Directional Filtering Using Multicomponent Seismic Arrays

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ABSTRACT

Three-component (3-C) geophone recordings can be used to infer the incidence direction of impinging seismic waves. Single 3-C station records have been analysed, in the past, using a variety of algorithms. Many of these methods are based on least-squares fitting of a line to an assumed polarization ellipsoid. To assist in direction estimation, I propose to use linear, in-line arrays of 3-C geophones. The multiplicity of recordings promises to provide better estimates of arrival direction and thus better enhancement or rejection of given events.

INTRODUCTION

We are interested in the use of three-component (3-C) recordings for a number of reasons: Enhancement of conventional P recordings by rejection of non-P or out-of-plane events, creation of a side-scan (off-line) image, development of various S-wave parameter estimates and sections. All of these processes are dependent upon identification/analysis of the various events recorded on the three channels. There are several characteristics of wave types that can be used for analysis: Event velocity, polarization, frequency-band, amplitude, and traveltime.

Most P-wave energy will reside on the vertical channel. This is usually the case, because the generally low-velocity near surface bends reflection energy toward the vertical. However, other events can be on the vertical channel, sometimes with distressingly high amplitudes: Ground roll (in line and scattered from off-line directions), side swipe (P events from out of the line's vertical plane), and mode-converted S waves can all be culprits. These, perhaps undesirable, events can often be distinguished by their lower velocities and lower frequencies. In principle, they will exhibit different directions of arrival or polarization shapes from vertically incident P waves.

Much of the S-wave energy, either source-generated or caused by conversion of P to S, will be recorded on the horizontal channels. If we are interested in making an S-wave section then we need to enhance S events and suppress undesirable coherencies or noise. Ground roll, shear-wave refractions, large statics, and relatively high-amplitude noise all can compromise the quality of the S-wave decipherability.

Thus the challenge is to develop analysis methods which separate P- or S-wave signals, of given arrival directions, from unwanted events or noise.

METHODS

A great deal of the polarization analysis effort has come from earthquake seismology and nuclear test ban treaty compliance (Flinn, 1965; Montalbetti and Kanasewich, 1970; Magotra et al., 1989; Walck and Chael, 1991). A primary problem in these studies is to determine the location (epicenter or hypocenter) and character of events.

Three-component recording and analysis has been an integral part of this work. Many of the 3-C processing methods are based on the covariance of the three channels with each other (Kanasewich, 1981; Jurkevics, 1988; Menke and Lerner-Lam, 1991).

My interest here is to use linear arrays of 3-C stations to assist in the estimation of event arrival direction. After Magotra et al. (1989), assume that the seismic recordings can be represented by a zero-mean (3x1) vector $\vec{x}$ in v, r, t, (vertical, radial, tangential coordinates. We seek a direction $\vec{u}$ in which the projection y of x has a maximum length. The scalar y is expressed as

$$y = U^T X = (u_1, u_2, u_3) \begin{pmatrix} v \\ r \\ n \end{pmatrix}, \tag{1}$$

where $U^T U = 1$.

Note as X is zero mean then

$$\bar{y} = E[y] = E[U^T X] = 0 \tag{2}$$

$$\sigma_y^2 = E[y^2] = U^T C U, \tag{3}$$

where E is the statistical expectation of the arguments over some time, σ_y^2 is the variance of y.

The matrix C in (3) is the convariance matrix:

$$C = \begin{bmatrix} \sigma_{vv}^2 & \sigma_{vr}^2 & \sigma_{vn}^2 \\ \sigma_{rv}^2 & \sigma_{rr}^2 & \sigma_{rn}^2 \\ \sigma_{nv}^2 & \sigma_{nr}^2 & \sigma_{nn}^2 \end{bmatrix} \tag{3}$$

The problem is to maximize σ_y^2 subject to $U^T U = 1$. A Lagrangian multiplier approach can be used to solve this problem. For multiple stations, we could just find the direction that maximizes the sum of the projection lengths of each station. This would be done by summing the covariances of the individual stations. Thus the final summed covariance matrix would be:

$$C_s = \begin{bmatrix} \Sigma\sigma_{vv}^2 & \Sigma\sigma_{vr}^2 & \Sigma\sigma_{vn}^2 \\ \Sigma\sigma_{rv}^2 & \Sigma\sigma_{rr}^2 & \Sigma\sigma_{rn}^2 \\ \Sigma\sigma_{nv}^2 & \Sigma\sigma_{nv}^2 & \Sigma\sigma_{nn}^2 \end{bmatrix} \tag{4}$$

Now use (4) instead of (3) in the Lagrangian problem to find U. Once U is found then we can project the data onto U or use this incidence arrival knowledge for some other purpose.

Another more sophisticated scheme for analysing 3-C arrays would be to attempt a full modal decomposition of the data. Similar to Dankbaar (1987) and Labonté (1990), we could assume that P and S waves comprise the seismic traces (v, r, n). The P waves are arriving from a direction $\widehat{\theta}_p$ through a layer with velocity V_p , whereas the S waves are described by $\widehat{\theta}_s$ through V_s. The recorded components can then be written as

$$\begin{aligned} v(t) &= P(t)\cdot f_v\left(v_p,\widehat{\theta}_p\right) + S(t)\cdot\ g_v\left(v_s,\widehat{\theta}_s\right) \\ r(t) &= P(t)\cdot f_r\left(v_p,\widehat{\theta}_p\right) + S(t)\cdot\ g_r\left(v_s,\widehat{\theta}_s\right) \\ n(t) &= P(t)\cdot f_n\left(v_p,\widehat{\theta}_p\right) + S(t)\cdot\ g_n\left(v_s,\widehat{\theta}_s\right). \end{aligned} \tag{5}$$

There are many ways to try to solve this problem. We could take a least-squares approach to find the scalar P and S values, then filter each mode according to direction of arrival. It might be better to transform these data into $(f,\ k_x)$ or $(\tau,\ p_x)$ space and design a filter there.

Note that it also might be appropriate to estimate the Rayleigh wave component which would add another term in (5):

$$\begin{aligned} v(t) &= P(t)\cdot f_v\left(v_p,\widehat{\theta}_p\right) + S(t)\cdot\ g_v\left(v_s,\widehat{\theta}_s\right) + R(t)\cdot h_v\left(v_R,\widehat{\theta}_R\right) \\ r(t) &= P(t)\cdot f_v\left(v_p,\widehat{\theta}_p\right) + S(t)\cdot\ g_r\left(v_s,\widehat{\theta}_s\right) + R(t)\cdot h_r\left(v_R,\widehat{\theta}_R\right) \\ n(t) &= P(t)\cdot f_v\left(v_p,\widehat{\theta}_p\right) + S(t)\cdot\ g_n\left(v_s,\widehat{\theta}_s\right) + R(t)\cdot h_n\left(v_R,\widehat{\theta}_R\right). \end{aligned} \tag{6}$$

These equations allow an independent estimate of the Rayleigh wave component. This procedure is likely to be complicated by a number of factors including P and S statics across the array and geophone balancing between channels and stations.

CONCLUSIONS

Three-component recording and analysis promises to improve conventional sections as well as provide shear-wave images. A key factor in realizing these advances is development of directional filters. Several approaches to the design of these filters have been outlined here.

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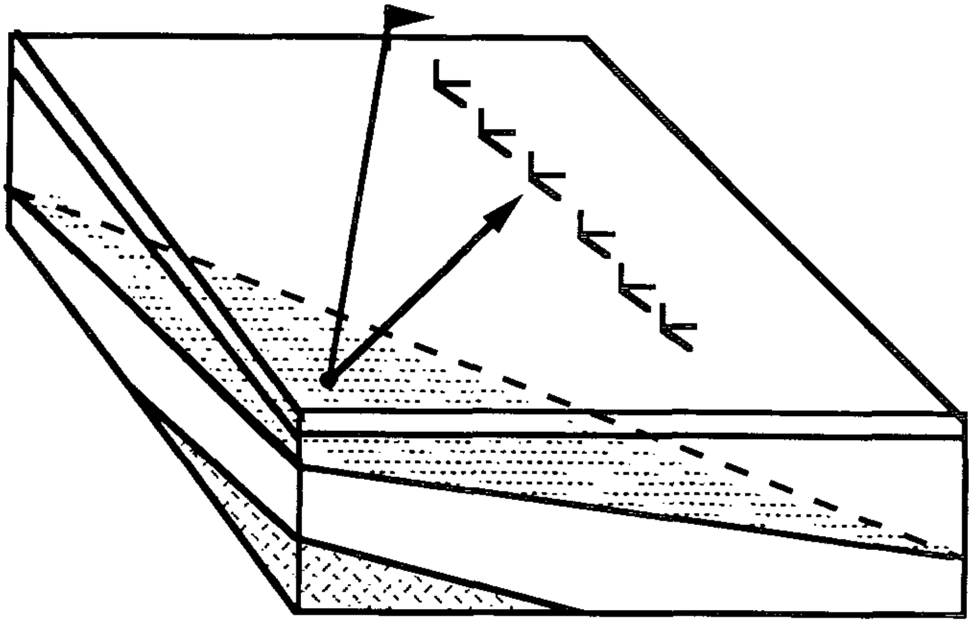

Figure 1. Schematic diagram for a reflection (side swipe) from out of the vertical plane through the shot and receiver.

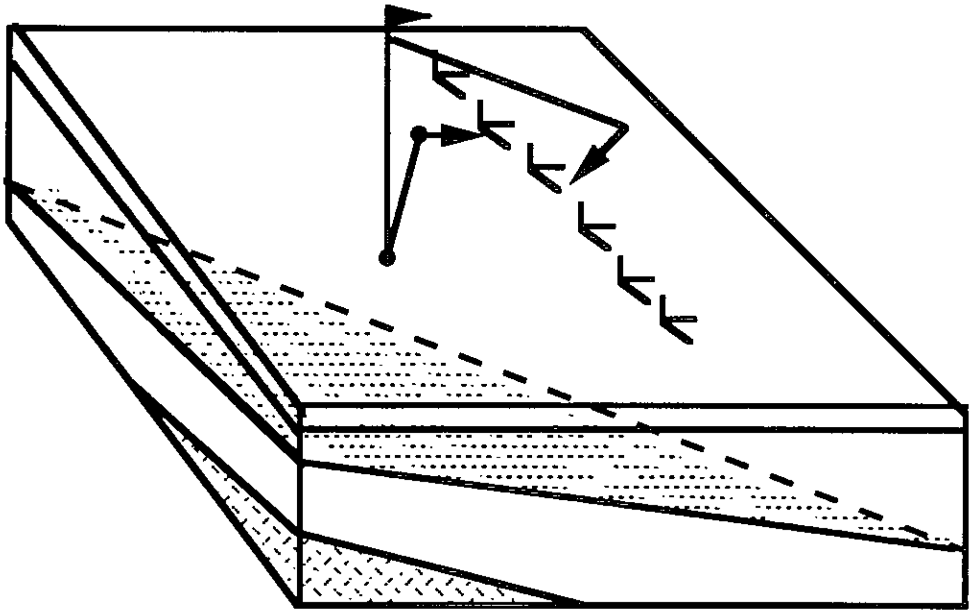

Figure 2. Schematic diagram of off-line noise and signal scattered into the line.