Constrained inversion of P-S seismic data

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ABSTRACT

A method to estimate S-wave interval velocity, using P-S seismic data is presented. The method is composed of three steps. First, the P-S data are converted to a relative change in S-velocity section. The section is then linearized. Constrained inversion is used to convert this last section to S-interval velocity. To illustrate the method, the S velocity of a field data set was derived, and a Vp/Vs section was computed. The constrained inversion was found to provide reasonable velocities and Vp/Vs.

INTRODUCTION

Accurate measurements of S-wave velocity (β), P-wave velocity (α) and Vp/Vs (α/β) can be used to describe the lithologic properties of rock. Such relationships can be complex, but when used in conjunction with well-log or other independent measurements, they can provide a coherent interpretation of reservoir properties (Tatham and McCormick, 1991). Until recently, seismic exploration has consisted mainly of the interpretation of P-P data, and P velocity inversion has long been part of the interpretation process (Oldenburg et al., 1983; Smith and Gidlow, 1987; Russell and Hampson, 1991). However, since the early 1980's, P-S seismic surveying has developed rapidly, and P-S data is increasingly available (Stewart, 1994). Because more P-S data is being acquired, and because S interval velocity is an important indicator of rock lithology, it is desirable to have practical S inversion methods for P-S data. In this paper, one method will be described which uses constrained linear-inversion to give an S interval velocity estimate. Description begins with an overview of the required input, followed by a chart of the basic inversion flow. The governing equations are described, and an inversion example is presented.

INVERSION OVERVIEW

The S velocity inversion presented here requires four sources of data: a P interval velocity in P-S time, an S interval velocity model in P-S time, an average α/β ratio, and seismic traces gathered into common-conversion points. Output is an S velocity section, where the amplitude of each sample is the S interval velocity at that point.

The P-S reflectivity data should be scaled to represent true reflectivity, and be as broad-band, and noise-free as possible. NMO correction should be applied. The P velocity should be obtained from P velocity inversion, and should be stretched to tie the P-S data. Thus it is converted from a log in P-P time, to a log in P-S time.

The average α/β is used during the inversion process to model angles at conversion points. If the input CCP gathers are asymptotically approximated, the Vp/Vs used in the approximation should be used in the inversion. The inversion flow is summarized in Figure 1.

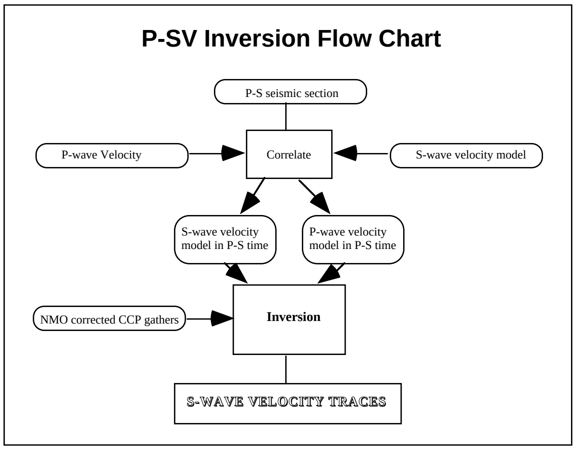

FIG. 1. Flow-chart for P-S inversion.

RELATIVE CHANGE IN S-WAVE VELOCITY

Estimation of S velocity from P-S seismic data requires a relationship between β and P-S reflectivity. A simple relationship, is the ratio of velocity change across an interface to the average velocity across the interface ($\Delta\beta/\beta$), or normalized change in velocity. This value can be estimated using the following equations. At each conversion point:

$$R^{ps} = 4c\frac{\Delta\rho}{\rho} + d\frac{\Delta\beta}{\beta}, \tag{1}$$

where

$$c = -\frac{\alpha\tan\phi}{8\beta}\left(1 - \frac{2\beta^2}{\alpha^2}\sin^2\theta + \frac{2\beta}{\alpha}\cos\theta\cos\phi\right),$$

$$d = \frac{\alpha\tan\phi}{2\beta}\left(\frac{4\beta^2}{\alpha^2}\sin^2\theta - \frac{4\beta}{\alpha}\cos\theta\cos\phi\right),$$

where R_{ps} is the P-S reflection coefficient. The angles θ and ϕ are calculated as averages across the interface. The values $\Delta\alpha/\alpha$, $\Delta\beta/\beta$, and $\Delta\rho/\rho$ are the relative changes in P-wave velocity, S-wave velocity, and density (Aki and Richards, 1980). Using an assumed relationship between ρ and α such as $\rho = \kappa\,\alpha^{0.25}$ (Gardner et al, 1974), the P-S coefficient equation is:

$$R^{ps} = c\frac{\Delta\alpha}{\alpha} + d\frac{\Delta\beta}{\beta}\,. \tag{2}$$

Equation (2) can be cast as a least squares problem and solved for $\Delta\beta/\beta$ (Stewart, 1990). The sum of the squares of the error at a single interface is:

$$\varepsilon = \sum (R - R^{ps})^2\,, \tag{3}$$

where R and R^{ps} are the recorded and modeled P-S reflectivity respectively. Summation is over trace offset. Expansion of (3) gives the result:

$$\begin{aligned}\varepsilon = {} & \sum R^2 - \frac{2\Delta\alpha}{\alpha}\sum Rc - \frac{2\Delta\beta}{\beta}\sum Rd \\ & + \frac{\Delta\alpha^2}{\alpha^2}\sum c^2 + \frac{\Delta\beta^2}{\beta^2}\sum d^2 + \frac{2\Delta\alpha\Delta\beta}{\alpha\beta}\sum cd\end{aligned} \tag{4}$$

Differentiation of (4) with respect to $\Delta\beta/\beta$ will give a value of $\Delta\beta/\beta$ that minimizes the error function:

$$\frac{\partial\varepsilon}{\partial\frac{\Delta\beta}{\beta}} = -2\sum Rd + \frac{2\Delta\beta}{\beta}\sum d^2 + \frac{2\Delta\alpha}{\alpha}\sum cd = 0. \tag{5}$$

Rearranging (5) to solve for $\Delta\beta/\beta$ gives:

$$\frac{\Delta\beta}{\beta} = \frac{\sum_{i=1}^{n\ \mathrm{offsets}} R_i^{ps}\, d_i - \frac{\Delta\alpha}{\alpha}\sum_{i=1}^{n\ \mathrm{offsets}} c_i\, d_i}{\sum_{i=1}^{n\ \mathrm{offsets}} d_i^2}\,. \tag{6}$$

The angles θ and ϕ (required by coefficients 'c' and 'd') are approximated using RMS velocities and Snell's law (Dix, 1955; Zheng and Stewart, 1991; Slawinski and Slawinski, 1994). The RMS velocities and $\Delta\alpha/\alpha$ can both be derived from the P-wave interval velocity model. Having established a relationship between P-S reflection data and the relative change in S velocity (equation 6), constrained linear-inversion can be used to convert $\Delta\beta/\beta$ to a straight-forward velocity.

CONSTRAINED LINEAR INVERSION

Velocity estimates from P-P seismic inversion have often been compromised by a lack of low-frequency information in typical seismic data (Hendrick and Hearn, 1993). That is, velocity estimates exhibit low-frequency instability. In the P-P case, linear inversion can be stabilized by constraining the velocity solution to be close to an initial guess (Russell and Hampson, 1991). The result is that velocity information not resolved by the data is provided by the initial velocity estimate. Similarly, a constrained algorithm can be used to estimate actual velocity from the normalized velocity changes. The first step is to linearize $\Delta\beta/\beta$ from equation (6). This can be done by scaling $\Delta\beta/\beta$ with a macro-velocity model (7b). The change in S velocity ($\Delta\beta$) across an interface can be represented by:

$$\Delta\beta_i = \beta_{i+1} - \beta_i \tag{7A}$$

or, using $\Delta\beta/\beta$ from equation (6),

$$\Delta\beta_i = \left(\frac{\Delta\beta}{\beta}\right)_i \alpha_i \left(\frac{\alpha}{\beta}\right)^{-1}_{mod} \tag{7B}$$

Here, $\Delta\beta/\beta$ has been modified to approximate $\Delta\beta$ using the model P velocity and the model α/β. Equation 7a is the *model response* and the modified form of $\Delta\beta/\beta$ (equation 7b) is the *observed data*.

The model response (7a) is a linear function of β and as such, can be represented by the following first-order Taylor expansion (underlined symbols indicate vectors):

$$\underline{\Delta\beta} = \underline{\Delta\beta}_o + \mathbf{Z}\left(\underline{\beta} - \underline{\beta}_o\right) \tag{8}$$

where $\underline{\Delta\beta}_o$ and $\underline{\beta}_o$ are initial guesses of the change in S velocity across an interface and the S interval velocity respectively, $\underline{\beta}$ is the model interval velocity and Z is an n x n+1 matrix of the partial derivatives of 7a:

$$Z = \begin{bmatrix} -1 & 1 & 0 & \cdots\cdots 0 \\ 0 & -1 & 1 & 0\cdots\cdots 0 \\ & & \vdots & \\ 0\cdots\cdots & 0 & -1 & 1 \end{bmatrix} \tag{9}$$

where a single element of Z is:

$$Z_{ij} = \frac{\partial \Delta\beta_i}{\partial \beta_j}$$

Setting $\underline{\delta} = \underline{\beta} - \underline{\beta}_o$ gives the change in S velocity vector:

$$\underline{\Delta\beta} = \underline{\Delta\beta}_o + \mathbf{Z}\,\underline{\delta} \tag{10}$$

Let error vector $\underline{e}$ represent the difference between (10) and the observed data from equation (7b):

$$\underline{e} = \underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o - \mathbf{Z}\underline{\delta} \tag{11}$$

The sum of the squares of (11) is:

$$\underline{e}^T\underline{e} = \left(\underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o - \mathbf{Z}\underline{\delta}\right)^T \left(\underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o - \mathbf{Z}\underline{\delta}\right) \tag{12}$$

At this point a constraint λ is introduced to the squared error function in the form of a Lagrange multiplier problem (Lines and Treitel, 1984).

$$S = \underline{e}^T\underline{e} + \lambda\left(\underline{\delta}^T\underline{\delta} - \delta_o^2\right) \tag{13}$$

where, S (eq. 13) is minimized subject to the constraint that the absolute value of $\underline{\delta}$ is constant (Lines and Treitel, 1984).

The use of λ has a twofold effect. Singularities in matrix inversion (equation 16) are avoided, and the resulting S velocity estimates are constrained to be close to an initial guess (Lines and Treitel, 1984). Differentiation of S (eq. 13) with respect to $\underline{\delta}$, and setting the result equal to zero, gives a modified form of the least-squared error:

$$\begin{aligned}\frac{\partial S}{\partial \underline{\delta}} = & \frac{\partial}{\partial \underline{\delta}}\left[\underline{\delta}^T\mathbf{Z}^T\mathbf{Z}\underline{\delta} - \left(\underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o\right)^T\mathbf{Z}\underline{\delta}\right] \\ & - \frac{\partial}{\partial \underline{\delta}}\left[\underline{\delta}^T\mathbf{Z}^T\left(\underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o\right)\right] \\ & + \frac{\partial}{\partial \underline{\delta}}\left[\left(\underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o\right)^T\left(\underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o\right)\right] \\ & + \frac{\partial}{\partial \underline{\delta}}\left[\lambda\left(\underline{\delta}^T\underline{\delta} - \delta_o^2\right)\right]\end{aligned} \tag{14}$$

Solving for $\underline{\delta}$:

$$\underline{\delta} = \left(\mathbf{Z}^T\mathbf{Z} + \lambda\mathbf{I}\right)^{-1}\mathbf{Z}^T\left(\underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o\right) \tag{15}$$

Replacing $\underline{\delta}$ with $\underline{\beta} - \underline{\beta}_o$:

$$\underline{\beta} = \left(\mathbf{Z}^T\mathbf{Z} + \lambda\mathbf{I}\right)^{-1}\mathbf{Z}^T\left(\underline{\Delta\beta}_{obs} - \underline{\Delta\beta}_o\right) + \underline{\beta}_o \tag{16}$$

Equation (16) is the constrained linear-inversion equation. It ensures that the input model provides as much low-frequency information to the velocity estimates as the user thinks is required.

P-S INVERSION APPLICATIONS

The P-S inversion method developed above was used to provide an estimate of the S velocity and Vp/Vs for line EKW-002 of the Lousana 3-C data set. This line was acquired as part of a two line, 3-C survey by Unocal (Miller et al, 1994). Available to this study were the vertical and radial component seismic data, and two sonic logs (P sonics only). Also available were horizon interpretations and macro Vp/Vs ratios from the work of Miller et. al 1994. The seismic data sets were processed to preserve true amplitude, and a broad frequency-band.

The first step in the inversion process was to derive an estimate of the P velocity for the entire line. Wells 12-20 and 16-09 (Figure 2) were used as initial P velocity models, and P-P inversion was accomplished using the Promax processing package, using a frequency-domain post-stack inversion (Figure 3). Then, the horizon picks, shown in Figure 4, were used to correlate the P velocity to P-S time (Figure 5); this step employed a sinc function interpolator to resample the stretched P velocity.

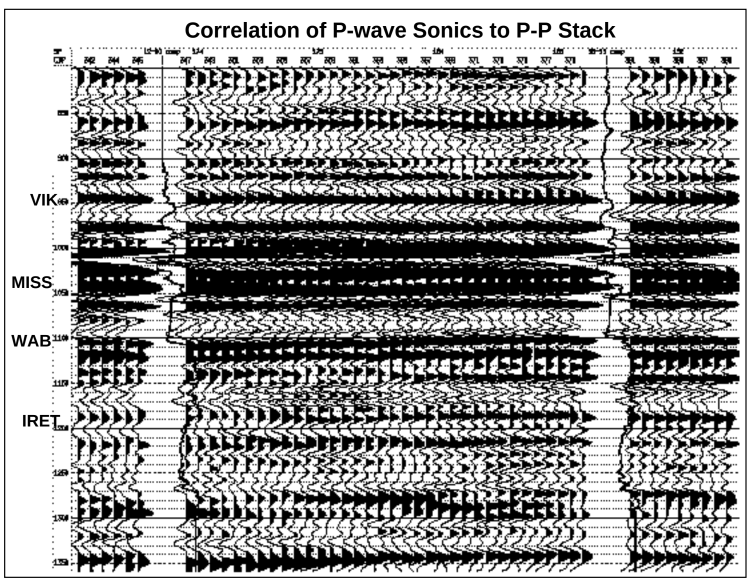

FIG. 2. P sonics correlated to P-P seismic data, well 2-20 is on the left and well 16-09 is on the right. The correlated logs were used as initial velocities for P-P inversion.

Macro Vp/Vs ratios (See Miller et al, 1994, pg. 7-21) were used to scale the P-sonic logs to S-sonic logs; these were used as initial S-velocity input for the P-S inversion (Figure 6). The inversion input used for line 002 is summarized:

1) P-velocity from P-P inversion

2) Correlation of P-wave velocity using horizon picks and sinc function resampling

3) Initial guess S-wave velocity from P-sonic logs scaled with macro Vp/Vs

Figure 7 is the P-S inversion of the data. At this stage, issues regarding the proper scaling of the seismic data, and how tightly the S-velocity solution should have been constrained to the initial velocity, were not uniquely known. The P velocity from P-P inversion, and the S velocity from P-S inversion were then used to estimate a continuos Vp/Vs for the line (Figure 8).

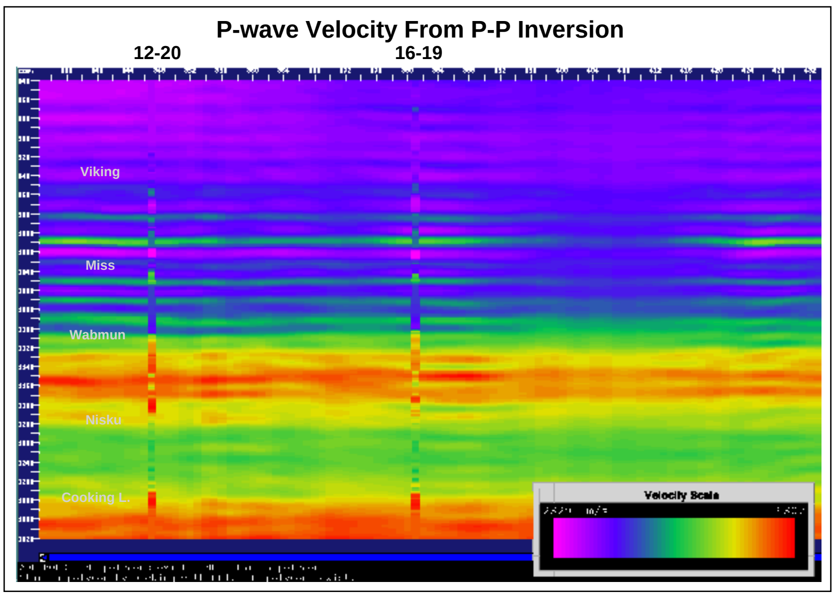

FIG. 3. P interval velocity from post-stack inversion. Initial velocities from P sonics are overlain at 12-20 and 16-18. The time axis is 840 - 1320 ms, the cdp range is 332 - 432.

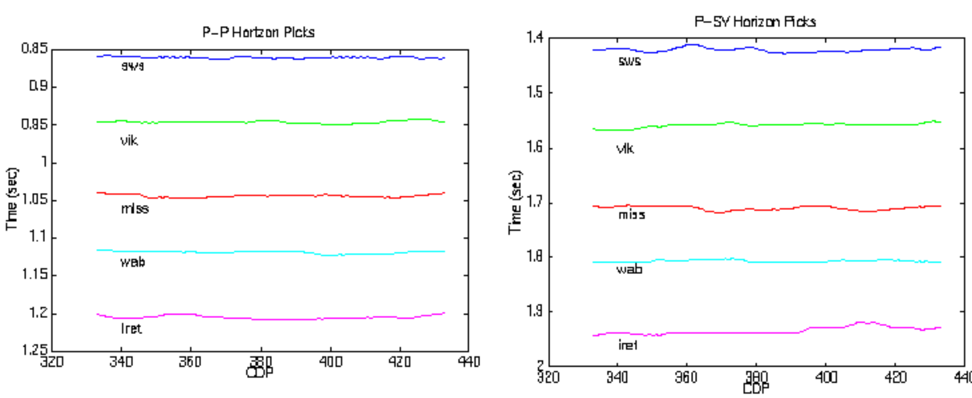

FIG. 4. Horizon picks. Picks were used to correlate the P-velocity section of Figure 3 to P-S time.

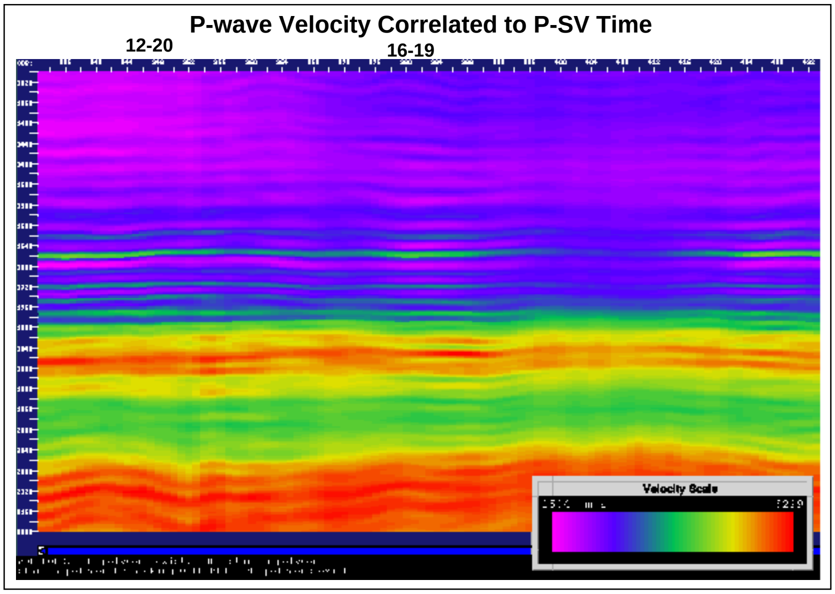

FIG. 5. P velocity from Figure 3 correlated to P-S time using the horizon picks. The time axis is 1300 - 2200 ms, the cdp range is 332 - 432.

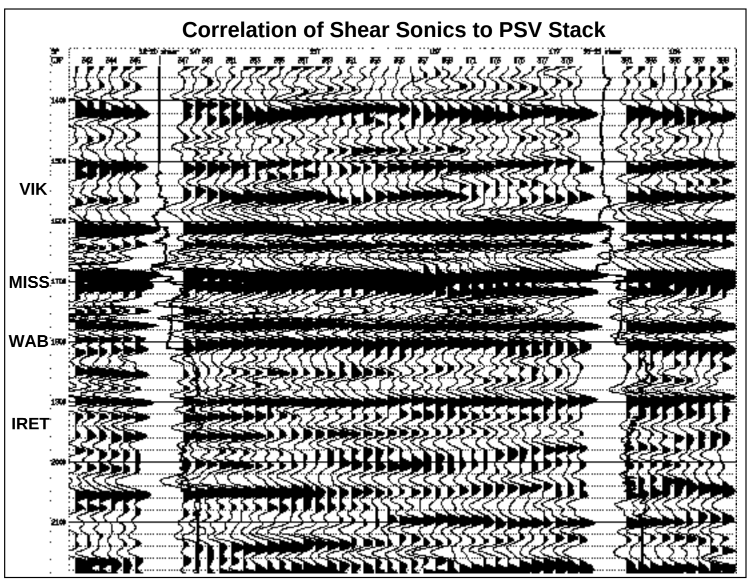

Fig.6. Correlation of S-sonic logs to P-S data. The correlated logs were used as initial S velocities for P-S inversion.

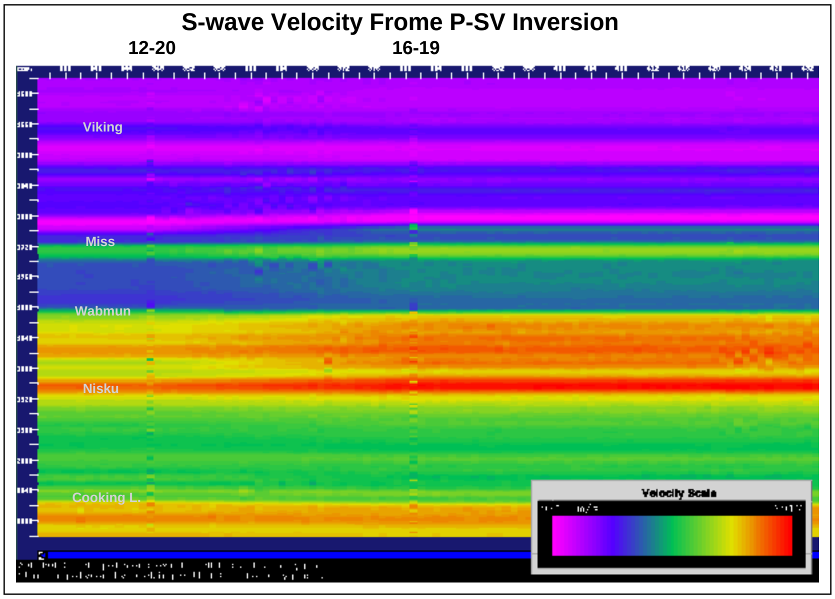

FIG. 7. S velocity from P-S inversion. The S sonics, which were used as initial S-velocity estimates, are indicated at 12-20 and 16-18.

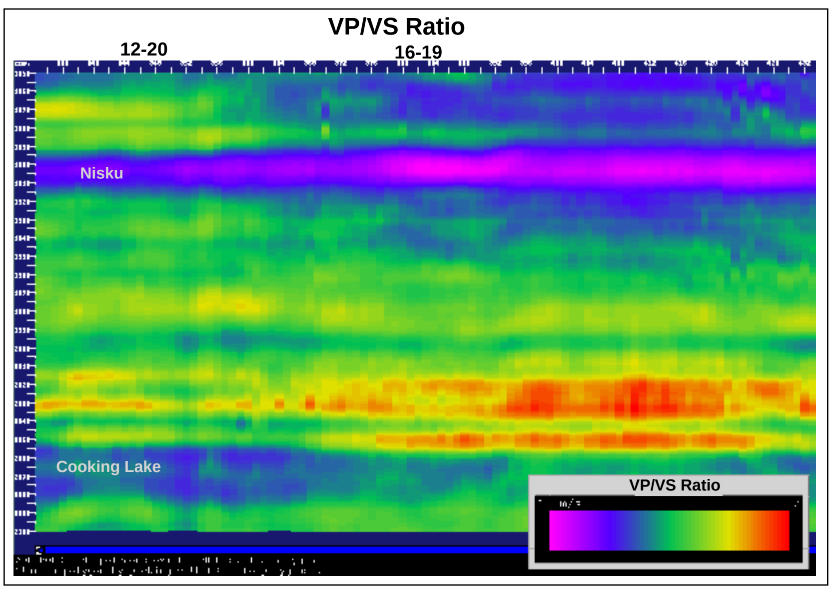

FIG. 8. Vp/Vs derived by dividing the P velocity (Figure 5) by the S (Figure 7). The time range is 1850 to 2100 ms to show greater detail. The Vp/Vs near 1 at the Nisku level is probably due to multiple interference originating in the Manville coals (Miller et al., 1994). An anomalous Vp/Vs increase to the right of 16-19 can be seen. The cdp range is 332 - 432.

DISCUSSION

Both of the P and S velocity estimates (Figure 3 and 7 respectively) show a fairly consistent lateral velocity, with no sudden velocity changes. The S velocity estimate may be somewhat over-constrained to the initial guess velocity. This means that the ratio of reflection data to background velocity is to highly biased to the back ground velocity. The effect can be seen in the smooth nature of the S velocity estimate (Figure 7), compared with the P estimate (Figure 3). The Vp/Vs section of Figure 8 is included as an example of the continuous Vp/Vs display, that is possible with both P and P-S inversion estimates. Examination of Figure 8 shows the effects of multiple contamination at the Nisku level (Miller et al., 1994), the Vp/Vs is near 1, and a Vp/Vs decrease corresponding with the zone just above the Cooking Lake level. The multiple interference at the Nisku level probably originates in the Manville coals (Miller et al., 1994), and the Vp/Vs increase to the right of 16-19 is as yet unexplained.

CONCLUSION

A method is presented which can be used to estimate S interval velocity from P-S seismic data. The method is composed of three steps: P-S data is converted to a relative change in velocity section, this section is scaled by a macro-velocity model to give a simple-change in velocity section, this last section is converted to S interval velocity using constrained linear-inversion. A field data set was inverted to derive S interval velocity and a Vp/Vs section. Contamination by multiples resulted in a Vp/Vs estimate of 1 at the Nisku level, and an anomalous increasing Vp/Vs was found above the Cooking lake level. These results are tempered by the non-unique nature, with regards to scaling and initial velocity constraint, of the inversion solution. More work must be done to determine if the results are valid, and how to set parameter boundaries accordingly.

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