Wavefield separation in transversely isotropic medium: A plane-wave decomposition approach

Lawrence H. T. Le, Donald T. Easley and Robert R. Stewart

INTRODUCTION

Devaney and Oristaglio (1986) described a method to decompose a two-dimensional elastic vector wavefield into compressional (*P*) and shear (*S*) waves for homogeneous isotropic layered structure. By two-dimensional wavefield, it is assumed that the disturbance is generated by a line source of infinite length. The method can be extended to a transversely isotropic (T.I.) material with a vertical axis of symmetry. In this case, the wavefields to be separated are quasi *P*- and quasi *S*-waves. The method is applicable to an array of receiver in both vertical and horizontal directions provided that the phase velocities are known in any direction. This is similar to the acquisition geometry of VSPs and surface seismic data respectively.

THEORY

The following is a brief review of the theory by Devaney and Oristaglio (1986). The two-dimensional elastic displacement wavefield $\mathbf{u}(x,z,t)$ can be decomposed into a spectrum of plane waves:

$$\mathbf{u}(x,z,t) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} e^{-i\omega t}\, d\omega \int_{-\infty}^{\infty} \left[A_{qP}(x,k_z,\omega)\,\hat{\mathbf{q}} + A_{qS}(x,k_z,\omega)\,\hat{\mathbf{s}} \right] e^{ik_z z}\, dk_z \tag{1}$$

where $\hat{\mathbf{q}}$ and $\hat{\mathbf{s}}$ are the orthonormal quasi-*P* and quasi-*S* polarization vectors in the saggital plane and A_{qP} and A_{qS} are the corresponding spectra. It should be noted that the integration variable k_z is the vertical wavenumber, which is appropriate for a vertical borehole-type geometry. The spatial and temporal Fourier transform of the wavefield $\mathbf{u}$ is given by

$$\tilde{\mathbf{u}}(x,k_z,\omega) = \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} \mathbf{u}(x,z,t)\, e^{-i(k_z z - \omega t)}\, dt \quad . \tag{2}$$

By substituting eqns. (1) into (2), we obtain

$$\tilde{\mathbf{u}}(x,k_z,\omega) = A_{qP}(x,k_z,\omega)\,\hat{\mathbf{q}} + A_{qS}(x,k_z,\omega)\,\hat{\mathbf{s}} \quad . \tag{3}$$

Taking the dot product of the transform wavefield $\tilde{\mathbf{u}}$ with $\hat{\mathbf{q}}$ and $\hat{\mathbf{s}}$ respectively, we obtain the spectra in terms of the polarization vectors and $\tilde{\mathbf{u}}$:

$$A_{qP}(x,k_z,\omega) = \frac{\hat{\mathbf{q}}}{\hat{\mathbf{q}} \cdot \hat{\mathbf{q}}} \cdot \tilde{\mathbf{u}}(x,k_z,\omega) \tag{4}$$

and

$$A_{qS}(x,k_z,\omega) = \frac{\hat{\mathbf{s}}}{\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}} \cdot \tilde{\mathbf{u}}(x,k_z,\omega) \quad . \tag{5}$$

Making use of eqns. (4) and (5) in (1), the separated quasi P-wave is given by

$$\mathbf{u}_{qP}(x,z,t) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} \frac{\hat{\mathbf{q}}\,\hat{\mathbf{q}}}{\hat{\mathbf{q}} \cdot \hat{\mathbf{q}}} \cdot \tilde{\mathbf{u}}(x,k_z,\omega)\, e^{i(k_z z - \omega t)}\, dk_z \tag{6}$$

and the separated quasi S-wave is

$$\mathbf{u}_{qS}(x,z,t) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} \frac{\hat{\mathbf{s}}\,\hat{\mathbf{s}}}{\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}} \cdot \tilde{\mathbf{u}}(x,k_z,\omega)\, e^{i(k_z z - \omega t)}\, dk_z \tag{7}$$

where $\hat{\mathbf{q}}\,\hat{\mathbf{q}}$ and $\hat{\mathbf{s}}\,\hat{\mathbf{s}}$ are dyadics.

The theory described above depends on the polarization vectors which are eigenvectors obtained by solving the characteristic system associated with the equation of motion (Daley and Hron, 1977)

$$\left(\Gamma_{jk} - \lambda\,\delta_{jk}\right) f_k = 0 \tag{8}$$

where λ is the eigenvalue, f_k is the eigenvector component and

$$\Gamma_{jk} = \frac{c_{ijkl}\, p_k\, p_l}{\rho}$$

(9)

such that

c_{ijkl} = elastic tensor

ρ = density

p_k = slowness vector .

For transversely isotropic material with a vertical axis of symmetry, the eigen-system in the saggital plane simplifies to

$$\begin{bmatrix} \Gamma_{11}-\lambda & \Gamma_{13} \\ \Gamma_{13} & \Gamma_{33}-\lambda \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = 0 \ .$$

(9)

The vanishing determinant of the coefficient matrix yields two eigenvalues λ_i ($i=1, 2$). These eigenvalues can be substituted into (9) to obtain the appropriate eigenvectors:

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}_i = \frac{1}{\sqrt{(\Gamma_{13})^2 + (\Gamma_{11}-\lambda_i)^2}} \begin{bmatrix} \Gamma_{13} \\ -(\Gamma_{11}-\lambda_i) \end{bmatrix} .$$

(10)

CONCLUDING REMARKS

An existing approach for wavefield separation in isotropic media has been extended to separate quasi wavefields in transversely isotropic media. Including the free surface effect within the developed theoretical framework does not seem to be a problem.

REFERENCES

Daley, P. F., and Hron, F., 1977, Reflection and transmission coefficients for transversely isotropic media: Bull. Seis. Soc. Am., **67**, 661-675.

Devaney, A. J., and Oristaglio, M. L., 1986, A plane-wave decomposition for elastic wave fields applied to the separation of P-waves and S-waves in vector seismic data: Geophysics, **51**, 419-423.