Cubic anisotropy and anisotropic salt

Zandong Sun and R. James Brown

ABSTRACT

This paper considers the question whether a salt layer is anisotropic or not and what its anisotropic features are. The geophysical classification of salt is carried out based on a combination of velocity measurement and salt-characteristic investigation through thin sections on different salt samples and previous work. A physical modelling was performed to characterize anisotropic salt. It has been found that there are three types of anisotropic salt. The anisotropic features of chevron-crystal salt (widely present in the Whitkow Member of the Prairie Evaporite Formation in the Western Canada Basin) match cubic symmetry quite well. Exact expressions for phase velocities in an arbitrary direction have been derived for cubic symmetry. Group velocity formulae are also developed in symmetry planes.

INTRODUCTION

There exists some controversy over the question whether salt layers in the subsurface may be anisotropic media or not. Results from the experiments of Sun et al. (1991) have shown that some types of salt exhibit shear-wave splitting.

Geologically, salt textures have been well documented. Strotzki and Welch (1983) studied the relation between salt texture and temperature and concluded that the preferred orientation developed in extruded salt is a function of extrusion temperature. Also the connection with the diapirism of salt domes were discussed. Larsen (1983) did textural analysis and crystallographic orientation study for a salt dome in Denmark. Spencer and Lowenstein (1990) systematically analyzed diagenesis and geological procedure of different types of salt and textural feature. Those types of salt which have preferred crystal orientation do widely exist.

In order to understand what types of salt are anisotropic, the features of salt anisotropy and how the anisotropy can be observed, we carried out a geological investigation including salt texture microscope study correlated with laboratory velocity measurement. Theoretical derivation of phase and group velocity for cubic symmetry has been completed. Group velocity against ray angle are observed and fit well with theoretical calculation for salt sample with chevron crystals from Whitkow Member of Prairie Evaporite Formation in Western Canada Basin. VSP numerical modelling is also carried out in the purpose of observing shear wave splitting.

SALT CLASSIFICATION AND VELOCITY PROPERTIES

Geophysically, the criterion for classifying salts is to observe how the different types of salt cause seismic waves to behave differently while the waves propagate through the salt units. Geologically, these properties of salt depend on the origins of evaporites and the influence of diagenesis or burial metamorphism.

Basically, salt can be classified into two group: pure and impure. Impure salt are those in which salt crystals mixed uniformly with clastic deposits which is shown isotropic in the laboratory measurement. Salt layers interbedded with other thin layers are considered as pure salt. Pure salt are classified as following types.

The first type of salt is (I) crystal-oriented salt . It can be produced in two different geological processes as described in the following.

IA: Chevron salt: syndepositional open-space growth on the bottom of a brine body. This type of salt is commonly preserved in modern (Casas et al., 1992) and ancient evaporites (Whitkow Member of Prairie Evaporite Formation in Western Canada Basin, Meijer Drees, 1986).

IB: Recrystallized salt: one of the case is that An initial floodwater causes extensive tubular networks of vertical and horizontal dissolution cavities. Subsequent evaporative concentration of the flood water results in halite-saturated brines and renewed crystal growth.

This type of salt (I) is layered and has syntaxially grown crystalline framework salt consisting of vertically oriented and vertically elongated crystals. Stratigraphically, a salt layer usually consists of many salt units and interbedded with thin shale (mud), anhydrite or carbonate.

The second type is (II) detrital-framework salt. The framework of grains with point contacts establishes a primary detrital texture in evaporites as clastic rocks. And the salt layer exhibits isotropic features (Lowenstein and Hardie, 1985; Weiler et al., 1974).

The third type of salt is (III) burial metamorphic salt. This type of salt can be anisotropic or isotropic. It can be the salt with strongly preferred crystal orientation (Strotzki and Welch, 1983; Larsen, 1983). It also can be strongly altered by the temperatures and pressures due to burial which is also called anhedral mosaic salt. It should be noted that a texture in salt domes will lead to an anisotropic behavior when strained thermomechanically. The basic feature of anhedral mosaic salt is polygonal mosaic texture which notably lacks vertical orientation, primary growth features have been destroyed and have disappeared. Instead, the anhedral mosaic consists of clear grains that meet at triple junctions that approach 120° angles

PHASE AND GROUP VELOCITIES OF CUBIC SYMMETRY

It can be extremely complicated to get the exact expressions for velocities with directional dependence which is one of fundamental properties of seismic anisotropy in an arbitrary off-symmetry plane. However, for the case of cubic symmetry we have derived the exact expressions for phase velocities in any direction. Group velocity formulae are also developed for propagation in symmetry planes.

In order to provide intuition into the phase velocity expressions, we introduce three anisotropy parameters ε, ω, and φ for cubic symmetry.

$$\varepsilon = (C_{11} - C_{44})^2(\alpha - \frac{1}{3}) - \alpha(C_{44} + C_{12})^2 \tag{1}$$

$$\omega = \beta[(C_{12} + C_{44})^2(3C_{11} - 2C_{12} - 5C_{44}) - (C_{11} - C_{44})^3] + \frac{1}{3}\alpha(C_{11} - C_{44})(C_{11} + C_{12})(C_{11} - C_{12} - 2C_{44}) - \frac{2}{27}(C_{11} - C_{44})^3 \tag{2}$$

$$\varphi = arccos[- \frac{3\sqrt{3}\,\omega}{2(-\varepsilon)^{3/2}}] \tag{3}$$

where $\alpha = n_1^2 n_2^2 + n_2^2 n_3^2 + n_3^2 n_1^2$, and $\beta = n_1^2 n_2^2 n_3^2$.

Recall that unit vectors n_1, n_2, n_3 are functions of phase angle (θ). We have the exact expressions for the phase velocities as functions of phase angle in any arbitrary direction (see Appendix).

The P-wave and two shear waves phase velocity can be expressed by:

$$v_P^2 = \frac{2\sqrt{(-3\varepsilon)}}{3\rho} cos\frac{\varphi}{3} + \frac{C_{11} + 2C_{44}}{3\rho}. \tag{4}$$

$$v_{SV}^2 = \frac{2\sqrt{(-3\varepsilon)}}{3\rho} cos(\frac{\varphi}{3} + \frac{2\pi}{3}) + \frac{C_{11} + 2C_{44}}{3\rho} \tag{5}$$

$$v_{SH}^2 = \frac{2\sqrt{(-3\varepsilon)}}{3\rho} cos(\frac{\varphi}{3} + \frac{4\pi}{3}) + \frac{C_{11} + 2C_{44}}{3\rho}. \tag{6}$$

The shear-wave singularities in salt are on the major symmetry axes. At these points the phase velocities are written as follows:

$$v_P^2 = \frac{\sqrt[3]{(-4\omega)}}{\rho} + \frac{C_{11} + 2C_{44}}{3\rho} \tag{7}$$

$$v_{SV}^2 = v_{SH}^2 = \frac{\sqrt[3]{4\omega}}{2\rho} + \frac{C_{11} + 2C_{44}}{3\rho}. \tag{8}$$

where n_i is the unit vector normal to the wavefront, $\lambda = \rho v^2$, ρ is the density, v is the phase velocity. In a symmetry plane the group velocity can be easily determined from phase velocity using well known relationships (Postma, 1955; Backus 1965).

$$V^2(\phi) = v^2(\theta) + \left(\frac{dv}{d\theta}\right)^2 \tag{9}$$

$$\tan\phi = \frac{v\tan\theta + \left(\frac{dv}{d\theta}\right)}{v - \left(\frac{dv}{d\theta}\right)\tan\theta}. \quad (10)$$

It is indicated in (9) and (10) that along a symmetry axis of a symmetry plane, group velocity is equal to phase velocity and phase angle (θ) or slowness direction is the same as the group angle (ϕ) or angle of incidence of the ray. This is because the derivative of phase velocity with respect to phase angle in that direction is zero, namely:

$$\frac{dv}{d\theta} = 0. \quad (11)$$

It is obvious that, on the symmetry axes, the derivative of velocity with respect to phase angle in that direction is zero. It is also indicated in (9) and (10) that group angle (Φ) is a function of phase angle (θ) (More detailed and intuitive discussion is given by Brown et al. (1991).

In one of the symmetry planes unit vectors n_1, n_2, n_3 are featured as:

$$n_1 = \cos\theta,\ n_2 = \sin\theta,\ n_3 = 0. \quad (12)$$

From (12) we have $\alpha = \cos^2\theta\sin^2\theta$ and $\beta = 0$. The following formulae are derived (Appendix) for group velocities (V_P, V_{SH}, V_{SV}) in a typical plane of cubic symmetry.

$$V_P^2 = v_P^2 - \frac{\cos^2(\varphi/3)\sin^2(4\theta)}{48\rho^2 v_P^2\varepsilon}(C_{11} + C_{12})^2(C_{11} - C_{12} - 2C_{44})^2 \quad (13)$$

$$V_{VS}^2 = v_{SV}^2 - \frac{\cos^2((\varphi + 2\pi)/3)\sin^2(4\theta)}{48\rho^2 v_{SV}^2\varepsilon}(C_{11} + C_{12})^2(C_{11} - C_{12} - 2C_{44})^2. \quad (14)$$

$$V_{SH}^2 = v_{SH}^2 - \frac{\cos^2((\varphi + 4\pi)/3)\sin^2(4\theta)}{48\rho^2 v_{SH}^2\varepsilon}(C_{11} + C_{12})^2(C_{11} - C_{12} - 2C_{44})^2. \quad (15)$$

Again, at those points where shear-wave has singularities group velocities are expressed by the following formulae:

$$V_P^2 = v_P^2 + \frac{(4\omega)^{2/3}\sin^2(4\theta)}{81\rho^2 v_P^2(4\omega)^2}(C_{11} + C_{12})^2(C_{11} + C_{44})^2(C_{11} - C_{12} - 2C_{44})^2. \quad (16)$$

$$V_{SH}^2 = V_{SV}^2 = v_{SH}^2 + \frac{(4\omega)^{2/3}\sin^2(4\theta)}{324\rho^2 v_{SH}^2(4\omega)^2}(C_{11} + C_{12})^2(C_{11} + C_{44})^2(C_{11} - C_{12} - 2C_{44})^2. \quad (17)$$

MODELING STUDIES

The physical modelling was carried out by recording shear waves propagating through a ball made of a salt sample which consists of equigranular (around 1-2 mm) crystals, white to colorless halite, and vertically oriented cloudy and milky pathes. Crystallographic orientation is not observable (salt type IA). The frequency scaling was 10,000:1, and distance and time scaling 1:10,000. The data were recorded every 15° per trace (from 0° to 180°) with 13 traces per record. The schematic diagram of the modelling geometry is shown in Figure 1. The perimeter of circle 1 is 11.7 cm which indicates the diameter of the ball is 3.724 cm. In order to observe the anisotropic feature in this salt medium, shear-wave transducers are oriented in three different ways:

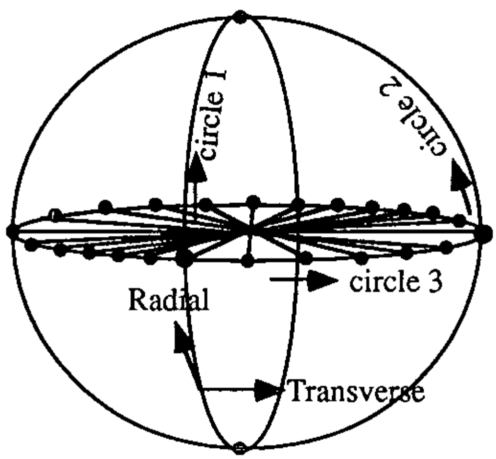


Figure 1. Physical modelling geometry

• Radial–Radial (R–R) polarization. Two transducers (source and receiver) are parallel to each other and oriented in radial (inline) direction (Figure 1).

• Transverse–Transverse (T–T) polarization. Two transducers are parallel to each other and oriented in transverse (crossline) direction (Figure 1).

• Radial–Transverse or Transverse–Radial (R–T or T–R) polarization. The polarization of source and receiver transducers are perpendicular to each other.

The theoretical curves of group velocities against ray angle are plotted in Figure 2 (calculated by applying equations 13–17)). It represents first arrivals of *P*-wave and shear waves (*SH* and *SV*) against ray angle at 1 second.

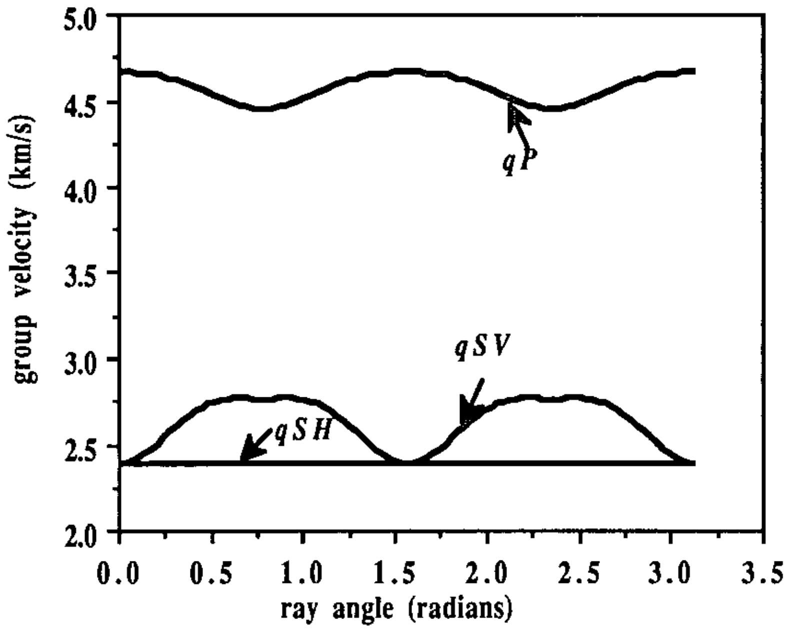
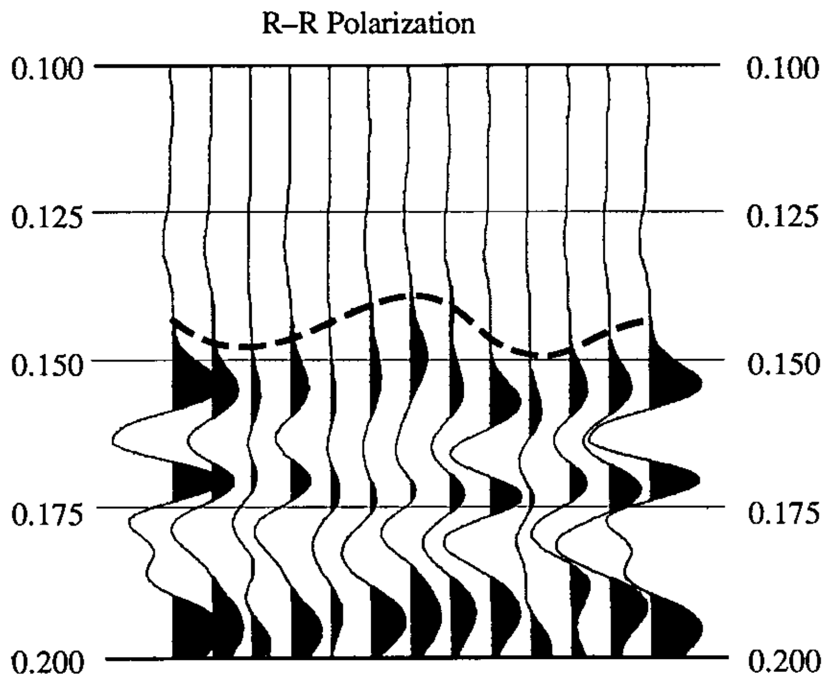
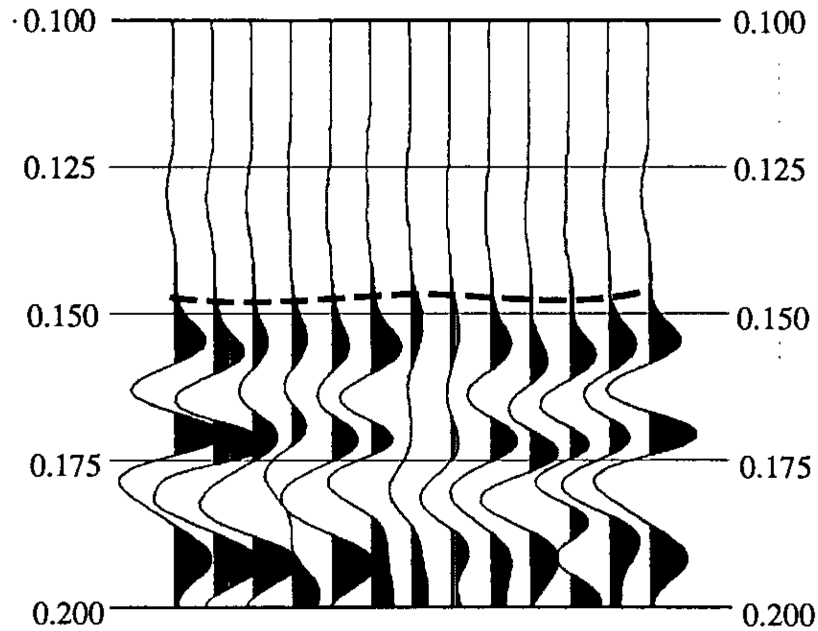


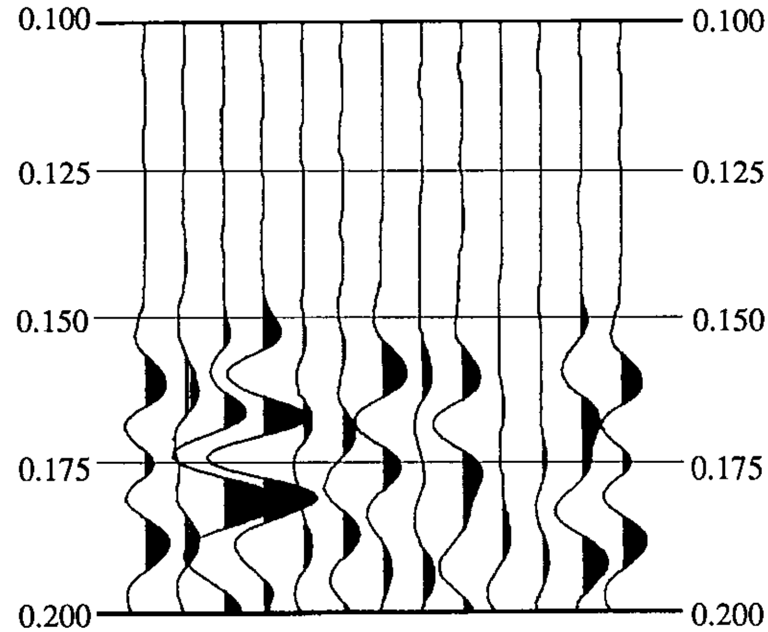
Figure 2. group velocity against ray angle



3a)



3b)



3c)

Figure 3. Zero-offset shear transmission record over chevron salt ball along circle 1.
a) R–R polarization
b) T–T polarization
c) T–R polarization

The shear waves transmission records along circle 1 are plotted in Figure 3. Figure 3a is the plot with transducers R–R polarization. In the plot the pattern of first arrivals is very similar with *SV* in Figure 2. Figure 3b shows that shear wave plot of T–T polarization. There is not much time–shift of first arrivals. And it is similar with *SH* curve in Figure 2. Little first arrival time–shift in some traces can be caused by two transducers being unparalleled also circle 1 is randomly selected and may not be exactly on a symmetry plane. Similar results were obtained along circle 2 and 3. Shear waves pattern showing in Figure 3a and b indicates that this type of salt probably exhibits cubic anisotropy.

Amplitudes are very low in the shear–wave record with R–T (or T–R) polarization because of the low energy projection.

CONCLUSION

This study has shown that there are at least three types of anisotropic salt. A salt layer may exhibit anisotropy, weak anisotropy, isotropy. Shear-wave feature of chevron crystal salt matches cubic anisotropy well. Phase and group velocity equations derived in this paper can be applied to this type of salt and any medium with cubic symmetry. A anisotropic behavior led by a texture in salt domes when strained thermomechanically may be close to cubic symmetry.

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APPENDIX

Phase velocity and group velocity in anisotropic medium of cubic symmetry

In the case of cubic symmetry, only three independent stiffness have nonzero values (Musgrave, 1970)

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \quad \text{(A-2)}$$

From well known Kelvin-Christoffel equation, we have Kelvin-Christoffel stiffness

$$\Gamma_{jk} = \sum_{i=1}^{3} \sum_{l=1}^{3} n_i\, n_l\, c_{ijkl}, \quad j,k = 1,2,3$$

where n_i is the unit vector normal to the wavefront, and c_{ijkl} is the tensor of elastic stiffnesses.

In a medium of cubic symmetry, Kelvin-Christoffel stiffnesses are

$$\Gamma_{11} = n_1^2\, C_{11} + (\, n_2^2 + n_3^2\,)\, C_{44}$$

$$\Gamma_{22} = n_2^2\, C_{11} + (\, n_1^2 + n_3^2\,)\, C_{44}$$

$$\Gamma_{33} = n_3^2\, C_{11} + (n_1^2 + n_2^2)\, C_{44}$$

$$\Gamma_{12} = \Gamma_{21} = M = n_1 n_2 (C_{44} + C_{12})$$

$$\Gamma_{13} = \Gamma_{31} = N = n_1 n_3 (C_{44} + C_{12})$$

$$\Gamma_{23} = \Gamma_{32} = L = n_2 n_3 (C_{44} + C_{12})$$

suppose $\lambda = \rho\, v^2$ (ρ is the density, v is the phase velocity), Kelvin-Christoffel equation becomes

$$\begin{pmatrix} \Gamma_{11}-\lambda & M & N \\ M & \Gamma_{22}-\lambda & L \\ N & L & \Gamma_{33}-\lambda \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = 0 \qquad \text{(A-3)}$$

where P_1, P_2, P_3 are particle motion or polarization vector. In order to decide the phase velocity we require

$$\begin{vmatrix} \Gamma_{11}-\lambda & M & N \\ M & \Gamma_{22}-\lambda & L \\ N & L & \Gamma_{33}-\lambda \end{vmatrix} = 0 \qquad \text{(A-4)}$$

So we have equation

$$(\Gamma_{11} - \lambda)[(\Gamma_{22} - \lambda)(\Gamma_{33} - \lambda) - L^2] - M[M(\Gamma_{33} - \lambda) - LN] + N[ML - N(\Gamma_{22} - \lambda)] = 0$$

It becomes a cubic equation

$$\lambda^3 - (\Gamma_{11} + \Gamma_{22} + \Gamma_{33})\lambda^2 + (\Gamma_{11}\Gamma_{22} + \Gamma_{22}\Gamma_{33} + \Gamma_{33}\Gamma_{11} - L^2 - N^2 - M^2)\lambda$$

$$- (\Gamma_{11}\Gamma_{22}\Gamma_{33} + 2LMN) + (L^2\Gamma_{11} + N^2\Gamma_{22} + M^2\Gamma_{33}) = 0$$

$$\Gamma_{11} + \Gamma_{22} + \Gamma_{33} = (n_1^2 + n_2^2 + n_3^2)C_{11} + 2(n_1^2 + n_2^2 + n_3^2)C_{44}$$

$$= C_{11} + 2C_{44}$$

where $n_1^2 + n_2^2 + n_3^2 = 1$

$$\Gamma_{11}\Gamma_{22} + \Gamma_{22}\Gamma_{33} + \Gamma_{33}\Gamma_{11} - (L^2 + N^2 + M^2)$$

$$= 2(n_1^2n_2^2 + n_2^2n_3^2 + n_3^2n_1^2)C_{44}^2 + (n_1^2n_2^2 + n_2^2n_3^2 + n_3^2n_1^2)C_{11}^2$$

$$+ 2(n_1^4 + n_2^4 + n_3^4)C_{11}C_{44} + 2(n_1^2n_2^2 + n_2^2n_3^2 + n_3^2n_1^2)C_{11}C_{44}$$

$$+ (n_1^4 + n_2^4 + n_3^4)C_{44}^2 - 2(n_1^2n_2^2 + n_2^2n_3^2 + n_3^2n_1^2)C_{12}C_{44}$$

$$- (n_1^2n_2^2 + n_2^2n_3^2 + n_3^2n_1^2)C_{12}^2$$

suppose $\alpha = n_1^2n_2^2 + n_2^2n_3^2 + n_3^2n_1^2$, $\beta = n_1^2n_2^2n_3^2$, then $n_1^4 + n_2^4 + n_3^4 = 1 - 2\alpha$.
And we have

$$\Gamma_{11}\Gamma_{22} + \Gamma_{22}\Gamma_{33} + \Gamma_{33}\Gamma_{11} - (L^2 + N^2 + M^2)$$

$$= (1 + \alpha)C_{44}^2 + \alpha C_{11}^2 + 2(1-\alpha)\, C_{11}C_{44} - \alpha(C_{44} + C_{12})^2$$

$$\beta = n_1^2 n_2^2 n_3^2$$

$$\Gamma_{11}\Gamma_{22}\Gamma_{33} = \beta C_{11}^3 + (\alpha - 3\beta)C_{11}^2 C_{44} + (1 + 3\beta - 2\alpha)C_{11}C_{44}^2 + (\alpha - \beta)C_{44}^3$$

$$2LMN = 2\beta(C_{44} + C_{12})^3$$

$$L^2\Gamma_{11} + N^2\Gamma_{22} + M^2\Gamma_{33} = (C_{44} + C_{12})^2[3\beta\, C_{11} + (\alpha - 3\beta)C_{44}]$$

$$(L^2\Gamma_{11} + N^2\Gamma_{22} + M^2\Gamma_{33}) - (\Gamma_{11}\Gamma_{22}\Gamma_{33} + 2LMN)$$

$$= (C_{44} + C_{12})^2[3\beta C_{11} + (\alpha - 3\beta)C_{44}] - 2\beta(C_{44} + C_{12})^3$$

$$- \beta C_{11}^3 - (\alpha - 3\beta)C_{11}^2 C_{44} - (1 + 3\beta - 2\alpha)C_{11}C_{44}^2 - (\alpha - \beta)C_{44}^3$$

We introduce three anisotropy parameters ε, ω, and φ for cubic symmetry

$$\varepsilon = (C_{11} - C_{44})^2(\alpha - \frac{1}{3}) - \alpha(C_{44} + C_{12})^2$$

$$\omega = \beta[(C_{12} + C_{44})^2(3C_{11} - 2C_{12} - 5C_{44}) - (C_{11} - C_{44})^3]$$

$$+ \frac{1}{3}\alpha(C_{11} - C_{44})(C_{11} + C_{12})(C_{11} - C_{12} - 2C_{44}) - \frac{2}{27}(C_{11} - C_{44})^3$$

$$\varphi = arccos[- \frac{3\sqrt{3}\,\omega}{2(-\varepsilon)^{3/2}}]$$

$$\delta = \frac{\varepsilon^2}{4} + \frac{\omega^3}{27}$$

Now, following general formulations give phase velocity of an arbitrary off-symmetry and symmetry planes in 3-D space.

P-wave phase velocity

when $\delta < 0$

$$v_P^2 = \frac{2\sqrt{(-3\varepsilon)}}{3\rho} cos\frac{\varphi}{3} + \frac{C_{11} + 2C_{44}}{3\rho}$$

(A-5)

SV-wave phase velocity

$$v_{SV}^2 = \frac{2\sqrt{(-3\varepsilon)}}{3\rho} cos(\frac{\varphi}{3} + \frac{2\pi}{3}) + \frac{C_{11} + 2C_{44}}{3\rho}$$
(A-6)

SH-wave phase velocity

$$v_{SH}^2 = \frac{2\sqrt{(-3\varepsilon)}}{3\rho} cos(\frac{\varphi}{3} + \frac{4\pi}{3}) + \frac{C_{11} + 2C_{44}}{3\rho}$$
(A-7)

when $\delta = 0$ two shear waves have same phase velocity, these points are called shear wave singularities. The formulations can be written as

$$v_P^2 = \frac{\sqrt[3]{(-4\omega)}}{\rho} + \frac{C_{11} + 2C_{44}}{3\rho}$$
(A-8)

$$v_{SV}^2 = v_{SH}^2 = \frac{\sqrt[3]{4\omega}}{2\rho} + \frac{C_{11} + 2C_{44}}{3\rho}$$
(A-9)

Group Velocity

In the symmetry plane group velocity can be determined from phase velocity by using relation

$$V^2(\phi) = v^2(\theta) + \left(\frac{dv}{d\theta}\right)^2 \qquad \text{(A-10)}$$

Because of the symmetric feature of unit vectors n_1, n_2, n_3, we suppose one of the symmetry plane (others are the similar)

$n_1 = cos\theta$, $n_2 = sin\theta$, $n_3 = 0$, then $\alpha = cos^2\theta sin^2\theta$, $\beta = 0$ (A-11)

$$V_P^2 = v_P^2 - \frac{cos^2(\varphi/3)sin^2(4\theta)}{48\rho^2 v_P^2 \varepsilon}(C_{11} + C_{12})^2(C_{11} - C_{12} - 2C_{44})^2 \qquad \text{(A-12)}$$

$$V_{SV}^2 = v_{SV}^2 - \frac{cos^2((\varphi + 2\pi)/3)sin^2(4\theta)}{48\rho^2 v_{SV}^2 \varepsilon}(C_{11} + C_{12})^2(C_{11} - C_{12} - 2C_{44})^2 \text{(A-13)}$$

$$V_{SH}^2 = v_{SH}^2 - \frac{cos^2((\varphi + 4\pi)/3)sin^2(4\theta)}{48\rho^2 v_{SH}^2 \varepsilon}(C_{11} + C_{12})^2(C_{11} - C_{12} - 2C_{44})^2 \qquad \text{(A-14)}$$

On those points where two shear waves have same phase velocity (singularities), two shear waves group velocity also are the same.

$$V_P^2 = v_P^2 + \frac{(4\omega)^{2/3} sin^2(4\theta)}{81\rho^2 v_P^2 (4\omega)^2}(C_{11} + C_{12})^2(C_{11} + C_{44})^2(C_{11} - C_{12} - 2C_{44})^2 \quad \text{(A-15)}$$

$$V_{SH}^2 = V_{SV}^2 = v_{SH}^2 + \frac{(4\omega)^{2/3} sin^2(4\theta)}{324\rho^2 v_{SH}^2 (4\omega)^2}(C_{11} + C_{12})^2(C_{11} + C_{44})^2(C_{11} - C_{12} - 2C_{44})^2 \quad \text{(A-16)}$$