Full 3-D versus rotated 2-D filters

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ABSTRACT

This paper demonstrates that a cylindrically symmetric filter in the 3-D frequency domain gives rise to a cylindrically symmetric operator in the 3-D time domain. We also show that a 2-D vertical slice of this symmetric 3-D time-domain operator is not equivalent to the 2-D operator realized from a vertical panel filter in the frequency domain: That is, the 3-D operator is not a rotated version of the 2-D operator.

INTRODUCTION

Recently, the application of f-k filters to 3-D data as a one pass 3-D operation has been compared to two orthogonal passes of the 2-D filter (Stewart and Schieck, 1993). It was noted that the 3-D time-domain operator used in that study was approximated by 2-D filter coefficients. To create a 3-D time-domain operator it may seem intuitively correct to rotate a 2-D operator (in x,t) by 2π, around the time axis, to sweep out a 3-D cylinder. However, the full 3-D operator is not equivalent to an axially rotated 2-D operator.

3-D f-k OPERATOR

The inverse transform of a 3-D filter F can be defined as:

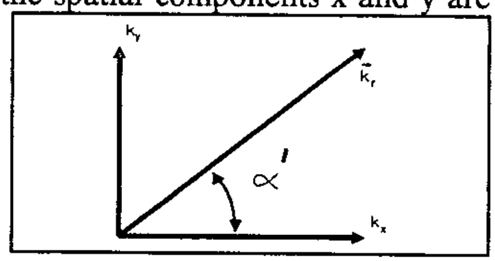
$$f(t,x,y)=\frac{1}{(2\pi)^3}\int\int\int_{-\infty}^{\infty} F(\omega,k_x,k_y)e^{i(\omega t+k_x x+k_y y)}dk_x dk_y d\omega \ . \qquad (1)$$

If the filter $F(w,k_x,k_y)$ is axially symmetric and the spatial components x and y are written as:

$$x=r\cos\phi \qquad y=r\sin\phi$$

and frequency components as :

$$k_x=k_r\cos\alpha' \qquad k_y=k_r\sin\alpha'$$



eq. (1) becomes:

$$f(t,r,\phi)=\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{0}^{2\pi}\int_{0}^{\infty}F(\omega,k_r)e^{i(\omega t+k_r r\cos(\alpha'-\phi))}k_r dk_r d\alpha' d\omega \qquad (2)$$

where $k_r=\sqrt{k_x^2+k_y^2}$, $r=\sqrt{x^2+y^2}$, $dk_x dk_y=k_r dk_r d\alpha'$, and α' is the angle between k_r and the k_x axis. Substituting $\alpha=(\alpha'-\phi)$ in eq. (2) we get the following integral:

$$f(t,r)=\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{0}^{\infty}F(\omega,k_r)k_r e^{i\omega t}\left[\int_{-\phi}^{2\pi-\phi}e^{ik_r r\cos\alpha}d\alpha\right]dk_r d\omega$$
$$f(t,r)=\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{0}^{\infty}F(\omega,k_r)k_r e^{i\omega t}\left[\int_{0}^{2\pi}e^{ik_r r\cos\alpha}d\alpha\right]dk_r d\omega. \qquad (3)$$

The integral formulation of a zero-order Bessel function (Abramowitz and Stegan, 1972) is:

$$2\pi J_0(u)=\int_0^{2\pi}e^{iu\cos\alpha}d\alpha \qquad (4)$$

Using (4) with (3) gives (Meskó,1984):

$$f(t,r)=\frac{1}{2\pi^2}\int_{-\infty}^{\infty}[\int_0^{\infty}F(\omega,k_r)k_r J_0(k_r r)dk_r]e^{i\omega t}d\omega, \qquad (5)$$

where the term in the square brackets is now a zero-order Hankel transform. Note that the filter f(t,r) is axi-symmetric (does not depend on ϕ). Suppose $F(\omega,k_r)$ is a cylinder of radius a:

$$F(\omega,k_r) = \frac{1}{2\pi a^2} \quad for \quad k_r \le a$$
$$= 0 \quad for \quad k_r > a. \qquad (6)$$

Substituting for $F(\omega,k_r)$, the filter operation becomes:

$$f(t,r)=\frac{1}{4\pi^3 a^2}\int_{-\infty}^{\infty}[\int_0^{a}k_r J_0(k_r r)dk_r]e^{i\omega t}d\omega \qquad (7)$$

We can write the Hankel transform in (5) as:

$$\frac{1}{2\pi a^2}\int_0^a \frac{k_r r}{r} J_0(k_r r)dk_r \tag{8}$$

The recurrence relation between the zero- and first-order Bessel function:

$$\int uJ_0(u)du = uJ_1(u) \tag{9}$$

so if $u=k_r r$, $dk_r = \frac{1}{r}$ du and $0 \leq u \leq ar$ then eq. (8) becomes:

$$= \frac{rJ_1(ar)}{2\pi ar} \tag{10}$$

Finally, eq. (5) can be reduced to

$$f(t,r) = \frac{J_1(ar)}{(2\pi)^3 ar}\int_{-\infty}^{\infty} e^{i\omega t}d\omega \;=\; \frac{J_1(ar)}{(2\pi)^3 ar}\delta(t), \tag{11}$$

2-D f-k OPERATOR

The expression for the integral equation of the inverse 2-D transform of the same f-k filter is given by:

$$f(t,R) = \frac{1}{2\pi^2}\int\int_{-\infty}^{\infty} F(\omega,k_R)e^{i(\omega t+k_R R)}dk_R d\omega, \tag{12}$$

where R is the radius in 2-D along the x or y directions. Consider an axial slice through the previous 3-D disk to define a panel with a width of 2a:

$$\begin{aligned} F(\omega,k_r) &= 1 \;\; for \;\; |k_r| \leq a \\ &= 0 \;\; for \;\; |k_r| > a \;, \end{aligned} \tag{13}$$

then

$$\begin{aligned} f(t,R) &= \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}[\int_{-a}^{a} e^{i(k_R R)}dk_R]e^{i\omega t}d\omega = \frac{1}{(2\pi)^2}sinc(2aR)\int_{-\infty}^{\infty} e^{i\omega t}d\omega \\ f(t,R) &= \frac{1}{(2\pi)^2}sinc(2aR)\delta(t) \;. \end{aligned} \tag{14}$$

DISCUSSION

The Bessel function is not equivalent to the sinc function as shown in Figure 1. Figure 1 compares the function of eq. (11) to that of eq. (14). The drop in amplitude with increasing distance are similar. The minimums of the 3-D operator approximately match every other trough of the 2-D operator. This indicates that an axial slice of the 3-D time-domain operator is not equivalent to the 2-D operator. However, as demonstrated by Stewart and Schieck (1993) this approximation appears to give reasonable results in the case of the median f-k filter. This may be due to this filters output of actual data values instead of the arithmetic mean of the weighted input data values as for mean type filters.

The filter time-domain response of these two equations is demonstrated in Figures 2 and 3 for a dip reject filter of 4 ms/trace. These are generated by taking the f-k transform of a 32 by 32 point matrix made up of 0 or 1 corresponding to the desired reject zone.

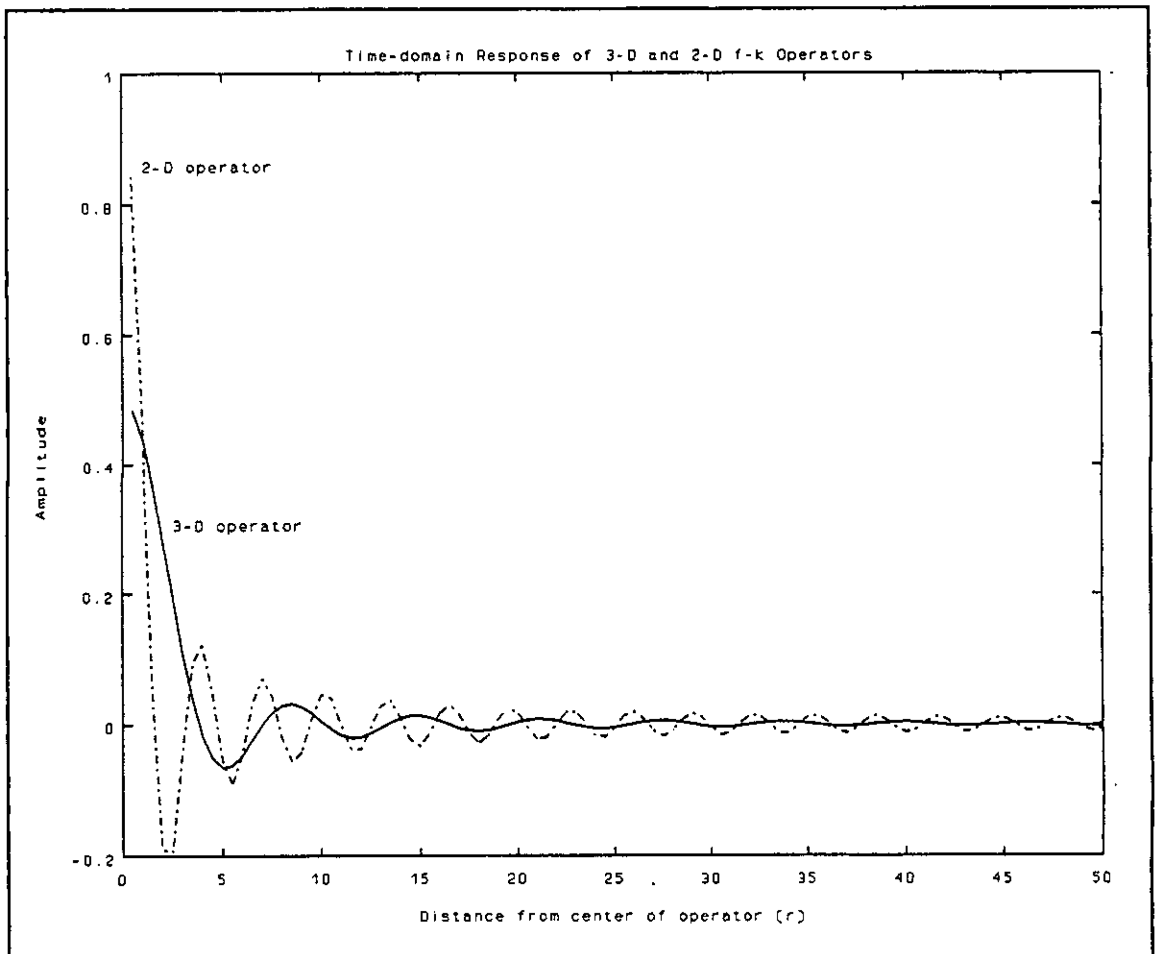


Figure 1 $J_1(r)/r$ function of 3-D operator compared with sinc(2R) for 2-D filtering.

CONCLUSIONS

While the two operators are similar they are not equivalent. The result of applying an axially rotated 2-D filter in place of the actual 3-D coefficients may be suitable for the 3-D median f-k filter.

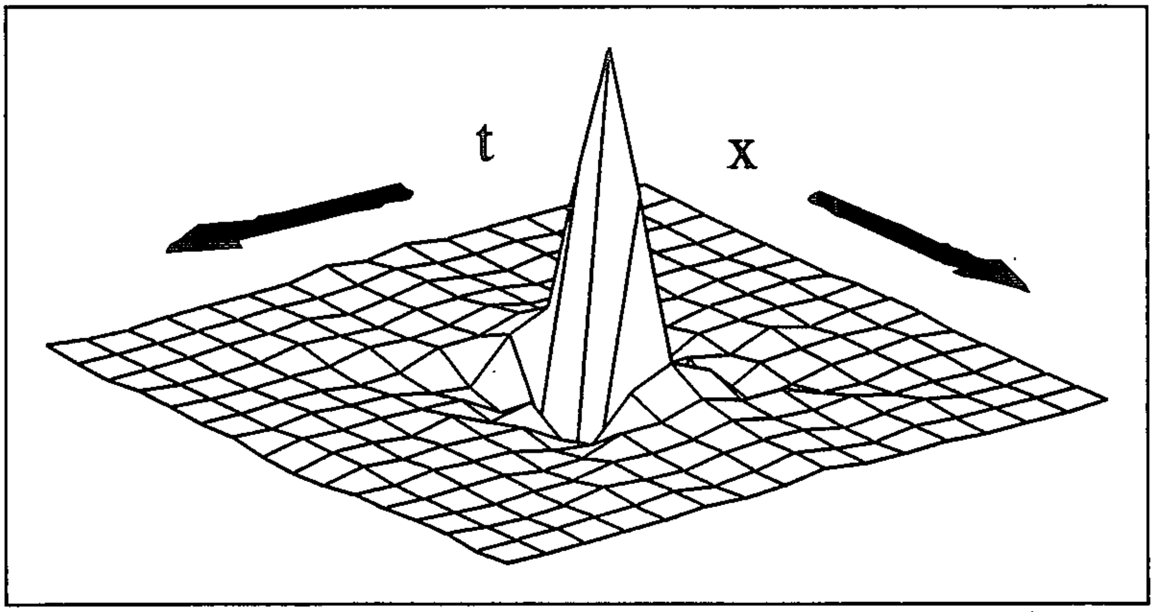


Figure 2 Axial slice of 3-D time-domain operator for a dip reject of 4 ms/trace.

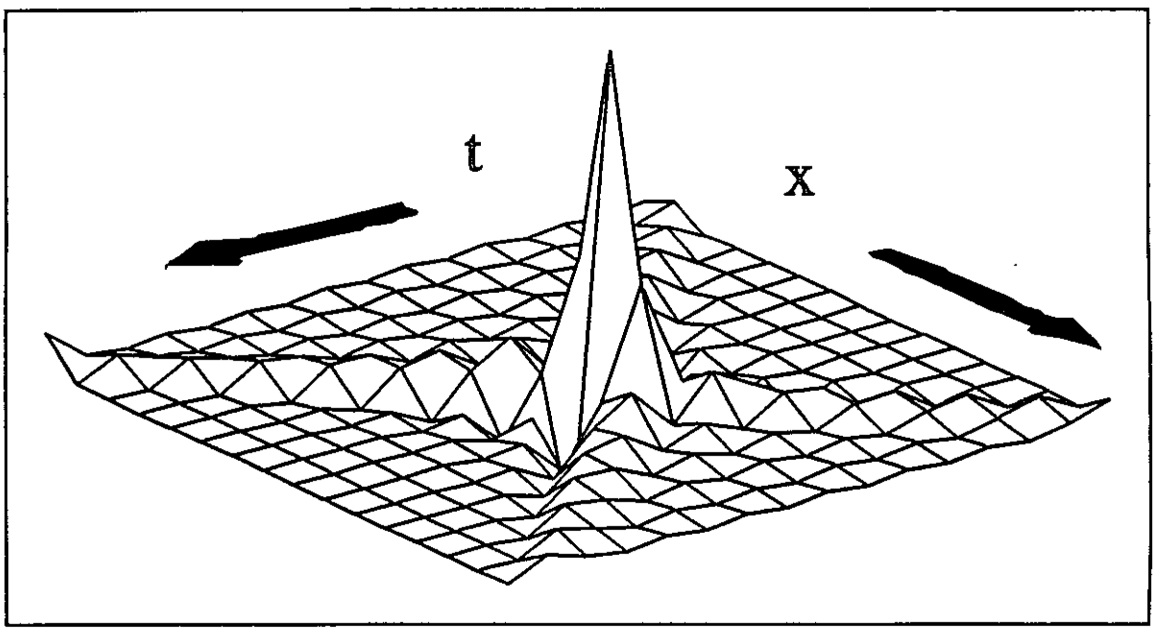


Figure 3 2-D time-domain operator for a dip reject of 4 ms/trace.

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