velocity model parameters, η and V_0, which represent reasonably the subsurface velocity structure.

The computation procedures are the following:

. Input a shot or receiver gather of crosswell data;

. Scan through a range of η and V_0 values;

- Give the time window width [$-\Delta t$, Δt], in which the first breaks of most traces in the gather lie; This window may not need to change for the same gather;
- For trace i:
  . Input geometrical parameters for trace i;
  . Calculate the traveltime, t(i), for the trace;
  . Find the amplitudes falling winthin the given time window for that trace;
- Repeat the above for all the other traces (N traces) in the gather;
- Calculate the semblence value;

. Plot the set of semblence values for this shot gather in a coordinate with η as horizontal axis and V_0 as vertical axis;

. Find the largest semblence value(s) in the plot; The η and V_0 corresponding to the largest semblence value are just the suitable parameters for the velocity model which we are looking for;

. Do this for all new shot or receiver gathers for sets of semblence values;

. Stop.

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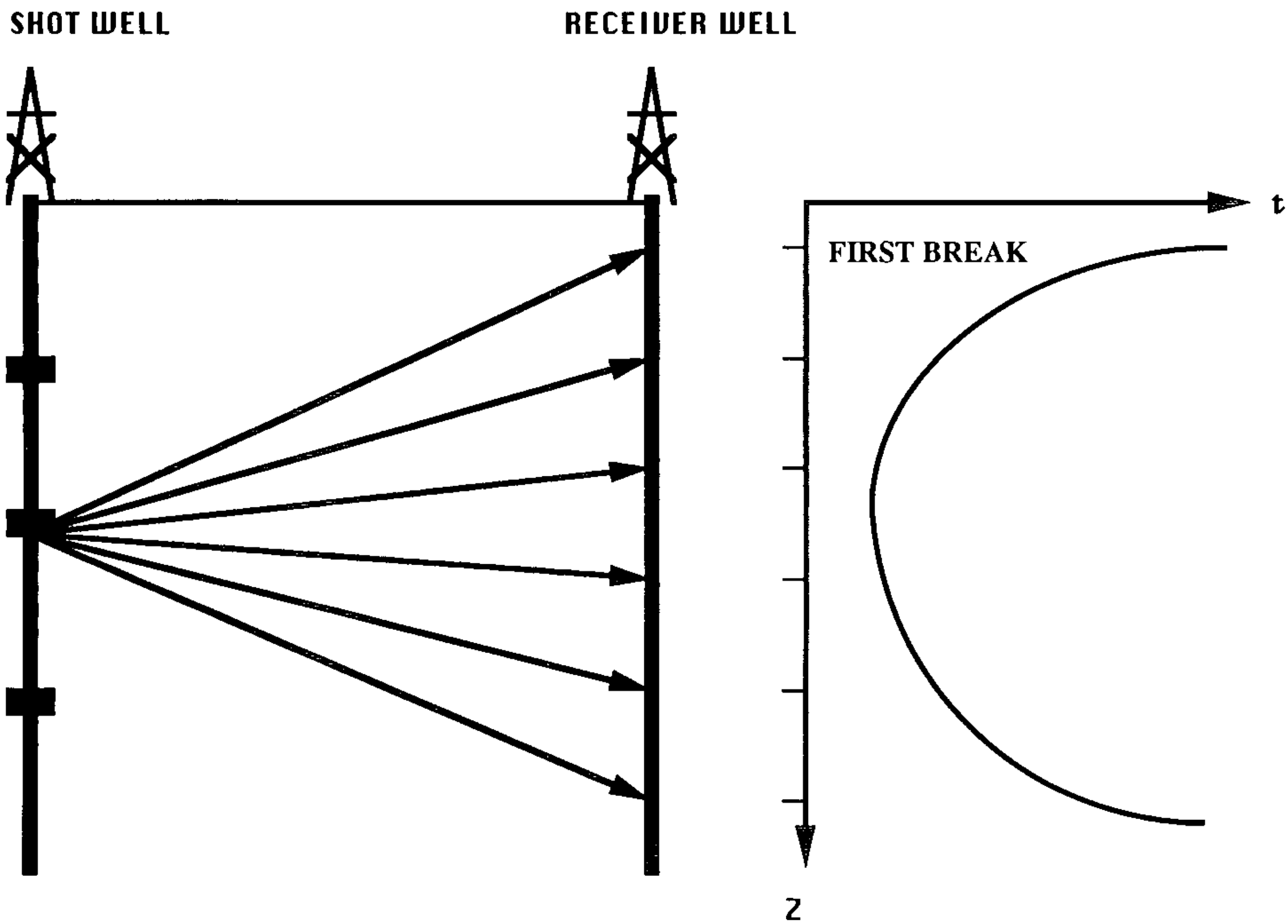

Fig. 1 The raypaths of the direct arrivals and their traveltime recorded in a typical crosswell seismic survey.

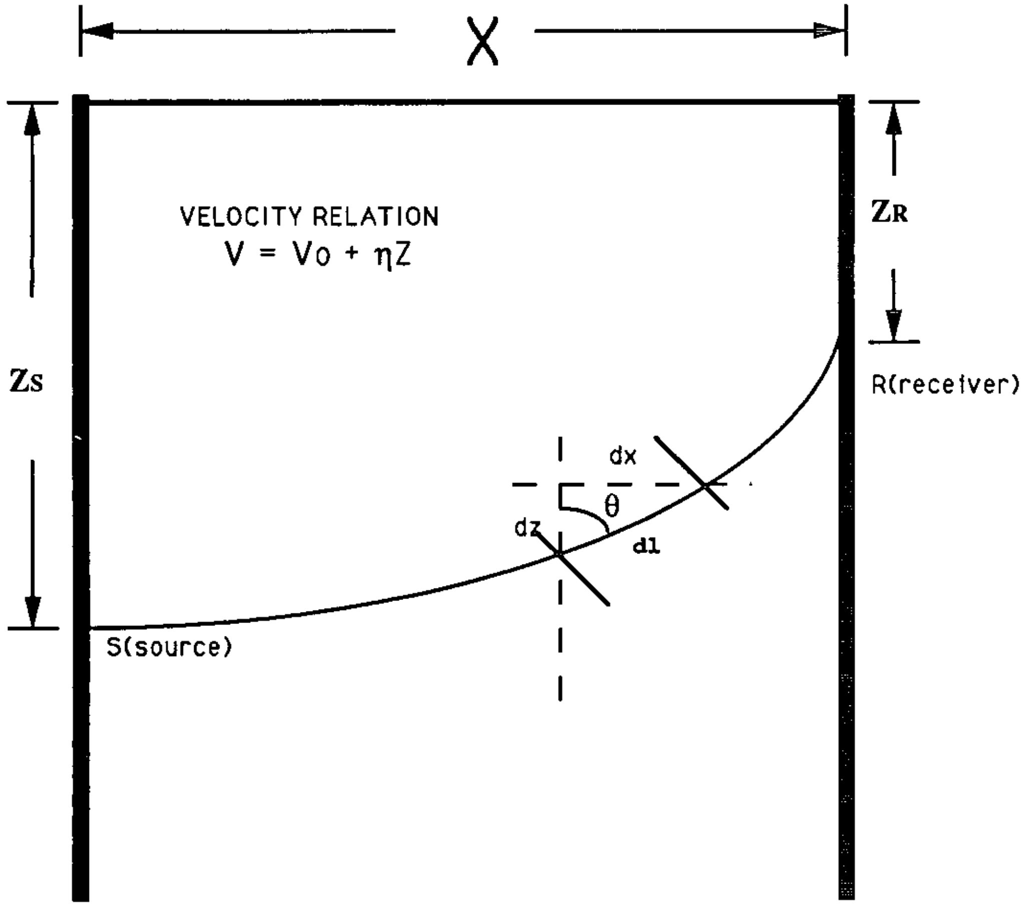

Fig. 2 Ray path of a seismic wave traveling from the source to the receiver.

APPENDIX 1 FORMULA DERIVATION

Fermat's Principle states that in an isotropic continuous elastic medium, the seismic waves propagate along the paths which are time minimal.

As illustrated in Figure 2, the traveltime for the ray of a seismic wave to travel from the source S at depth Z_S to the receiver R with a depth of Z_R, along the curved raypath SR, is given by

$$t = \int_{\widehat{SR}} \frac{dl}{v(z)} = \int_{Z_S}^{Z_R} \frac{dz}{v(z)\cos\theta} = \int_{Z_S}^{Z_R} \frac{\left(1 + \tan^2\theta\right)^{1/2}}{v(z)} dz \quad , \qquad (1\text{-}1)$$

and the corresponding offset X is

$$X = \int_{\widehat{SR}} dx = \int_{Z_S}^{Z_R} \tan\theta dz \quad , \qquad (1\text{-}2)$$

where the velocity v(z) and angle θ are a function of depth Z.

Fermat's Principle is equivalent to the statement that under the constraint (1-2), the traveltime of a wave, t, should be minimal, i.e.,

$$\varepsilon = t - \sqrt{\lambda} X \rightarrow \min,$$

where $\sqrt{\lambda}$ is the Lagrange factor (an arbitary constant). Since $t - \sqrt{\lambda}X$ varies with the quantity $\beta = \tan\theta$, to minimize $\varepsilon = t - \sqrt{\lambda}X$ is just to let

$$\frac{d\varepsilon}{d\beta} = \int_{Z_S}^{Z_R} \frac{d}{d\beta}\left[\frac{\left(1+\beta^2\right)^{1/2}}{v(z)} - \sqrt{\lambda}\beta\right] dz = 0 \quad , \qquad (1\text{-}3)$$

which results in

$$\frac{\tan\theta}{v(z)\sqrt{1+\tan^2\theta}} - \sqrt{\lambda} = 0 \quad , \qquad (1\text{-}4)$$

or

$$\frac{\sin\theta}{v(z)} = \sqrt{\lambda} = P \quad . \qquad (1\text{-}5)$$

We can see that this is the well-known Snell's law which governs the propagation direction of a wave in the medium.

In order to relate the traveltime t and offset X, equations (1-1) and (1-2) need to be expanded in series. The following formula is used to do so:

$$(1 - x)^{-1/2} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \ldots, \qquad |x|<1 \tag{1-6}$$

It is always true that $\sin^2\theta < 1$, except $\theta = \pi/2$. So

$$\begin{aligned}\left(1+\tan^2\theta\right)^{1/2} &= \left(1-\sin^2\theta\right)^{-1/2}\\ &= 1 + \frac{1}{2}\sin^2\theta + \frac{3}{8}\sin^4\theta + \ldots\\ &= 1 + \frac{1}{2}v(z)^2P^2 + \frac{3}{8}v(z)^4P^4 + \ldots\end{aligned} \tag{1-7}$$

$$\begin{aligned}\tan\theta &= \sin\theta\left(1-\sin^2\theta\right)^{-1/2}\\ &= \sin\theta + \frac{1}{2}\sin^3\theta + \frac{3}{8}\sin^5\theta + \ldots\\ &= v(z)P + \frac{1}{2}v(z)^3P^3 + \frac{3}{8}v(z)^5P^5 + \ldots\end{aligned} \tag{1-8}$$

Now equations (1-1) and (1-2) can be expressed as

$$t = \int_{z_S}^{z_R} \frac{dz}{v(z)} + \frac{P^2}{2}\int_{z_S}^{z_R} v(z)dz + \frac{3}{8}P^4\int_{z_S}^{z_R} v(z)^3dz + \ldots \tag{1-9}$$

$$X = P\int_{z_S}^{z_R} v(z)dz + \frac{1}{2}P^3\int_{z_S}^{z_R} v(z)^3dz + \frac{3}{8}P^5\int_{z_S}^{z_R} v(z)^5dz + \ldots \tag{1-10}$$

Suppose that

$$t^2 = c_0 + c_1X^2 + c_2X^4 + \ldots \tag{1-11}$$

where the parameters c_0, c_1, c_2,..., are to be determined(Mu, 1981). Comparing equations (1-9) and (1-10) with (1-11) and dropping the higher-order terms, gives

$$c_0 = \left(\int_{z_S}^{z_R} \frac{dz}{v(z)}\right)^2, \tag{1-12}$$

and

$$c_1 = \frac{\int_{z_S}^{z_R} \frac{dz}{v(z)}}{\int_{z_S}^{z_R} v(z)dz} \quad . \tag{1-13}$$

Thus we obtain a second-order approximated traveltime equation for a direct arrival:

$$t^2 = \left(\int_{z_S}^{z_R} \frac{dz}{v(z)}\right)^2 + \frac{\int_{z_S}^{z_R} \frac{dz}{v(z)}}{\int_{z_S}^{z_R} v(z)dz} X^2 \quad . \tag{1-14}$$

If a linear relationship of velocity with depth exists, that is,

$$v(z) = v_0 + \eta z \tag{1-15}$$

where v_0 and η are constants, then in equation (1-14),

$$\int_{z_S}^{z_R} \frac{dz}{v(z)} = \int_{z_S}^{z_R} \frac{dz}{v_0 + \eta z}$$

$$= \frac{1}{\eta}\int_{z_S}^{z_R} \frac{d(v_0 + \eta z)}{v_0 + \eta z}$$

$$= \frac{1}{\eta}\ln\left(\frac{v_0 + \eta z_R}{v_0 + \eta z_S}\right) \; ; \tag{1-16}$$

$$\int_{z_S}^{z_R} v(z)dz = \int_{z_S}^{z_R} (v_0 + \eta z)dz$$

$$= v_0 (z_R - z_S) + \frac{\eta}{2}(z_R^2 - z_S^2) \quad , \tag{1-17}$$

so (1-14) becomes

$$t^2 = \left[\frac{1}{\eta}\ln\left(\frac{v_0 + \eta z_R}{v_0 + \eta z_S}\right)\right]^2 + \frac{\frac{1}{\eta}\ln\left(\frac{v_0 + \eta z_R}{v_0 + \eta z_S}\right)}{v_0 (z_R - z_S) + \frac{\eta}{2}(z_R^2 - z_S^2)} X^2 \quad . \tag{1-18}$$

When $z_R = z_S$, the following formula should be used:

$$t = \frac{X}{v_0 + \eta z_R} \quad . \qquad (1\text{-}19)$$