Estimating anisotropic permeability from attenuation anisotropy using 3C-2D data

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ABSTRACT

It has been experimentally proved that the attenuation, Q, rather than velocity, is strongly correlated with permeability (Klimentos and McCann, 1990; Akbar et al., 1993). Thus it should be possible to relate attenuation anisotropy with anisotropic permeability in partially or completely saturated rocks (Gelinsky et al., 1994).

A mathematical approach is presented for estimating anisotropic permeability from 3C-2D data, using the narrow relationship between seismic attenuation and permeability as predicted by Biot's laws for isotropic saturated porous media.

This approach could be applied to media with anisotropic permeability due to transverse isotropy produced by a stack of horizontal layers or azimuthal anisotropy caused by vertical fractures.

INTRODUCTION

Recently, Gibson and Toksöz (1990) predicted how the permeability would vary with direction in fractured rocks based on seismic velocity anisotropy. Further, the experimental data of Han (1987) and Klimentos and McCann (1990) obtained on sandstone samples show that the attenuation coefficient is more strongly related to clay content than velocity. On the other hand, Klimentos and McCann show a strong systematic relation between clay content and permeability and they conclude that attenuation is the key factor in determining permeability.

Additionally, the work of Gelinsky and Shapiro (1994a, b) shows that, for seismic frequencies, the attenuation anisotropy is proportional to the permeability anisotropy. In their study, anisotropy is considered twofold, consisting first of a weak anisotropy of the elastic constants and second of a much stronger anisotropy of the system's permeability. They showed that the absolute value of the qSV-wave attenuation is larger than that of the qP waves for all frequencies in a material with anisotropic permeability and a homogeneous and isotropic frame.

Then it is feasible to obtain valuable information about the reservoir anisotropy from the study of the behavior of the seismic attenuation with direction using 3C-3D seismic data. It is expected that by estimating the attenuation for P, SV and SH waves one can obtain a better image of the spatial distribution of the permeability in the reservoir.

THEORY

In general terms, a reservoir can present at least two different types of anisotropy: (i) transverse isotropy, caused by fine horizontal layering (Fig. 1a), and (ii) azimuthal anisotropy due to parallel (or nearly parallel) vertical aligned cracks or fractures, or due to unequal horizontal stresses (Fig. 1b).

For both anisotropies the elastic properties vary depending on the direction of measurement. In the first case, waves generally travel faster horizontally, along layers, than vertically across layers. For materials showing azimuthal anisotropy, waves traveling along the fracture direction — but within the competent rock — generally travel faster than waves crossing the fractures.

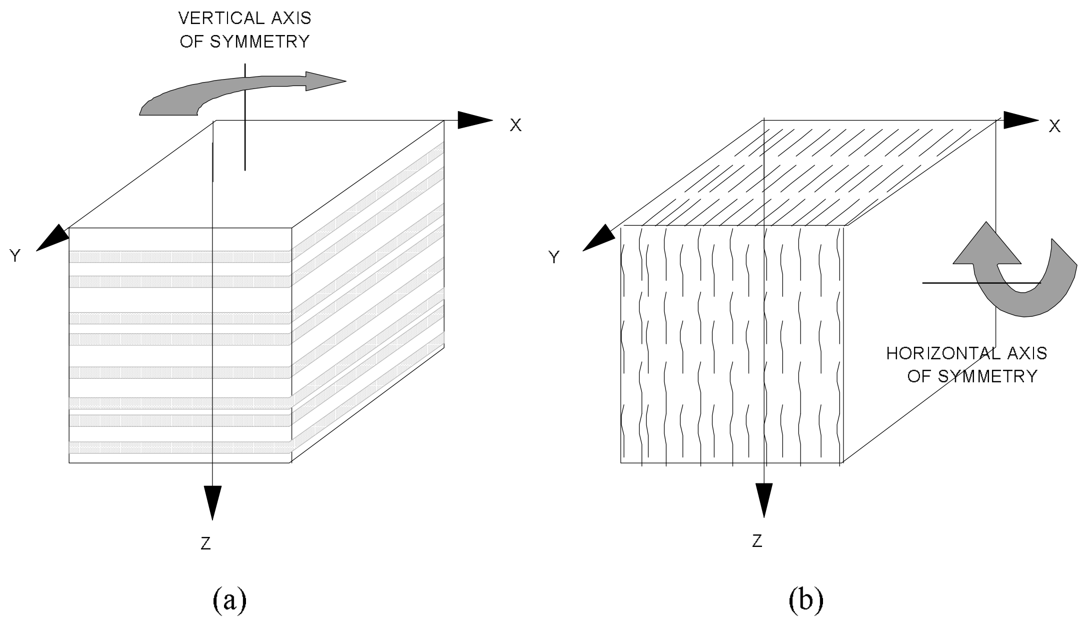

Fig. 1. Simple geometries assumed for elastic anisotropy.(a) In layered rock, elastic properties are uniform within horizontal layers, but may vary vertically and from layer to layer.(b) In vertically cracked rocks, elastic properties are uniform in vertical planes pa-rallel to the cracks, but may vary in the direction across the cracks.

In both cases, however if there exists any anisotropy caused by horizontal fine layering or fractures, it means that, besides the anisotropic static poroelastic stiffness tensor, the material will show a dynamic effect of anisotropic permeability as well.

In fact, for materials showing transverse isotropy, the permeability — the ease with fluids flow through rock — measured parallel to the layers of porous sedimentary rocks can be greater than the vertically measured permeability ($k_h > k_v$). In the other case, the vertical fractures acts as barriers to the fluid flow and the permeability measured perpendicular to the fracture planes is smaller than the permeability of the rock matrix measured parallel to the fractures ($k_h < k_v$).

As a result of anisotropy, at a given point in the medium the direction of the pressure-gradient vector is, in general, different from that of the velocity vector. In the general case of an anisotropic medium, there will result three different flow rates in each of the *x*, *y*, and *z* directions, whereas in an isotropic medium the flow is equal along all of these directions. Darcy's law for anisotropic media (Dullien, 1979) should be written as follows:

$$v_i = -\frac{1}{\mu}\left(k_{i1}\frac{\partial P}{\partial x_1} + k_{i2}\frac{\partial P}{\partial x_2} + k_{i3}\frac{\partial P}{\partial x_3}\right) \qquad (I = 1, 2, 3), \tag{1}$$

or

$$\vec{v} = -(\overline{\overline{k}}/\mu)\nabla\vec{P} \tag{2}$$

where 1, 2, and 3 represent the *x*, *y*, and *z* coordinates; k_{ij} form the elements of a second-order tensor, the values of which depend on the orientation of the medium with respect to the coordinate system; $\vec{v}$ is the velocity vector, and $\nabla\vec{P}$ is the pressure-gradient vector.

Assuming that anisotropic porous media are orthorhombic, i.e., they have three mutually orthogonal principal axes, the permeability tensor $\overline{\overline{K}}$ is symmetric ($k_{ij} = k_{ji}$) and rotation of the coordinate system will produce a diagonal matrix when the three coordinate axes are aligned with the principal axes of the medium. For this particular orientation of the medium, the pressure gradient and the velocity have the same direction and, therefore, in this case Darcy's law becomes:

$$v_i = -\left(k_i/\mu\right)\left(\partial P/\partial x_i\right) \qquad (I = 1, 2, 3), \tag{3}$$

where the three different values of k_i are still, in general, not equal.

The theory of Biot (1956, 1962) of wave propagation in an isotropic porous solid shows that there are two kinds of *P* waves (fast and slow) and a single *S* wave. Both compressional waves have different attenuations. The fast *P* wave has virtually constant phase velocity, although there is a small term varying as the square of frequency. The *S* wave also has constant velocity, as well as the same behavior of the attenuation with frequency ($\approx f^2$). This theory is valid only for a low-frequency range defined by the condition: $f < 0.15\left(\eta\phi/2\pi\rho_f k\right)$ where ρ and ρ_f are the densities of the saturated rock and of the fluid in the pore space, respectively, ϕ is the porosity, η is the viscosity of the fluid, k is the permeability, and f is the frequency.

The anisotropic permeability enters the Biot equations through a dissipation term. Because of the anisotropy, *P* and *S* waves do not separate anymore completely. An *SH* wave appears now. Then, the results of Biot (White, 1965) show that at low frequencies, when we consider wave propagation in an anisotropic porous medium, the attenuation coefficient γ for P , SV and SH waves can be expressed as:

$$\gamma_P = \frac{2\pi^2}{v_P}\left(\frac{\rho_f}{\rho} - \sigma_i\right)^2 \left(\frac{\rho}{\eta}\right) k_P f^2$$

$$\gamma_{SV} = \frac{2\pi^2}{v_{sv}}\left(\frac{\rho_f}{\rho}\right)^2 \left(\frac{\rho}{\eta}\right) k_{SV} f^2 \tag{4}$$

$$\gamma_{SH} = \frac{2\pi^2}{v_{SH}}\left(\frac{\rho_f}{\rho}\right)^2 \left(\frac{\rho}{\eta}\right) k_{SH} f^2$$

where v is the phase velocity, γ is the attenuation coefficient, and σ_i is a dimensionless rock modulus. The P, SV, and SH index for permeability and phase velocity represents the corresponding values of both parameters associated with each wave mode.

To analyze the seismic effects of global fluid flow in a system with anisotropic permeability, we initially consider a simple model which in the static limit is homogeneous and isotropic. Anisotropy of wave propagation is then a dynamic effect caused only by anisotropic permeability. In this way we can extend Biot's results for porous media to the case of anisotropic permeability. Figure 2 shows a sketch of an anisotropic porous solid. The axis vertical to the Earth's surface is the z-axis and the angle θ is defined between the slowness vector, $\vec{p}$, and the axis of symmetry. If the principal axes of the permeability tensor are chosen as coordinate axes, the permeability tensor becomes diagonal. Denoting the "principal" permeabilities as k_1, k_2, and k_3 and the angles made by the propagation slowness vector, $\vec{p}$, with the principal axes as α, β, and γ, we can express the magnitude of the permeability vector $\vec{k}$ as:

$$k = k_1 \cos^2 \alpha + k_2 \cos^2 \beta + k_3 \cos^2 \gamma \ , \tag{5}$$

As the pressure gradients produced by the traveling wave and the phase velocity have the same direction for orthorhombic anisotropic porous media, we have:

$$k_P = k_1 \cos^2 \theta \sin^2 \xi + k_2 \sin^2 \theta \sin^2 \xi + k_3 \cos^2 \theta$$

$$k_{SV} = k_1 \cos^2 \theta \cos^2 \xi + k_2 \cos^2 \theta \sin^2 \xi + k_3 \sin^2 \theta \tag{6}$$

and

$$k_{SH} = k_1 \sin^2 \xi + k_2 \cos^2 \xi$$

where ξ is the azimuthal angle of the plane containing the P and SV waves with respect to the x axis.

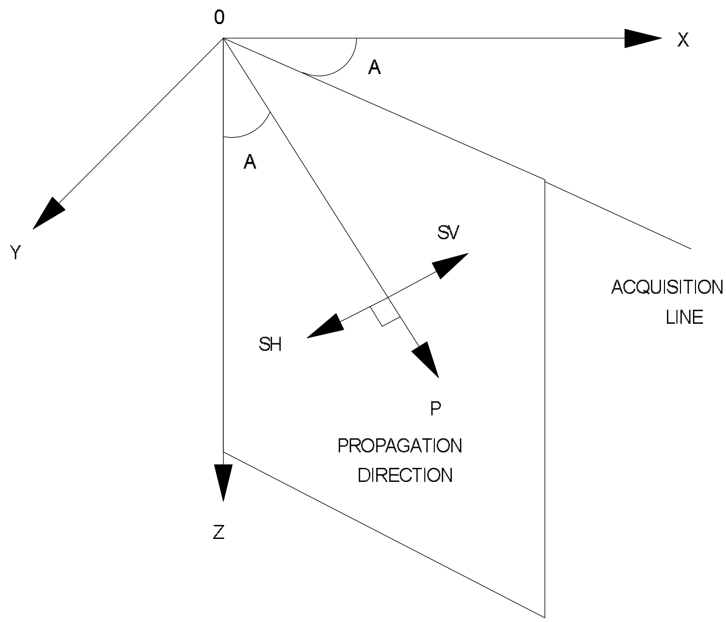

Fig. 2. Propagation direction for *P*, *SV*, and *SH* waves in an anisotropic porous media.

The equations (4) and (6) permit us relate the "principal" permeability components k_1, k_2, and k_3 with the attenuation coefficients γ_P, γ_{SV}, and γ_{SH} associated with the three P, SV, and SH wave modes.

Combining these equations we obtain a linear system of equations with three unknowns k_1, k_2, and k_3 as follows:

$$\begin{bmatrix} \sin^2\theta\cos^2\xi & \sin^2\theta\sin^2\xi & \cos^2\theta \\ \cos^2\theta\cos^2\xi & \cos^2\theta\sin^2\xi & \sin^2\theta \\ \sin^2\xi & \cos^2\xi & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} \tag{7}$$

where

$$A_1 = \gamma_P \frac{v_p}{2\pi^2}\left(\frac{\eta}{\rho}\right)\frac{1}{(\rho_f/\rho - \sigma_i)^2}$$

$$A_2 = \gamma_{SV} \frac{v_{SV}}{2\pi^2}\left(\frac{\eta}{\rho}\right)\frac{1}{(\rho_f / \rho)^2} \tag{8}$$

$$A_3 = \gamma_{SH} \frac{v_{SH}}{2\pi^2}\left(\frac{\eta}{\rho}\right)\frac{1}{(\rho_f / \rho)^2}$$

The solutions k_1, k_2, and k_3 are the principal permeabilities for a material whose anisotropy of wave propagation is only due to anisotropic permeability. But, as was related before, cracks or layering lead to an additional anisotropy of the elastic poroelastic stiffness tensor. This static poroelastic anisotropy may be described by poroelastic *P*-, *SV*-, and *SH*-wave phase velocities v_P, v_{SV}, and v_{SH} and by the Thomsen (1986) parameters (ε, γ, δ), specified by a formalism for fluid-filled, aligned cracks (Schoenberg and Douma, 1988). This will directly affect the phase velocities and become dominant for seismic frequencies affecting the seismic attenuation as well.

CONCLUSIONS

A mathematical background has been presented for estimating the principal permeabilities k_1, k_2, and k_3 by measurements of the seismic attenuation for *P*, *SV*, and *SH* waves in 3C-2D data.

The estimated permeabilities k_1, k_2, and k_3 for an homogeneous, isotropic, porous medium with a dynamic anisotropic permeability, caused by fractures or a stack of layers, depend on the seismic attenuation, phase velocity, θ (the angle of wave propagation), and ξ (the azimuthal direction of the seismic line).

For a more detailed description of the permeability behavior, it is necessary to consider the anisotropy of the static poroelastic stiffness tensor as given by the Thomsen parameters for fluid-filled, aligned cracks.

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