P–SV Wave Modeling In Transversely Isotropic Media

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PROPOSAL

We here propose a method for computing synthetic seismograms based on a direct integration in the ω-k domain for a buried explosive point source with receivers on the free surface. The medium is assumed to be transversely isotropic, vertically inhomogeneous and anelastic with any desired continuous or discontinuous velocity function. The eigenvalue problem studying the wave propagation in stratified anelastic media is formulated as a set of first order ordinary differential equations. The coefficient matrix of the systems thus formed depends on the elastic parameters of the media. Without recourse to the assumption that the coefficient matrix is constant, the displacement components on the free surface can be calculated by integrating directly the differential systems involved using a higher order Runge-Kutta method. In case that the medium is homogemeous, the propagator propagates the wavefields across the homogeneous zones in one step, bypassing the expensive Runge-Kutta integration scheme and reduces the computation time significantly.

The proposed method has the virtue of algorithmic simplicity. With minor modification, the solution for a a vertical point force acting on the free surface (Vibroseis) or a *VSP* configuration can be obtained.

THE PROBLEM AND THE SOLUTIONS

Consider an inhomogeneous and anelastic layered half-space as shown in Figure 1. The interfaces are located at $z = z_j$, $j = 0,1,...,N\text{-}1$, with $z_0 = 0$ corresponding to the bottom interface, N being the number of sublayers. A point-source generating P-waves is activated at an initial time $t = 0$.

The displacement components at the free surface, $z = H$ satisfying the necessary initial and boundary conditions can be expressed, in terms of cylindrical coordinates, as

$$s_z = \frac{V_o}{2\pi^2} \Re e \int_0^\infty G(\omega)\, e^{i\omega t}\, d\omega \int_0^\infty J_0(kr)\, \widetilde{S}_0(\omega,k,z_s)\, dk \tag{1}$$

$$s_r = -\frac{V_o}{2\pi^2} \Re e \int_0^\infty G(\omega)\, e^{i\omega t}\, d\omega \int_0^\infty J_1(kr)\, \widetilde{S}_1(\omega,k,z_s)\, dk \tag{2}$$

where s_z and s_r are the vertical and horizontal (radial) displacements respectively, ω is angular frequency, k, the horizontal wavenumber, J_o and J_1, the ordinary Bessel function of order zero and one, V_o, the volume generated at the source in one second, $G(\omega)$, the Fourier transform of the time-dependence of the source, $g(t)$ being equal to zero for $t < 0$ and $\Re e$ stands for real part of a quantity.

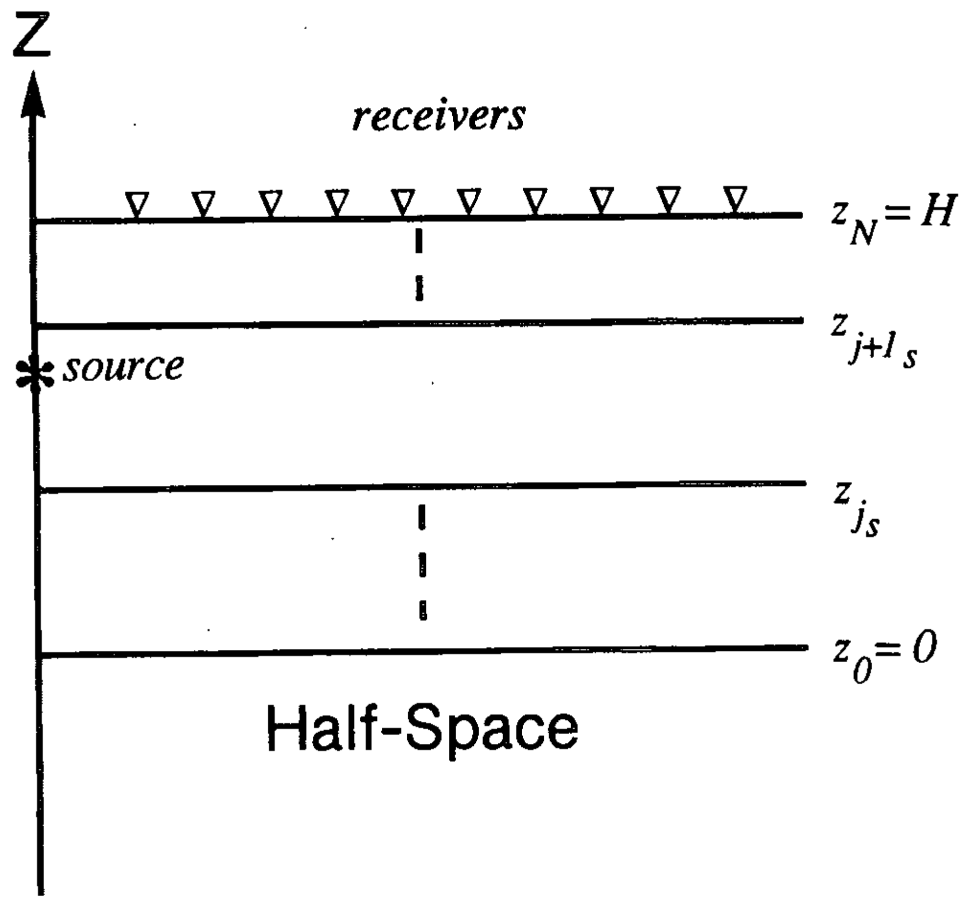

FIG.1. The geometry of the model. The source layer is bounded by z_{j+1s} and z_{js} .

The *reflectivity functions*, (where $n = 0$ is the vertical component and $n=1$ is the horizontal component) which compute the transmission and reflection responses have the following expressions:

$$\widetilde{S}_{n}(\omega,k,z_s)=\begin{cases} k\dfrac{T_{w_s}G\begin{pmatrix}1&4\\1&2\end{pmatrix}-T_{q_s}G\begin{pmatrix}1&3\\1&2\end{pmatrix}}{G\begin{pmatrix}3&4\\1&2\end{pmatrix}}-kw_s & \text{for } n=0\\ k\dfrac{T_{w_s}G\begin{pmatrix}2&4\\1&2\end{pmatrix}-T_{q_s}G\begin{pmatrix}2&3\\1&2\end{pmatrix}}{G\begin{pmatrix}3&4\\1&2\end{pmatrix}}-kq_s & \text{for } n=1\end{cases} \quad (3)$$

The displacement-stress vector of the source $\mathbf{f_s}=(\,w_s\,,\;q_s\,,\;T_{w_s},\;T_{q_s}\,)^{T}$ can be obtained from the level of the source by integrating the fourth-order system:

$$\frac{d\,\mathbf{f_s}}{dz}=\mathbf{A}\mathbf{f_s} \quad (4)$$

where

$$\mathbf{A}=\begin{pmatrix} 0 & k\dfrac{C_{13}}{C_{33}} & \dfrac{1}{C_{33}} & 0\\ -k & 0 & 0 & \dfrac{1}{C_{55}}\\ -\rho\omega^2 & 0 & 0 & k\\ 0 & X & -k\dfrac{C_{13}}{C_{33}} & 0 \end{pmatrix} \quad (5)$$

and

$$X=-\rho\,\omega^2+k^2\left(C_{11}-\frac{C_{13}^{\,2}}{C_{33}}\right) \quad (6)$$

with the initial values for the system being:

$$\mathbf{f}_s(z)=\frac{1}{2K_\alpha}\begin{pmatrix} -K_\alpha\\ k\\ (\lambda+2\mu)\,K_\alpha^2-\lambda k^2\\ -2\mu kK_\alpha \end{pmatrix} \quad (7)$$

and

$$K_\alpha = \begin{cases} \sqrt{k^2 - \dfrac{\omega^2}{\alpha^2}} & \text{if } k > \left|\dfrac{\omega}{\alpha}\right| \\ i\sqrt{\dfrac{\omega^2}{\alpha^2} - k^2} & \text{if } k < \left|\dfrac{\omega}{\alpha}\right| \end{cases} . \tag{8}$$

The values of the elastic parameters and density for equations (7)–(8) are those corresponding to the source.

The second-order minors can be obtained directly by integrating numerically the sixth-order differential system from the bottom interface to the free surface, $z = H$:

$$\frac{d\mathbf{G}}{dz} = \Omega\mathbf{G} \tag{9}$$

with

$$\mathbf{G} = \left\{ G\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, G\begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}, G\begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}, G\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}, G\begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}, G\begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} \right\}^T , \tag{10}$$

$$\Omega = \begin{bmatrix} 0 & 0 & \dfrac{1}{C_{55}} & -\dfrac{1}{C_{33}} & 0 & 0 \\ 0 & 0 & k & k\dfrac{C_{13}}{C_{33}} & 0 & 0 \\ X & -k\dfrac{C_{13}}{C_{33}} & 0 & 0 & k\dfrac{C_{13}}{C_{33}} & \dfrac{1}{C_{33}} \\ \rho\omega^2 & -k & 0 & 0 & k & -\dfrac{1}{C_{55}} \\ 0 & 0 & -k & -k\dfrac{C_{13}}{C_{33}} & 0 & 0 \\ 0 & 0 & -\rho\omega^2 & -X & 0 & 0 \end{bmatrix} , \tag{11}$$

and X being given by (6). The initial values at $z = 0$ are :

$$\mathbf{G}_{z=0} = \begin{bmatrix} K_\alpha K_\beta - k^2 \\ k\left\{ 2\mu_o K_\alpha K_\beta - (2\mu_o k^2 - \rho_o \omega^2) \right\} \\ -\omega^2 \rho_o K_\alpha \\ \omega^2 \rho_o K_\beta \\ -k\left\{ 2\mu_o K_\alpha K_\beta - (2\mu_o k^2 - \rho_o \omega^2) \right\} \\ \left(2\mu_o k^2 - \rho_o \omega^2 \right)^2 - 4k^2 \mu_o^2 K_\alpha K_\beta \end{bmatrix} \tag{12}$$

where λ, μ, ρ as well as the P- and S-wave velocities are those of the homogeneous half-space, the square roots for K_α is defined by (8) and K_β is defined correspondingly with P-wave velocity, α replaced by S-wave velocity, β.