Polarization filter: Design and testing

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ABSTRACT

A polarization filter is designed to process three-component seismic data. The design is based on covariance matrix analysis. Rectilinearly polarized waves are enhanced and non-rectilinearly polarized waves are attenuated. This filter has been successfully applied to field data to reduce random noise and groundroll. After applying the polarization filter, different types of waves (such as P, SV and SH wave) can be distinguished. The direction of an impinging wave may be determined if directional analysis is applied to the polarization-filtered data. However, because spherically polarized noise also affects the trajectory direction of the data, directional filtering after polarization filtering should be used very carefully. Only in some circumstances, can directional filtering get good results. If the signal-to-noise ratio is low on the original data, an erroneous result may occur. This technique may be helpful for shear wave and converted wave exploration.

INTRODUCTION

P-waves and S-waves have been used effectively in exploration seismology. Both waves are linearly polarized (Kanasewich, 1981). The trajectory of particle motion of a P-wave is along the direction of the wave propagation. And the trajectory caused by S-wave is perpendicular to the direction of the wave propagation. Meanwhile, a seismic source may generate other kinds of waves, for example, ground roll. Ground roll is one type of Rayleigh wave that arises because of the coupling of compressional waves and shear waves (SV) that propagate along the free surface (Yilmaz, 1987). Ground roll is elliptically polarized. Moreover, seismic recordings are often contaminated by noise which makes the detection and interpretation of small seismic events difficult (Kanasewich, 1981). This noise can be divided into mechanical and electrical noise. The former includes the vibration of equipment and vehicles. The latter can be caused by power lines and other electronic instruments. Fortunately, many types of noise have a low degree of linear polarization. The polarization difference between the signal and noise provides the possibility of using a polarization filtering technique to separate them.

If there are three-component seismic recordings, a polarization filter can be designed to enhance the signal to noise ratio by taking advantage of the polarization properties. This is another approach to improve the quality of seismic recording. A polarization filter can be designed to enhance rectilinearly polarized waves and to attenuate other energy. Or, if we are interested in surface waves, a polarization filter also can be designed to enhance this type of wave and to reject body waves.

A polarization filter may under some circumstances be also used for detecting the direction of impinging seismic waves . It is sometimes important to distinguish in-line energy and off-line energy. The existence of the off-line energy (French, 1974; Hospers, 1985) may result in the misinterpretation of the seismic data using conventional analysis. Moreover, it is not always true that the seismic line is just over the geology we are interested in. Therefore, the off-line energy may be of interest. Three-component recording provides the possibility of using off-line energy to construct images not directly below the line (Ebrom et al, 1989; Stewart, 1991).

It is necessary to enhance the rectilinear waves before applying a directional filter to them as non-rectilinearly polarized waves will contaminate the accuracy of the estimate.

Additionally, the polarization filter can be used to separate P-waves and S-waves, or to enhance Rayleigh waves. Generally, if the two type of waves have different polarization properties, an appropriate polarization filter can be designed to separate them or to enhance one and to attenuate the another.

METHOD

Based on the matrix analysis of polarization described by Flinn (1965), Kanasewich (1981) and Jurkevics (1988), a polarization filter is presented in the following discussion.

We express the observed seismic data as being composed of rectilinearly polarized signal and noise as below (Bataille, 1991):

$$\overline{V}(t) = u(t)\,\overline{p} + \overline{n}(t) \qquad (1)$$

here, $\overline{V}(t) = [V_1(t), V_2(t), V_3(t)]^T$ is the seismic data vector recorded at a three-component receiver. $u(t)$ is a scalar series of time t. $\overline{p}$ is a unit vector, indicating the polarized direction of wave $u(t)$. $\overline{n}(t) = [n_1(t), n_2(t), n_3(t)]^T$ is the noise. T refers to the transpose. Indices 1 to 3 refer to the Cartesian coordinates in the order of vertical, radial and transverse, respectively.

I assume that the signal $u(t)\overline{p}$ and the noise $\overline{n}(t) = [n_1(t), n_2(t), n_3(t)]^T$ are not correlated and that the expectation value of the noise is zero, that is:

$$\Phi_{un_i} = \int_{t_1}^{t_2} u(t) n_i(\tau + t)\, dt = 0$$

$$\frac{1}{t_1 - t_2} \int_{t_1}^{t_2} n_i^2(t)\, dt = 0 \qquad (2)$$

where t_1, t_2 are the ends of the time window.

With these assumptions, we can construct the covariance matrix of the observed data over a time window $M\Delta t$, where M is the width of the window and Δt is the sampling interval.

$$\mathbf{S} = U\bar{p}\,\bar{p}^T + \mathbf{N} \tag{3}$$

S is described as below:

$$\mathbf{S} = \begin{bmatrix} Var(1,1) & Cov(1,2) & Cov(1,3) \\ Cov(2,1) & Var(2,2) & Cov(2,3) \\ Cov(3,1) & Cov(3,2) & Var(3,3) \end{bmatrix} \tag{4}$$

here $Var(j,j) = \frac{1}{N}\sum_{i=1}^{N} (V_j(t_i) - v_j)^2$,

$Cov(j,k) = \frac{1}{N}\sum_{i=1}^{N} (V_j(t_i) - v_j)(V_k(t_i) - v_k)$,

v_j is the expected value of $V_j(t)$.

$$U = \frac{1}{N}\sum_{i=1}^{N} (u(t_i) - \mu)^2$$

$$\mu = \frac{1}{N}\sum_{i=1}^{N} u(t_i)$$

$$N_{ij} = \frac{1}{N}\sum_{k=1}^{N} (n_i(t_k) - \mu_i)(n_j(t_k) - \mu_j)$$

$$\mu_i = \frac{1}{N}\sum_{k=1}^{N} n_i(t_k)$$

Now we try to get the direction vector from equation (3). Because $\bar{p}$ is a unit vector, $\bar{p}^T\,\bar{p} = 1$ we can rewrite (3) as:

$$(\mathbf{S} - \mathbf{N})\cdot\bar{p} = U\cdot\bar{p} \tag{5}$$

This is a eigenvalue problem. u is a eigenvalue of matrix (**S** - **N**) and $\bar{p}$ is the associated eigenvector. Because we do not know the noise **N** and only **S** can be calculated. The thing we can do is to use **S** to substitute (**S** - **N**) and assume the eigenvalue and eigenvector of **S** are close to those of (**S** - **N**). If the signal-to-noise ratio is not too bad (say 3 or 5), this approximation will be valid. Generally, even if the noise is spherically polarized (isotropic), the direction of the eigenvector of **S** is not equal to that of (**S** - **N**). We will discuss this problem later.

Under some circumstances, we can still use the eigenvalues and the principal eigenvector to process the signal contaminated by the spherically polarized noise for separating the signal and noise and approximately determining the polarization direction of the signal. Some examples of synthetic data will be given later.

Supposing λ_1, λ_2, λ_3 are the three eigenvalues of **S** and that there is following relation between the eigenvalues:

$$\lambda_1 \geq \lambda_2 \geq \lambda_3$$

and $\overline{e_1}$ is the eigenvector associated with λ_1.

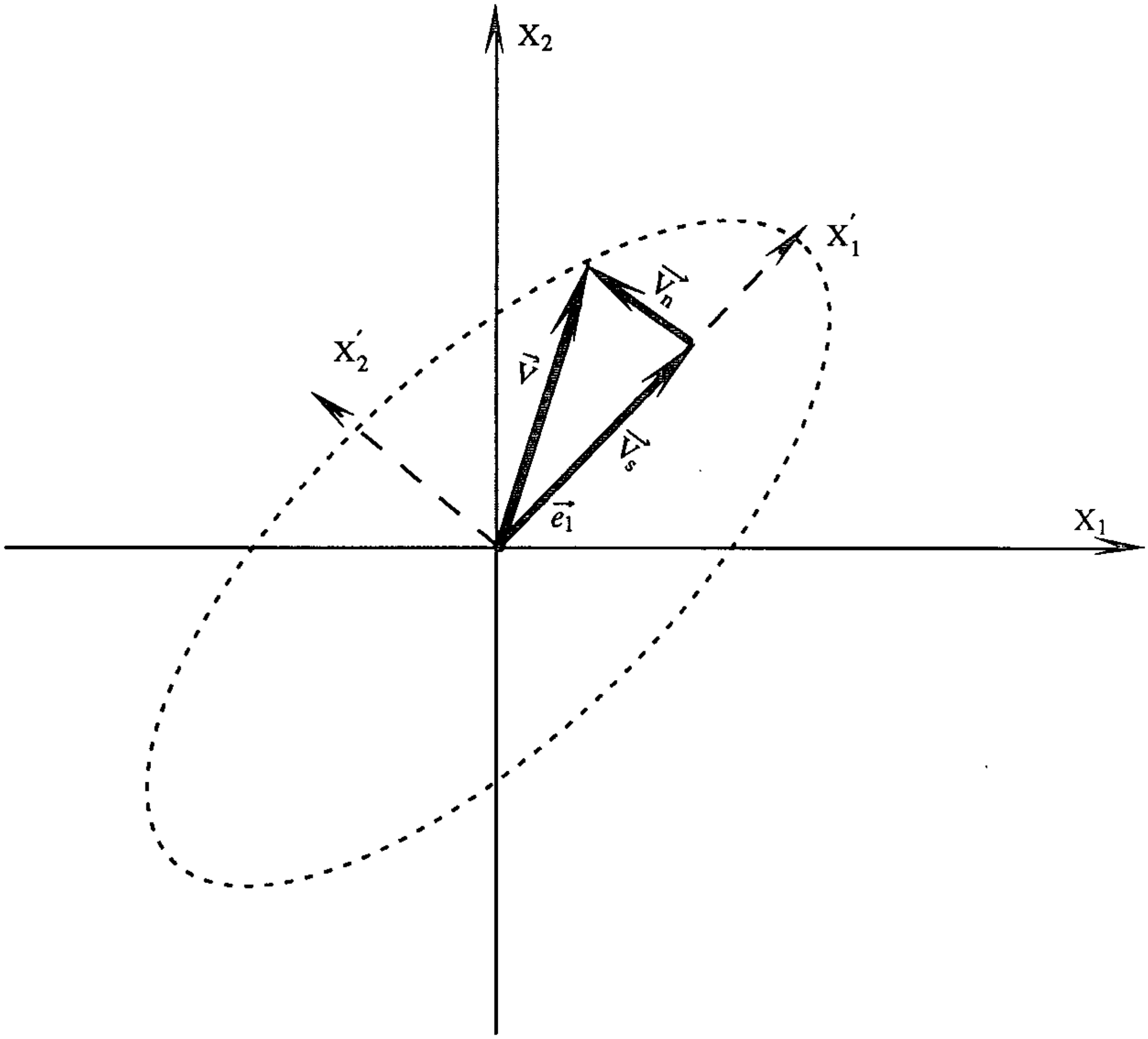

FIG. 1: Decomposition of a displacement vector into rectilinear and non-rectilinear components

Figure 1 shows the relationship among the direction of the eigenvector $\overline{e_1}$, trajectory of the particle over the time window $N\Delta t$ and the vector $\overline{V}$ of displacement at the mid-point of the time window. $\overline{V}$ can be divided into two vectors, $\overline{V_s}$ and $\overline{V_n}$. We assume that $\overline{V_s}$ is caused by signal and $\overline{V_n}$ is caused by noise.

We can define the first filter factor:

$$G_1 = 1 - (\lambda_2 + \lambda_3)/(2\lambda_1) \qquad (6)$$

G_1, varying between 0 and 1, indicates the rectilinearity of the particle trajectory. If G_1 is close to 1, the particle motion is rectilinear polarized in very high degree. If G_1 is close to 0, it means the wave has very low degree of rectilinearity. So G_1 can be thought as the rectilinearity of the data. This is a scalar factor applied same to all three components to reduce the amplitude caused by noise from the observed data.

The second factor is a vector factor, defined as:

$$\overline{G_2} = (\overline{V} \cdot \overline{e_1})\, \overline{e_1} \qquad (7)$$

The function of $\overline{G_2}$ is to reject the noise $\overline{V_n}$ by projecting the displacement vector $\overline{V}$ to the direction of eigenvector $\overline{e_1}$, which we consider is the polarization direction of the rectilinear signal.

We apply the factors from the whole window at the mid-point of the time window. Finally, the filtered data at the mid-point of the time window, respect to the original coordinate system, is:

$$\overline{S'} = G_1\overline{G_2} = [1 - (\lambda_2 + \lambda_3)/(2\lambda_1)](\overline{S} \cdot \overline{e_1})\overline{e_1} \qquad (8)$$

Then the window is moved with one time point, and the above procedure is repeated. After the window is moved from the beginning to end of the recording, the polarization filter is completed. This filter is a point-by-point non-linear filter.

DIRECTION ERROR FROM SPHERICALLY POLARIZED NOISE

Generally, when a rectilinearly polarized signal is contaminated by noise, even if the noise is a spherically polarized, its polarization direction will be changed. Let's discuss this in 2-dimensions.

After Gal'perin, suppose we have a rectilinear signal

$$\overline{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} A\sin\omega t \\ B\sin\omega t \end{pmatrix} \qquad (9)$$

with noise

$$\overline{Y} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} C\sin(\omega t+\beta) \\ D\sin(\omega t+\gamma) \end{pmatrix} \qquad (10)$$

The observed data will be

$$\overline{Z} = \overline{X} + \overline{Y} = \begin{pmatrix} A\sin\omega t + C\sin(\omega t+\beta) \\ B\sin\omega t + D\sin(\omega t+\gamma) \end{pmatrix} \qquad (11)$$

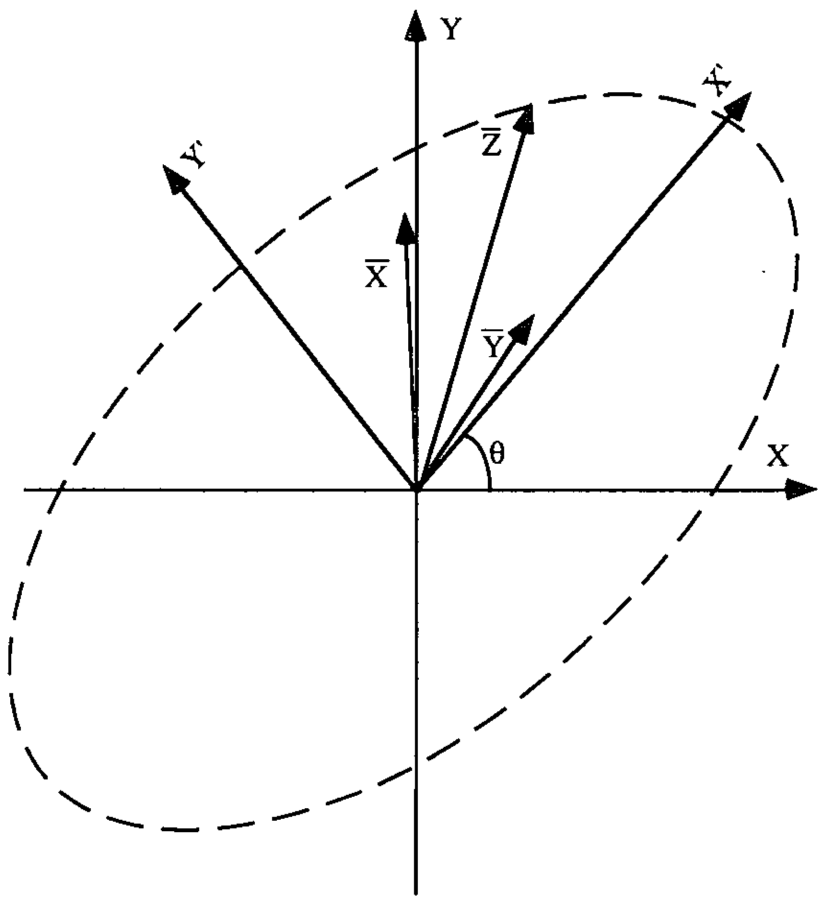

FIG. 2. Projection of signals to the coordinates correspondendt to the axis of the ellipse of the particle motion trajectary.

The trajectory of $\overline{Z}$ will be an ellipse. Supposing X' and Y' are the axes of the ellipsoid (Figure 2). Projection of $\overline{Z}$ on X' and Y' are:

$$C_\eta \sin(\omega t+\eta) = A\cos\theta\sin\omega t + C\cos\theta\sin(\omega t+\beta) + B\sin\theta\sin\omega t + D\sin\theta\sin(\omega t+\gamma)$$

$C_\xi \sin(\omega t+\xi) = -A\sin\theta\sin\omega t - C\sin\theta\sin(\omega t+\beta) + B\cos\theta\sin\omega t + D\cos\theta\sin(\omega t+\gamma)$

where C_η is the amplitude of the component of $\overline{Z}$ on the axis of X',

C_ξ is the amplitude of the component of $\overline{Z}$ on the axis of Y',

θ is the angle between the axes of x and Y'.

When $\omega t = 0$:

$$C_\eta \sin\eta = C\cos\theta\sin\beta + D\sin\theta\sin\gamma$$

$$C_\xi \sin\eta = -C\sin\theta\sin\beta + D\cos\theta\sin\gamma$$

When $\omega t = \frac{\pi}{2}$:

$$C_\eta \cos\eta = A\cos\theta + C\cos\theta\cos\beta + B\sin\theta + D\sin\theta\cos\gamma$$

$$C_\xi \cos\xi = -A\sin\theta - C\sin\theta\cos\beta + B\cos\theta + D\cos\theta\cos\gamma$$

So we get:

$$\tan\eta = \frac{C\cos\theta\sin\beta + D\sin\theta\sin\gamma}{A'\cos\theta + B'\sin\theta}$$

$$\cot\xi = \frac{A'\sin\theta - B'\cos\theta}{C\sin\theta\sin\beta - D\cos\theta\sin\gamma} \qquad (12)$$

where, $A' = A + C\cos\beta$

$B' = B + D\cos\gamma$

Because X' and Y' are on the axes of the trajectory ellipsoid, so the phase difference of the two components on X' and Y' is $\frac{\pi}{2}$, that is:

$$\tan\eta = -\cot\xi \qquad (13)$$

Substituting (11) to (12) and simplifying the equation (12), we get:

$$\tan 2\theta = 2\frac{A'B' + CD\sin\beta\sin\gamma}{A'^2 - B'^2 + C^2\sin^2\beta - D^2\sin^2\gamma} \qquad (14)$$

Right now, the direction of the axes of the ellipsoid has been found.

The direction of the trajectory for signal $\overline{X}$ is:

$$\tan\theta' = \frac{B}{A}$$

or $$\tan 2\theta' = 2\frac{AB}{A^2 - B^2} \quad (15)$$

Usually, $\theta' \neq \theta$.

Let's discuss two specific cases.

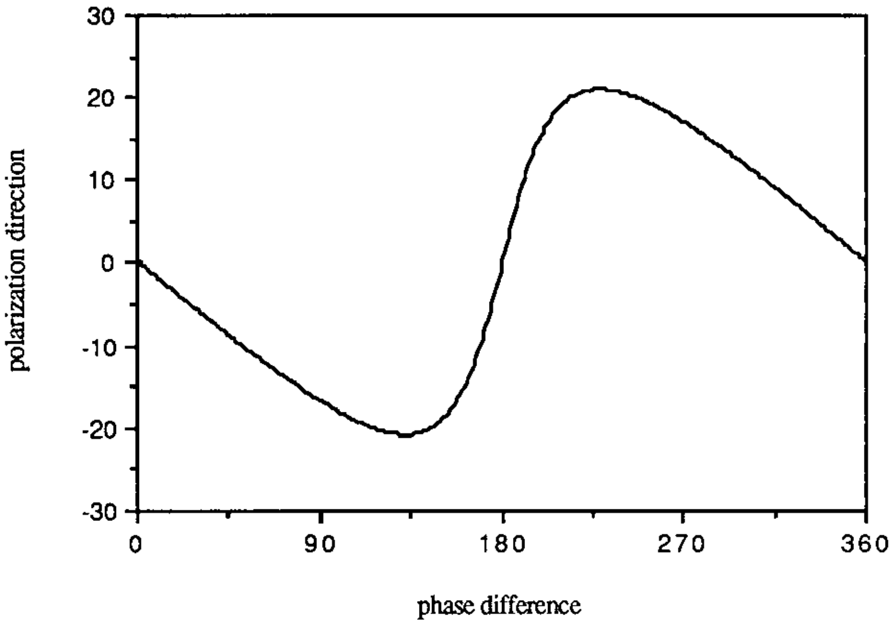

FIG. 3. Direction perturbation for a normal incident wave, S/N = 3.

CASE I: when $\beta = \gamma = 0$, the noise $\overline{Y}$ is another rectilinear wave.

$$\tan 2\theta = 2\frac{(A+C)(B+D)}{(A+C)^2 - (B+D)^2} = \frac{2\frac{B+D}{A+C}}{1 - (\frac{B+D}{A+C})^2} = \frac{2\tan\theta}{1 - \tan^2\theta}$$

So: $$\tan\theta = \frac{B+D}{A+C} \quad (16)$$

When A>>C and B>>D, or $\frac{A}{B} = \frac{C}{D}$, $\theta = \theta'$. This means, when the signal-to-noise ratio is large or the noise has the same direction as the signal, we can get the accurate direction of signal $\overline{X}$ from the observed data $\overline{Z}$.

CASE II: when C=D=1, $\beta-\gamma = \frac{\pi}{2}$, the noise $\overline{Y}$ is a spherically polarized wave.

$$\tan 2\theta = 2\frac{AB - A\sin\beta + B\cos\beta}{A^2 - B^2 + 2A\cos\beta + 2B\sin\beta} \tag{17}$$

The direction varies with the phase difference β.

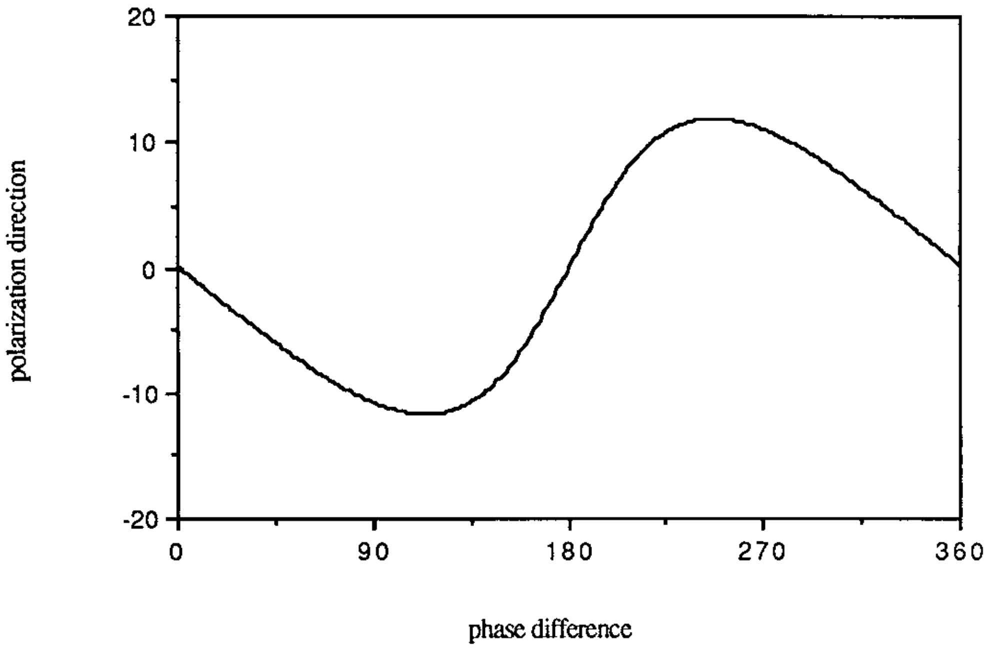

FIG. 4. Direction perturbation for a normal incident wave, S/N = 5

We calculated the direction perturbation for waves from the different directions and with different signal-to-noise ratios.

Figures 3 and 4 show the direction of circularly polarized noise perturbation for waves of incident angle of zero (normal incident), but with different signal-to-noise ratios, 3 and 5, respectively. Only when β is 0 or π, we can get the exact direction of the rectilinear signal $\overline{X}$ from the observed data. The maximum perturbation is 20° for the signal-to-noise ratio of 3, and 12° for 5.

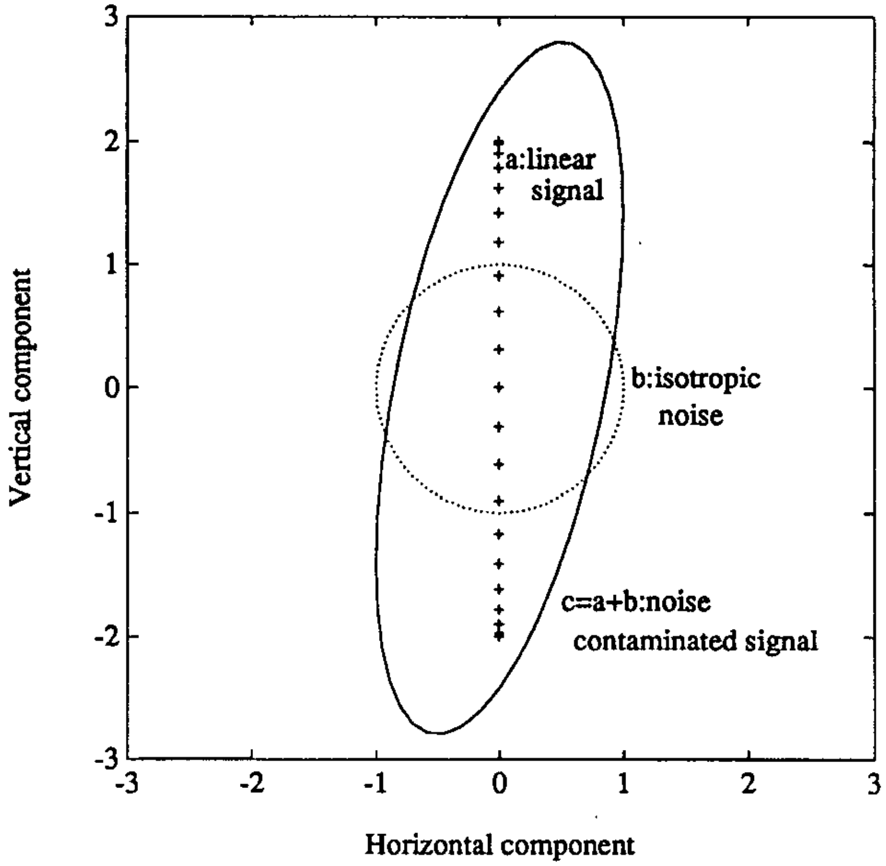

FIG. 5. When a rectilinear wave contaminated by a circularly polarized noise, the direction of the major axis of the trajectory is usually not as same as the direction of the rectilinear wave.

In summary, if the observed data contain noise, regardless of whether it is rectilinear or non-rectilinear, the predominant direction from covariance matrix is not the same as the direction of trajectory of the rectilinear signal in the data, unless the signal-to-noise ratio is so high that the noise is negligible, or the noise is a spherically polarized with certain specific phase differences.

Synthetic data are used to verify the above result (Figure 5). The signal and noise have the same frequency of 30 Hz. The signal-to-noise ratio is 2. The signal is a normally incident P wave. The noise is a circularly polarized with the phase difference (β) of 45° from the signal on the vertical component. The trajectory of the combined data of the signal and noise is an ellipsoid. There is an angle of 10° between the major axis of the ellipsoid and the direction of the signal.

SYNTHETIC DATA

Some different types of synthetic data are used to verify the polarization filter. Because we know every thing about the data, we can know how effective the filter is, from the filtered data. For simplicity, here we just discuss 2-D data.

The Ricker wavelet (Sheriff, 1984) is used to simulate the real seismic wavelet. It is a zero-phase wavelet and very similar to the wavelet of vibrator. The frequency of the wavelet is 60 Hz and three wavelets are coming consequently with the amplitude of 1, 0.5, and 0.8 respectively. The wavelets are centered at the time of 320, 370 and 450 ms, respectively. Some noise is added to the signal and the polarization filter is applied to the noise contaminated signal to see how the filter works.

Figure 6 shows random noise with the amplitude of 0.3 is added to the signal. (a) is the pure signal and (b) is the noise contaminated signal. It is hard to distinguish the second wavelet in (b), because of the noise. After the polarization filter applied to the data of (b), the signal-to-noise ratio is improved obviously and we can see the signal clearly. Same data is input to a bandpass filter (40 - 100 Hz) and the result is shown in Figure 7. It is sure that the result of polarization filter is better than bandpass filter on this case, because on the bandpass filtered data, the first wavelet and the second are joined together. And some noise is remained on the bandpass filtered data, since the random noise has pretty wide frequency range and some of the frequency component are within the band passed through the filter.

Let's test the most important property of the polarization filter. The signal is the same as that in Figure 6 (a), but the circularly polarized with the same frequency as that of the signal is added to the data (Figure 8 (a)). At the same time we although added 200 Hz circularly polarized noise in the data. The polarization filter works excellent on this case (Figure 8(c)). The output is almost same as the input(Figure 8 (a)). The bandpass filter (40 - 100 Hz) is also applied to the data. It is expected that only the 200 Hz circularly polarized noise is eliminated and the 60 Hz noise is still on the traces (Figure 9).

FIELD DATA

Field data are used to verify the polarization filter. The three-component data are from Rumsey area, Alberta, provided by Gulf Canada Resources Inc. The seismic data are collected on two lines in the Rumsey region of Alberta. Both lines run north to south. On line 1, there are 41 shot-points and 157 receiving points. On line 2, there are 46 shot-points and 157 receiving points. The data of the all shots were collected at each station for each line. Two kilograms of dynamite were used for each shot. The depth of most shots is 18 m. All receivers were three-component geophones.

The lines lie on the central Alberta plain over preserved Devonian sediments. During the Devonian period, this area suffered many transgressive-regressive pulses correlating to the sea level change. As relative sea level rises, there is some token mound-building and active reef building. The existence of these reefs provides good scatterers for seismic wave as well as potential structures for oil accumulation.

On the raw data, there is some strong random noise on some traces and strong groundroll on all records (Figures 10 - 11). Rayleigh waves (groundroll) are elliptically polarized. Because of this difference from P- and S-waves, we can apply the polarization filter to separate these two kinds of waves. We expect to find reflections within the groundroll contaminated area.

Figures 12 - 13 show the output of the filter from the raw data of figures 9 - 10. We can see that the random noise and the groundroll is reduced considerably. Within the groundroll area, some reflections can be seen. And reflections within the whole record have been enhanced.

CONCLUSION

Polarization filtering is an effective technique for three-component data for separating the waves with different polarization properties and enhancing the signal-to-noise ratio. Especially, when the noise has a similar frequency as the signal, a polarization filter can still be used. The polarization filtering can separate different kinds of waves and with a directional filter can determine the direction of impinging waves.

Because the polarization direction is changed by noise, even circularly polarized noise, it is necessary to check the original data before applying the polarization filter and directional filter. The conventional technique is recommended to be applied to enhance signal-to-noise ratio before polarization filter is applied. If the signal-to-noise ratio is too low, the result of directional filter will be useless. So when we use the polarization and directional filter, we should be careful.

ACKNOWLEDGMENTS

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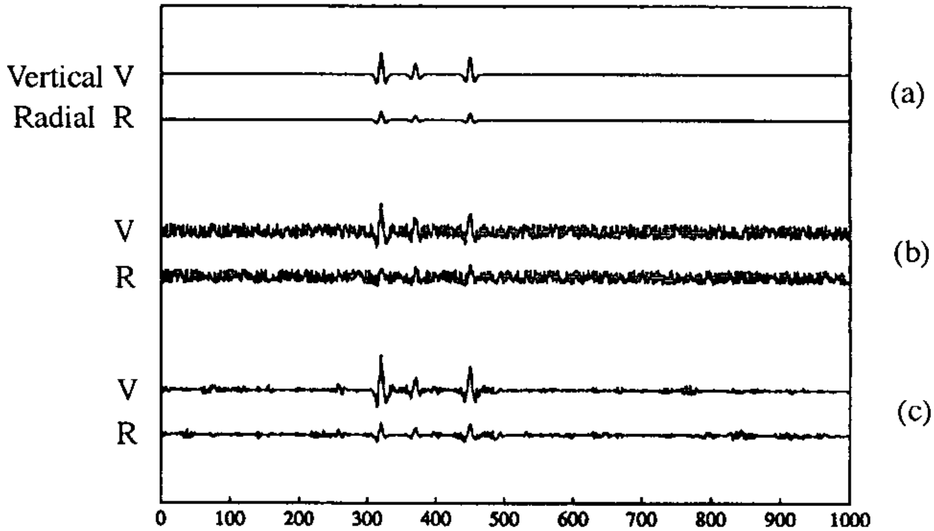

FIG. 6. (a) Pure signal of three wavelets centered at time 320, 370 and 420 ms with amplitude of 1.0, 0.5 and 0.8, respectively. (b) Random noise with amplitude of 0.3 is added to the signal. (c) Polarization filtered data. The S/N ratio is improved.

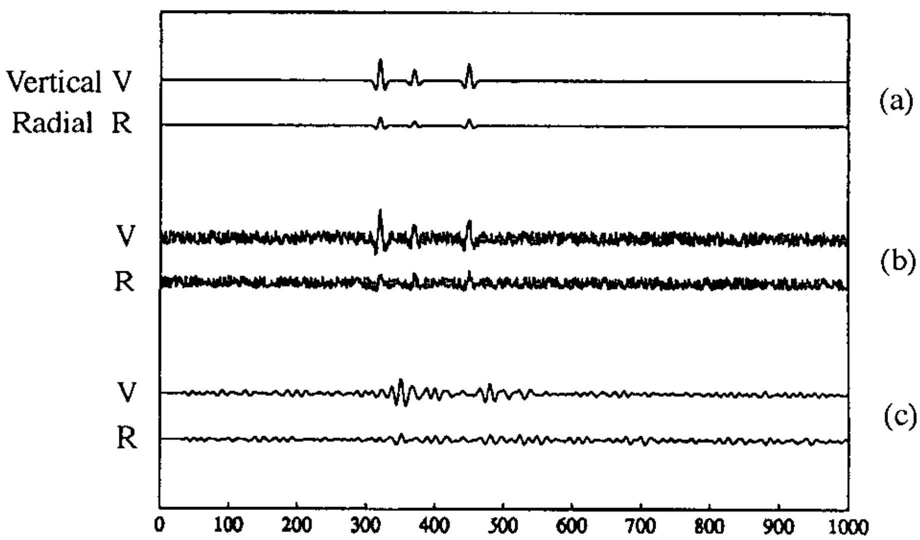

FIG. 7. (a) and (b) are the same as (a) and (b) of FIG. 6.
(c) Bandpass filtered data. There is still some noise left and the wavelets are distorted.

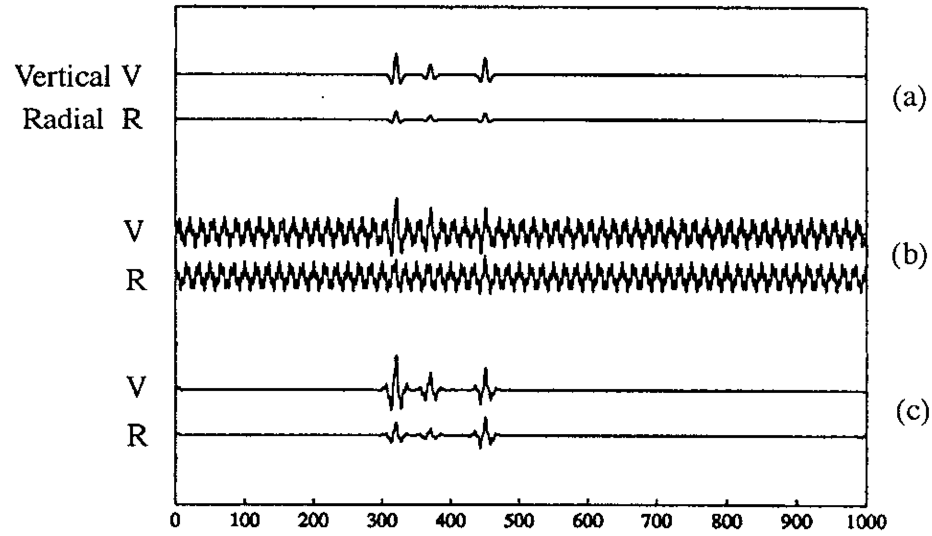

FIG. 8. (a) Same as (a) of FIG. 6. (b) 60 and 200 Hz circularly polarized noise with the amplitude of 0.4 and 0.3, respectively, are added to the pure signal. (c) Output of the polarization filter is almost exactly same as the pure signal.

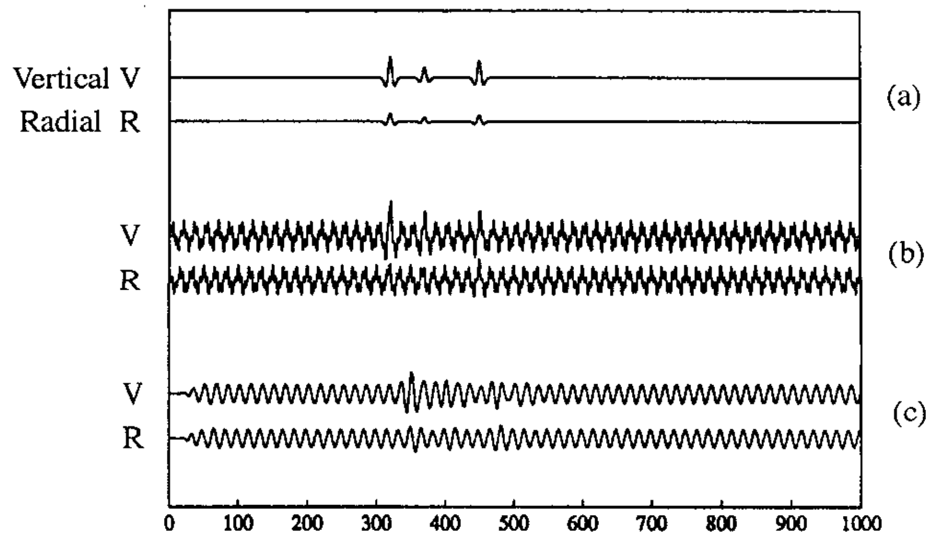

FIG. 9. (a) and (b) are the same as the (a) and (b) of FIG. 8. (c) Bandpass filtered data of (b). The 60 Hz noise is left at all. It is hard to distinguish signal from (c).

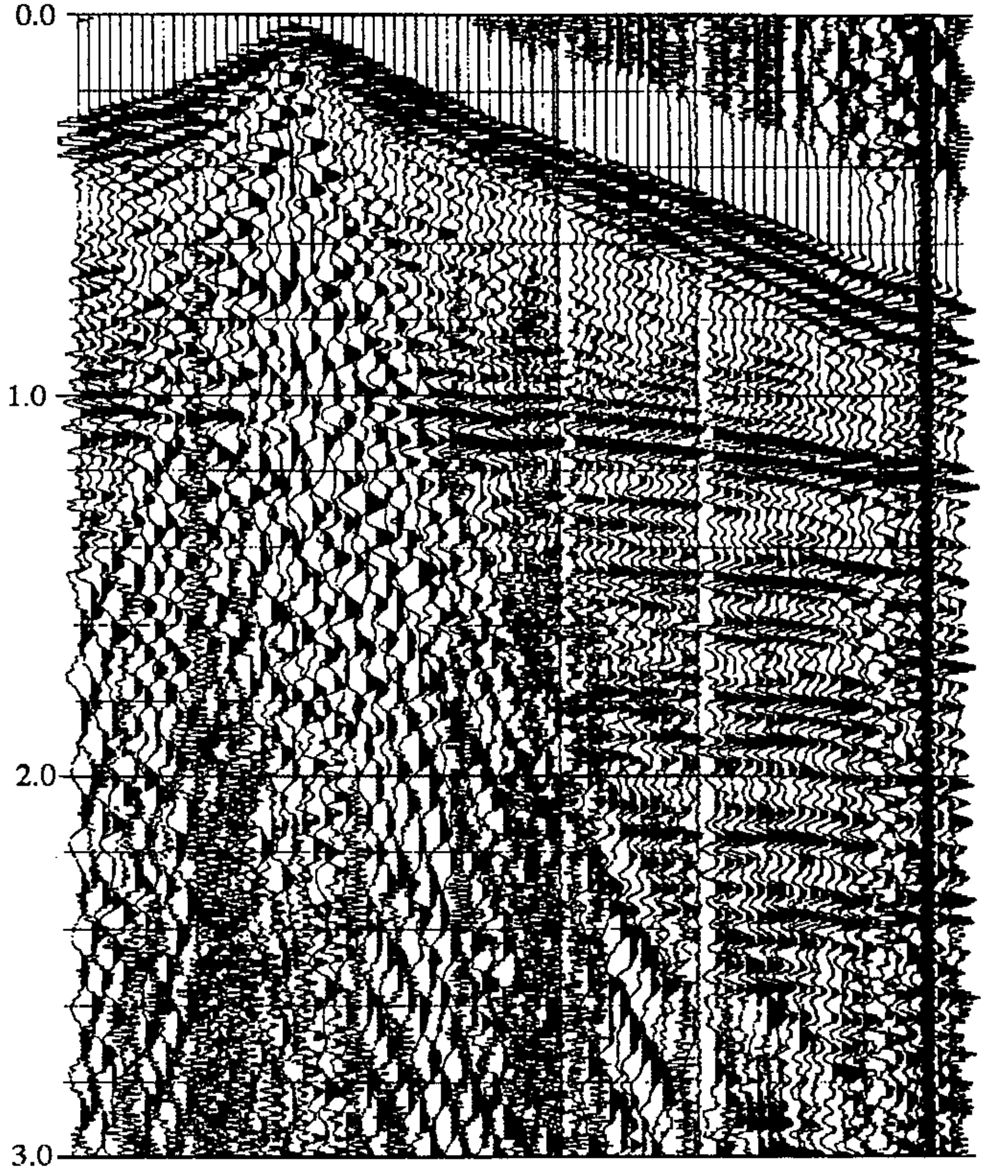

FIG. 10. Vertical component data of a common-shot gather from Rumsey, Alberta, provided by Gulf Canada Resources Inc. There is some random noise on some traces and groundroll is quite strong.

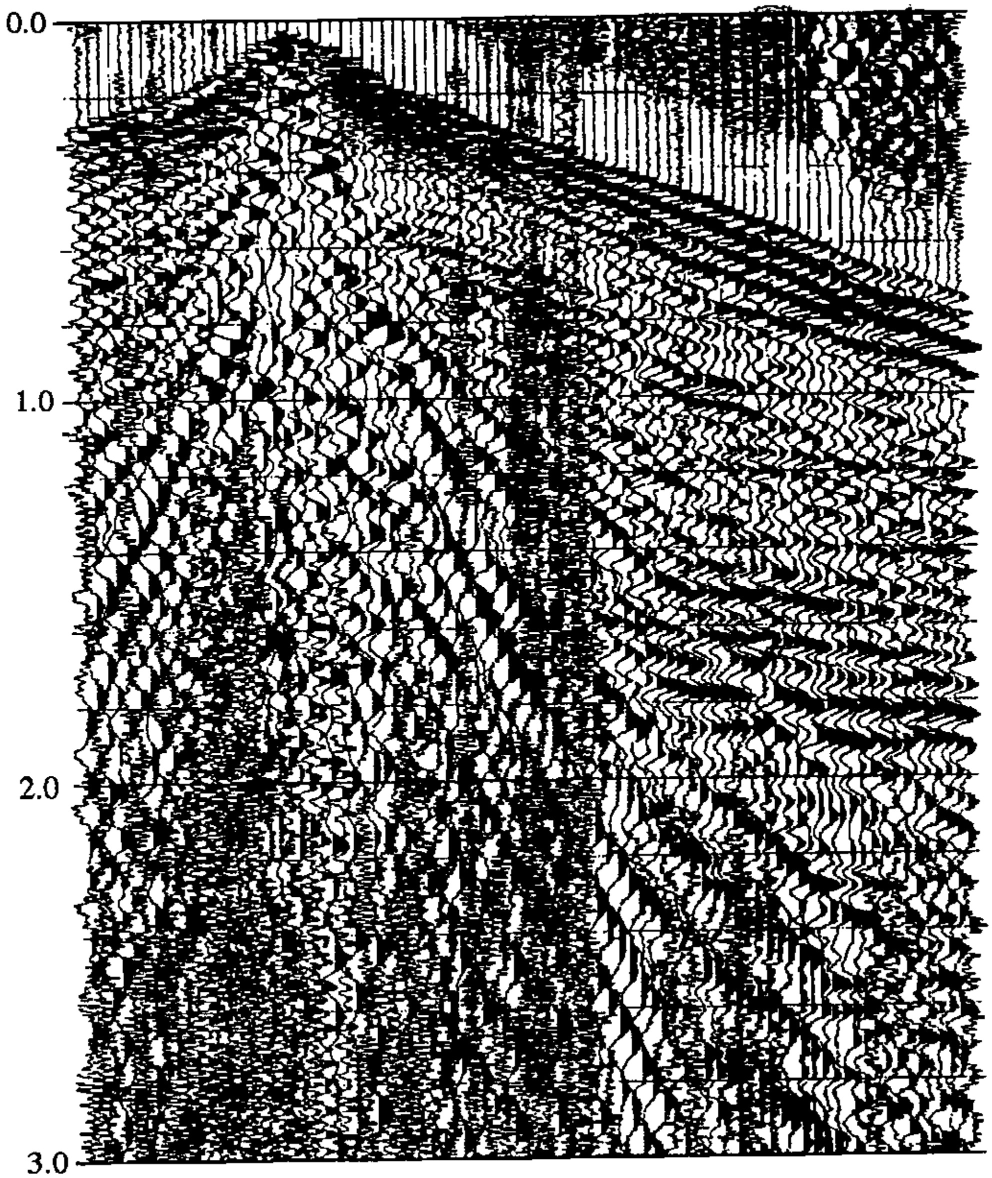

FIG. 11. In-line component data of the same gather as FIG. 10.

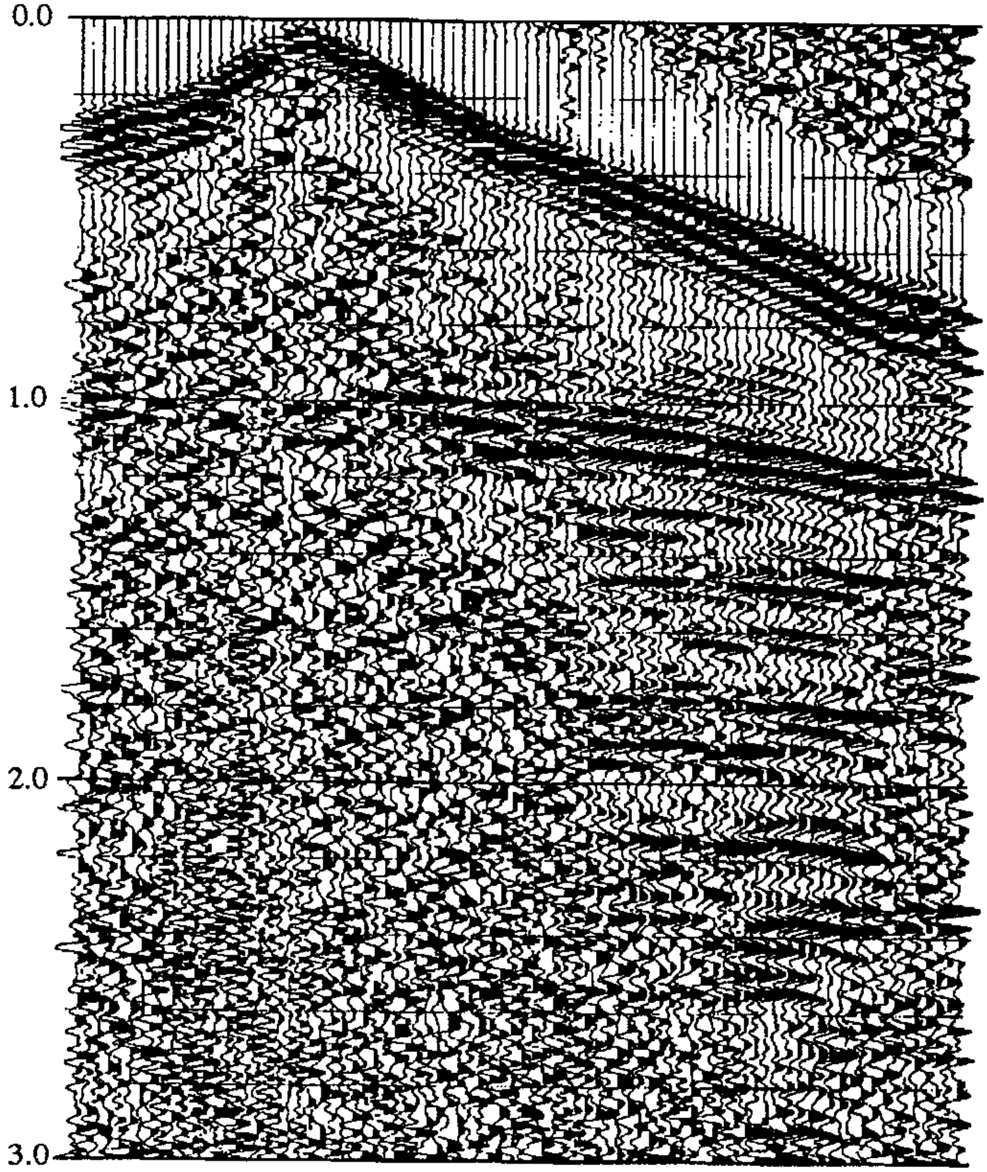

FIG. 12. Polarization filtered vertical component data of the data gather of Figs. 10 and 11. Random noise and groundroll are reduced. Some reflection are recovered within the groundroll contaminated area.

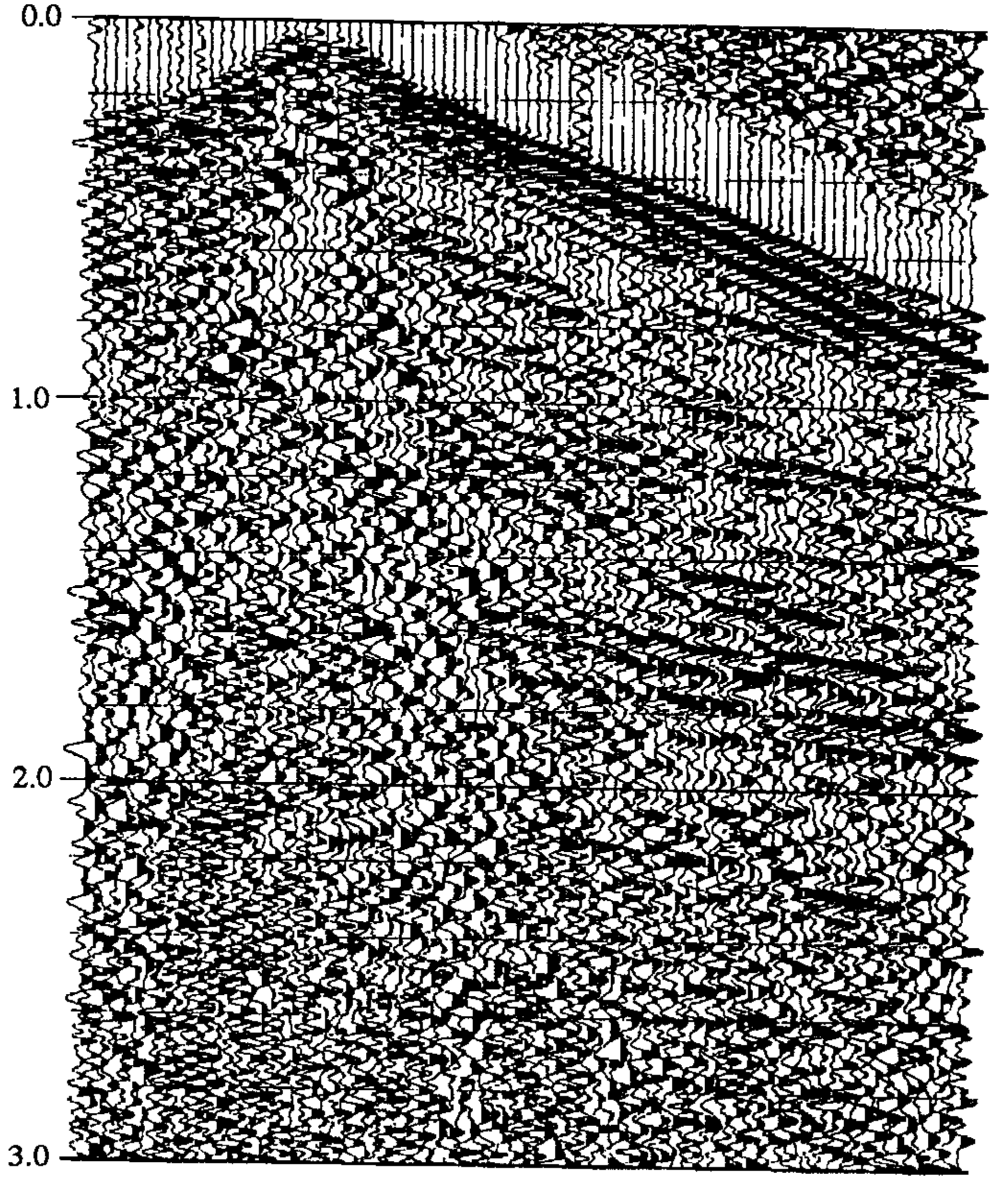

FIG. 13. In-line component of the polarization data.