Joint P and P-SV inversion: Application and testing

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ABSTRACT

Stewart (1990) outlined a method of inverting amplitudes of both P-P reflections and converted P-SV reflections to find changes in P and S-wave velocity. A program has been written to perform an inversion on interpreted P and P-SV seismic data and tests have been performed in order to assess the quality of the lithologic prediction and the range of values that will work in the algorithm.

From the error analysis, we find that the inversion should produce accurate results even for large changes in velocity and relatively long offsets. The inversion produces traces of relative changes in Poisson's ratio and a fluid factor in time. The inverted traces show lithologic or fluid changes in the rock which, in this study, matched the model reasonably well.

INTRODUCTION

Smith and Gidlow paper (1987) developed an amplitude inversion technique that extracts relative changes in the P and S–wave velocities, $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ respectively, from P-wave reflectivity data. They used an empirical relationship between α and β. Using the additional measurement of the converted P to S–wave reflectivity R^{ps} these parameters can be estimated without the need to rely on an empirical relationship between α and β.

With the two measurements, R^{pp} and R^{ps}, we estimate $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ without assuming a relationship between α and β. Carefully interpreted P–P and P–SV sections can be used for an amplitude inversion for rock properties using this algorithm. The code has now been written to perform this inversion and synthetic data has been used in the inversion program to test the algorithm.

THEORY

We use the approximations of $R^{pp}(\theta)$ and $R^{ps}(\theta)$ from Aki and Richards (1980) which assume small changes in elastic properties across an interface. Stewart (1990) has used a least squares approach in solving Aki and Richards' set of linear equations where $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ vary with R^{pp}, R^{ps}, and the reflected wave angles θ for P-P and ϕ for P-SV as well as the average P and S wave velocities.

The Aki and Richards equations are

$$R^{pp}(\theta) \sim \frac{1}{2}\left(1 - 4\frac{\beta^2}{\alpha^2}\sin^2\theta\right)\frac{\Delta\rho}{\rho} + \frac{1}{2\cos^2\theta}\frac{\Delta\alpha}{\alpha} - \frac{4\beta^2}{\alpha^2}\sin^2\theta\,\frac{\Delta\beta}{\beta} \tag{1}$$

for the P-P reflection coefficient and

$$R^{ps}(\theta) \sim \frac{-\alpha\tan\phi}{2\beta}\left[\left(1 - 2\frac{\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\phi\right)\frac{\Delta\rho}{\rho}\right.$$

$$\left. - \left(4\frac{\beta^2}{\alpha^2}\sin^2\theta - 4\frac{\beta}{\alpha}\cos\theta\cos\phi\right)\frac{\Delta\beta}{\beta}\right] \tag{2}$$

for the converted wave reflectivity. Using Gardner's (1974) relationship which empirically related density ρ and P wave velocity α, the Aki and Richard's equations are shown by Stewart (1990) as

$$R^{pp}(\theta) = a\frac{\Delta\alpha}{\alpha} + b\frac{\Delta\beta}{\beta} \tag{3}$$

and

$$R^{ps}(\theta) = c\frac{\Delta\alpha}{\alpha} + d\frac{\Delta\beta}{\beta} \tag{4}$$

where a, b, c, and d are a simple algebraic substitution. This yields a set of two linear equations with two unknown variables. Stewart (1990) gives the least squares solution to these equations as

$$\frac{\Delta\alpha}{\alpha} = \frac{\Sigma(d^2 - b^2)\,\Sigma(aR^{pp} - cR^{ps}) - \Sigma(cd - ab)\,\Sigma(bR^{pp} - dR^{ps})}{\gamma} \tag{5}$$

for the relative change in P–wave velocity and

$$\frac{\Delta\beta}{\beta} = \frac{\Sigma(c^2 - a^2)\,\Sigma(bR^{pp} - dR^{ps}) - \Sigma(cd - ab)\,\Sigma(aR^{pp} - cR^{ps})}{\gamma}$$

for the relative change in S–wave velocity where

$$\gamma = \Sigma(c^2 - a^2)\,\Sigma(d^2 - b^2) - [\Sigma(cd - ab)]^2.$$

The summations are taken over all of the offsets. These equations are what has been coded in C to invert the reflection amplitudes.

METHOD

The above equations from Stewart (1990) have produced the resulting inversion routine that has been tested for a single interface using input Zoeppritz amplitudes and a test was performed on synthetic CMP gathers. Error in the inverted rock velocity perturbations were calculated using the difference between the calculated $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ and those from the theoretical model. After the completion of the error analysis, the inversion of the offset synthetic was plotted as seismic traces of rock property reflectivity.

Error analysis

In the error analysis, Zoeppritz amplitudes as well as average α and β across the interface and respective reflection angles θ and φ were input to the inversion function. The resulting Δα/α and Δβ/β were then be compared to those input into Zoeppritz's equations using this simple formula:

$$\text{error in parameter} = \left| \frac{\text{measured parameter - actual value}}{\text{actual value}} \right| \times 100\%. \tag{6}$$

Values were found for a range of values of Δα/α and Δβ/β and the resulting errors were mapped in figure 1 for the errors in Δα/α and Δβ/β respectively. Note the less than 10% error operating range that follows a line of constant slope which is the ratio α/β for the upper medium in this case. The small elastic property change assumption in the Aki and Richards' approximations does not appear to be sensitive to large changes in α and β, but it is sensitive to large changes in α/β.

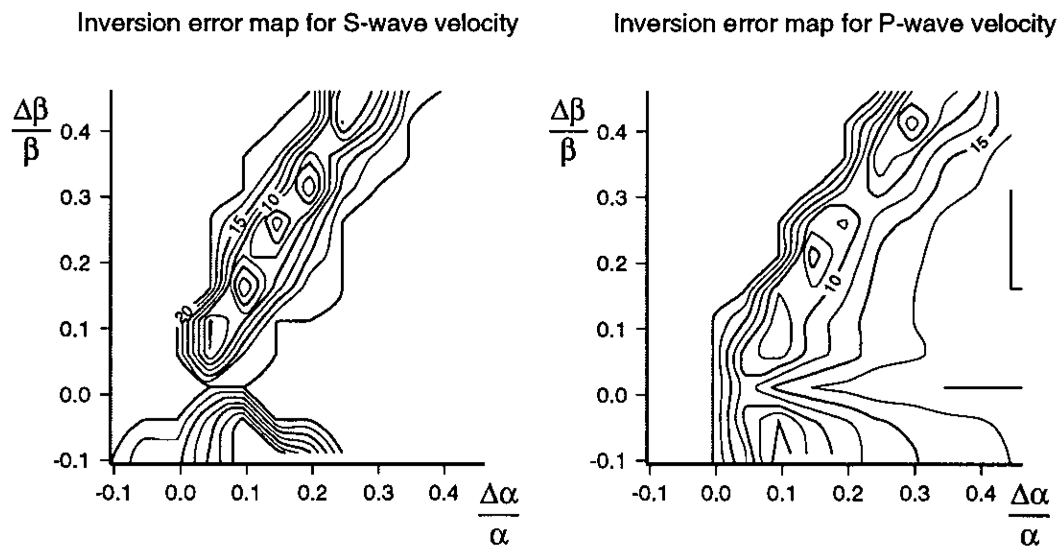

FIG. 1. Plots of error in relative changes across the interface as a function of Δα/α and Δβ/β. The operational area of <10% error is shaded on each plot.

Synthetic seismogram inversion

The program is designed to have an interpreted NMO corrected CMP gather as input, it then raytraces through the interpreted model to calculate the appropriate reflection angles, θ and φ, at each offset. Once raytracing is complete, the picked amplitudes for the respective horizons are read in and the inversion is performed. The outputs are then plotted in seismic trace form.

A simple bisection raytracing method was used where the inputs to the raytracing function are a velocity versus depth function and the offsets that were used in recording the data. The velocities are average interval velocities between the interpreted horizons and the depths are the approximate depths of the interfaces. Offsets are input as the first and last offsets and the offset spacing. Horizontal layers are assumed and raytracing is performed through the velocity function for each offset and for both P-P and P-SV raypaths to obtain θ and φ.

Once $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ are calculated, the difference can be taken as a pseudo Poisson's ratio reflectivity defined by Smith and Gidlow (1987) as

$$\frac{\Delta q}{q} = \frac{\Delta\alpha}{\alpha} - \frac{\Delta\beta}{\beta}. \tag{7}$$

A fluid factor is also defined in the same paper (Smith and Gidlow, 1987) as

$$\Delta F = \frac{\Delta\alpha}{\alpha} - 1.16\,\frac{\beta}{\alpha}\frac{\Delta\beta}{\beta} \tag{8}$$

which will give a large negative amplitude at the top of gas filled clastic silicate and a number close to zero for water filled media. Each of these four parameters are plotted as a seismic trace. Note that the fluid factor in equation (8) relies on β/α. Another important use of the P-SV section with the P-wave section is to find T_P/T_S ratios. These long wavelength estimates are important in standard AVO analysis as well.

RESULTS

The synthetic data used for the test were generated from a blocked velocity log shown in Figure 2. From this data, synthetic CMP gathers were generated for offsets ranging from 100 m to 2 km and plotted in figure 3. The resulting inversion is shown in figure 4.

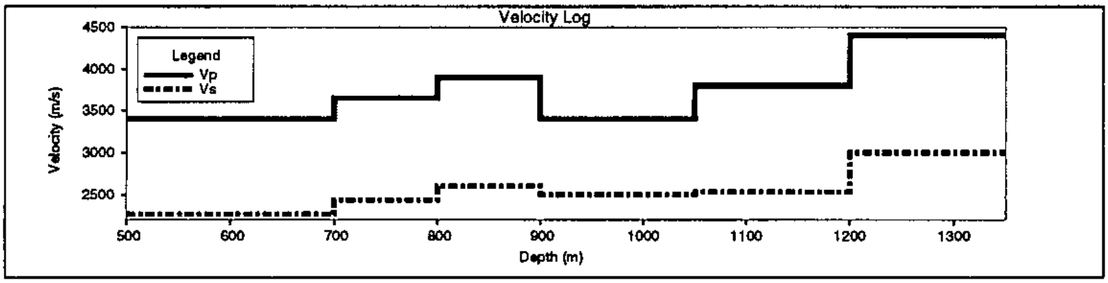

FIG. 2. Velocity versus depth function for the synthetic model. Note the large drop in α between 900 and 1050 m where there is little change in β. Here α/β drops from 1.5 to 1.34.

The inverted traces show where there is a substantial change in α/β. Both the $\Delta q/q$ and the ΔF traces show clearly the large decrease in α/β in the fluid filled zone which is in the interval between 900 and 1050 m in depth. There is some error in the traces where there is no change in α/β but the relative amplitudes clearly indicate where there is a major change in lithology or fluid content. The inverted velocity changes are compared to those of the model in Figure 5.

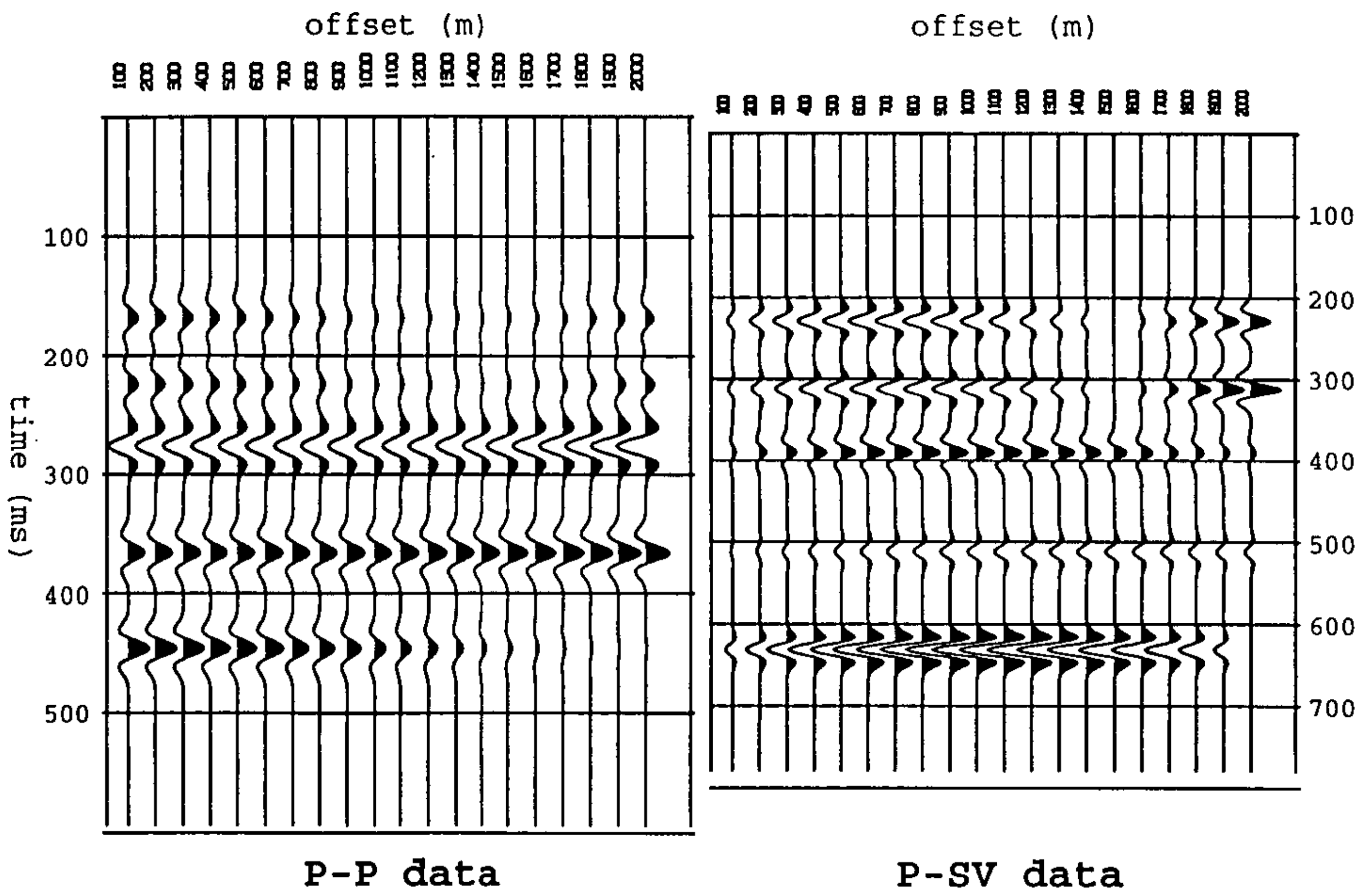

FIG. 3. Synthetic CMP gathers using Zoeppritz equation solutions generated from the velocity information in Figure 2.

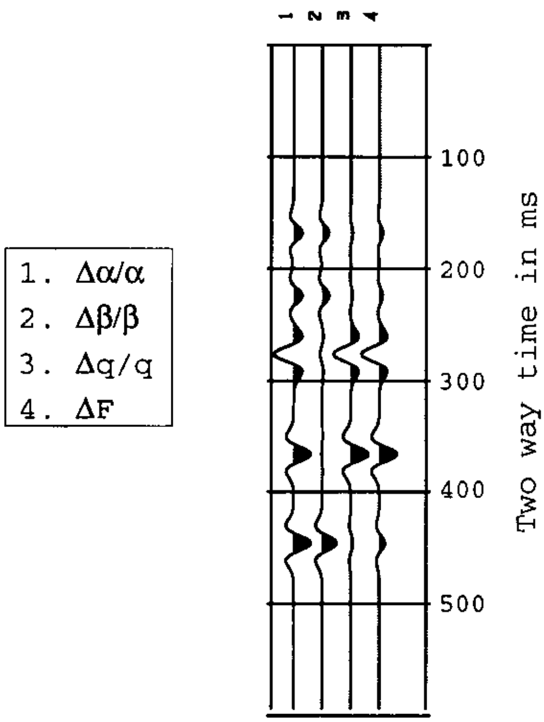

FIG. 4. The inverted property seismic traces. Note the relative amplitudes match those required by the model.

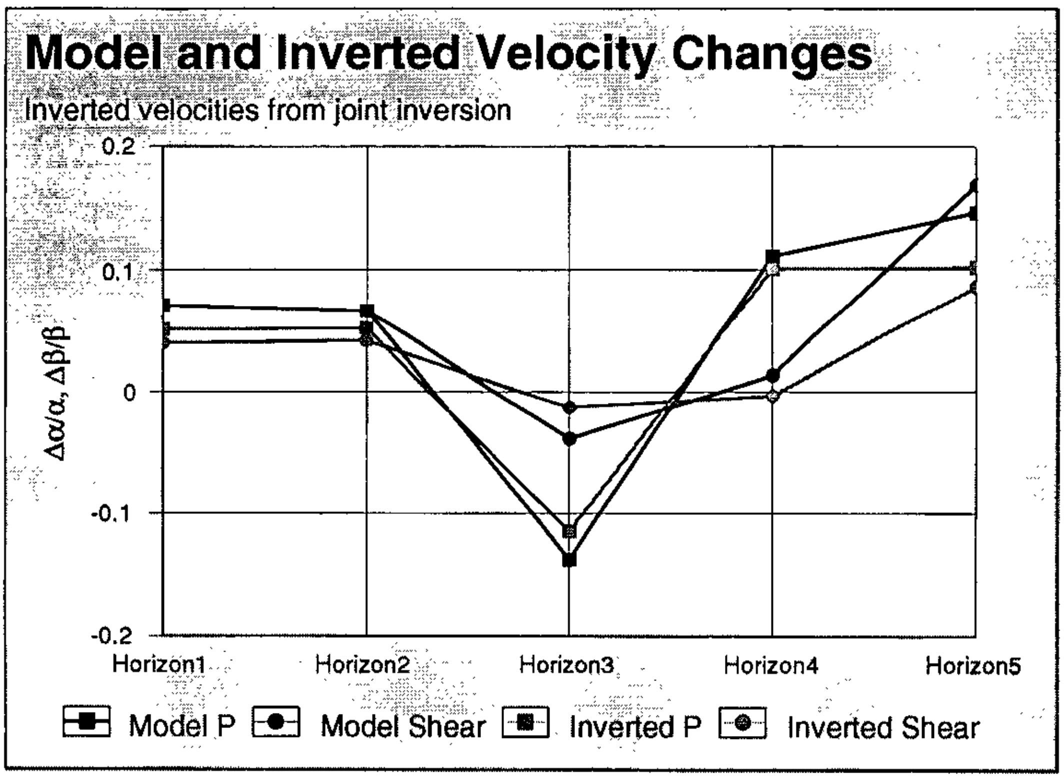

FIG. 5. Graph of $\Delta\alpha/\alpha$ and $\Delta\beta/\beta$ from the model and from the inversion.

This example uses fairly large reflection coefficients, beyond the assumption of small rock properties made in the original Aki and Richard's approximation. This is reasonable since the operating range found in the error analysis does not seem to limit the size of increase in α and β. The model chosen includes incident angles as high as 55° for the P-P waves and 65° for the P-SV waves which are also beyond the limitations of the initial approximations. The model includes a substantial change in α/β in the section which shows up clearly in the inverted traces. Although there are large changes in elastic parameters and high angles of incidence the polarity and relative amplitude of the inverted traces accurately depicts the relative lithology changes of the model.

CONCLUSIONS

Equations for joint P and P-SV amplitude inversion were derived by Stewart (1990) to estimate changes in P and S-wave velocities across an interface. The algorithm is now been tested on synthetic data and shows that the method is more robust than initially thought. The procedure is ready for testing with field data.

REFERENCES

Aki, K. and Richards, P.G., 1980, Quantitative seismology: Theory and methods, v.1: W.H. Freeman and Co.

Gardner, G.H.F.,Gardner, L.W., and Gregory, A.R., Formation velocity and density-the diagnostic basics of stratigraphic traps: Geophysics, 39, 770-780.

Press, W.H. et al, 1988, Numerical Recipes in C, Cambridge University Press.

Smith, G.C. and Gidlow, P.M., 1987, Weighted stacking for rock property estimation and detection of gas: Geophys. Prosp., 35, 993-1014.

Stewart, R.R., 1990, Joint P and P-SV inversion: CREWES Project Research Report, v.2, 112-115.

Young, G.B., and Braile, L.W., 1976, A computer program for the application of Zoeppritz's amplitude equations and Knott's energy equations, Bull. Seism. Soc. Am. 66, pp 1881-1885.